«»«»«»«»«» 57:022 Principles of Design II «»«»«»«»«»

Homework Solutions, Spring 1992

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«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #1 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. The foreman of a casting section in a certain factory finds that on the average, 1 in every 5 castings made is defective. ***N_8 = # defects among 8 castings. N_8 has the Binomial Distribution.***

a. If the section makes 8 castings a day, what is the probability that 2 of these will be defective

$$P\{N_8 = 2\} = \binom{8}{2}(0.2)^2(0.8)^6$$

b. What is the probability that 5 or more defective castings are made in one day?

$$P\{N_8 \geq 5\} = \sum_{i=5}^{8} P\{N_8 = i\} = 0.0104$$

2. A certain production process has a fraction defective of 8%. Three good pieces are required. Pieces are produced until the 3 good pieces are obtained. ***Use Pascal Distribution, with p=0.92.***

a. What are the expected value and standard deviation of the number of pieces produced?

$$\mu = \frac{k}{p} = \frac{3}{0.92} = 3.261$$

$$\sigma = \left[\frac{k(1-p)}{p^2}\right]^{0.5} = 0.532$$

b. Compute the probabilities of producing no more than 5 pieces.

$$P\{T_3 \leq 5\} = P\{T_3 = 3\} + P\{T_3 = 4\} + P\{T_3 = 5\}$$
$$= \binom{2}{2}(0.92)^3(0.08)^0 + \binom{3}{2}(0.92)^3(0.08)^{4-3} + \binom{4}{2}(0.92)^3(0.08)^{5-3}$$
$$= 0.9955$$

c. Suppose that instead of producing until 3 good pieces are obtained, a batch of size **n** is produced, and then the pieces are inspected. How large should **n** be in order to be 95% certain of obtaining at least 3 good pieces?
Choose smallest n such that $P\{T3 \leq n\} \geq 0.95$. From (b), when n=3, $P\{T_3 \leq 3\}=0.7787$; when n=4, $P\{T_3 \leq 4\}=0.7787+ 0.1869=0.9656 > 0.95$. Therefore, n=4.

3. A light bulb in an apartment entrance fails randomly, with an expected lifetime of 15 days, and is replaced immediately by the custodian. Assume that this lifetime has an *exponential* distribution.

a. What is the probability that a bulb lasts longer than its expected lifetime?

$$P\{T>15\}=1-F(15)$$
$$= 1-\left(1 - e^{-(1/15)(15)}\right)$$
$$= 0.3679$$

b. If the current bulb was inserted 10 days ago, what is the probability that its lifetime (since it was inserted) will exceed the expected lifetime of 15 days?

$P\{T>5\}=1-F(5)$
$= 1-\left(1 - e^{-(1/15)(5)}\right)$
$= 0.7165$

c. If you were to test 5 of these bulbs, what is the probability that more than half will exceed the expected lifetime? ***Use Binomial Distribution, with p=0.3679 from (a).***

$P\{N_5 \geq 2.5\} = P\{N_5 =3\}+P\{N_5 =4\}+P\{N_5 =5\}$
$= 0.2636$

d. If the custodian has 2 spare bulbs, what is the probability that these will be sufficient for the next 40 days? ***What is the probability that there are ≤ 3 failures in 40 days? (Use Poisson Distribution!)***
$P\{N_{40} \leq 3\} = 0.7213$

4. A city's population is 60% in favor of a school bond issue. What is the probability that, if a random sample of ten citizens are polled, the majority of those polled will oppose the issue? ***Use Binomial distribution with n=10, p=0.6. P{N10≤5} = 0.1662***

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #2 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. Trucks arrive at an unloading dock of a warehouse according to a Poisson process, at the rate of 4/hour.
   a. What is the probability distribution of the arrival time of the first truck? ***EXPONENTIAL***
      ...the arrival time of the 10th truck? ***ERLANG***
      ...the time between the arrivals of the first and second truck? ***EXPONENTIAL***
   b. According to the Central Limit Theorem, the arrival time of the 10th truck has approximately what distribution? (What is its mean and standard deviation?) ***NORMAL, with mean 10/4 and variance 10/4² (obtained from the fact that the actual arrival time has the Erlang distribution).***
   c. What is the probability distribution function of the maximum times between arrivals of the first 10 trucks? ***GUMBEL***
   d. Five minutes are required to unload a truck. What is the probability that one of the first ten trucks will arrive within less than five minutes of the preceeding truck? (In the case of the first truck, within five minutes of the beginning of the day.) ***Compute the probability that at least one of the first ten trucks will arrive within less than 5 minutes of the preceding truck.***

$1-P\{\tau_i \geq 5 \quad i=1,\ldots 10\}$
$= 1-P\{\tau_1 \geq 5 \text{ and } \tau_2 \geq 5 \text{ and } \ldots \tau_{10} \geq 5\}$
$= 1-\left[P\{\tau_1 \geq 5\}\times P\{\tau_2 \geq 5\}\times\cdots P\{\tau_{10} \geq 5\}\right]$
$= 1-\left[e^{-5\lambda}\right]^{10} = 0.9643$

We wish to simulate this Poisson process by generating the arrival times of the trucks.

   e. Generate 3 uniformly-distributed 4-digit numbers, using the Midsquare Method, with the last 4 digits of your ID# as the "seed".
   f. Use the 3 numbers which you obtained in (e) to compute the arrival times of the first 3 trucks by the inverse transformation technique.
   g. Why cannot the Rejection Technique be used in (f)? ***Because the exponential distribution density function has a tail which extends indefinitely to the right, and also because the density function is unbounded at t=0.***

2. An electronic device consists of many components, each subject to failure. The failure time of the device is, therefore, the minimum of the failure times of the individual components (all nonnegative random variables). We will therefore assume that the failure time of the device has approximately a Weibull distribution. Suppose that data has been collected on the failure times of a large number of these devices, and it is found that the average failure time is 200 days, with a standard deviation of 50 days.

a. What is the coefficient of variation of the failure time?

$$\frac{\sigma_Y}{\mu_Y} = \frac{50}{200} = 0.25$$

b. Using the plot or the table in the Hypercard stack on Extreme Value Distributions, estimate the parameter k of the Weibull distribution of failure time. ***k is approximately 4.54.***

c. Compute the parameter u of the Weibull distribution of failure time.

$$u = \frac{\mu_Y}{\Gamma\left(1+\frac{1}{k}\right)} = \frac{200}{0.22!} = 219$$

0.22! = 0.913 may be evaluated by APL, or you may find G(1.22) from a table in various books of mathematical tables.

d. What is the probability that the next device selected from the shelf will fail before its 90-day warranty expires?

$$P\{Y<90\} = F(90) = 1 - e^{-\left(90/219\right)^{4.54}} = 0.0175$$

... that it will still be operating one year from now?

P{Y>365} = 1-F(365) = 0.0000384

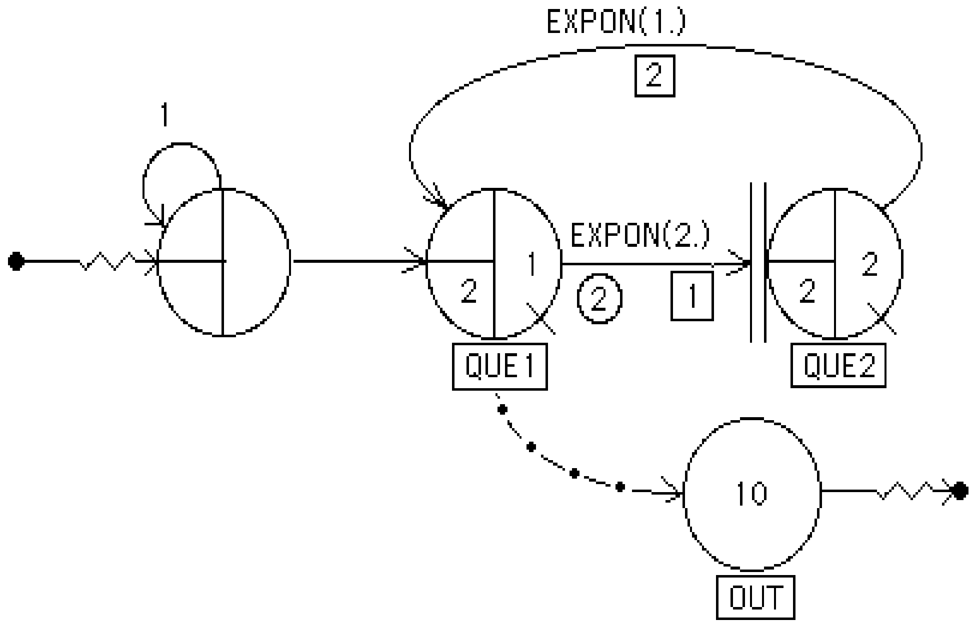
e. If your company buys 10 of these devices, what is the probability that no more than two of them fail within 90 days? ***Use Binomial Distribution, with n=10, p=0.0175. P{N10≤2} = 0.9994.***

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #3 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #4 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. The SLAM statements for the network model below are:

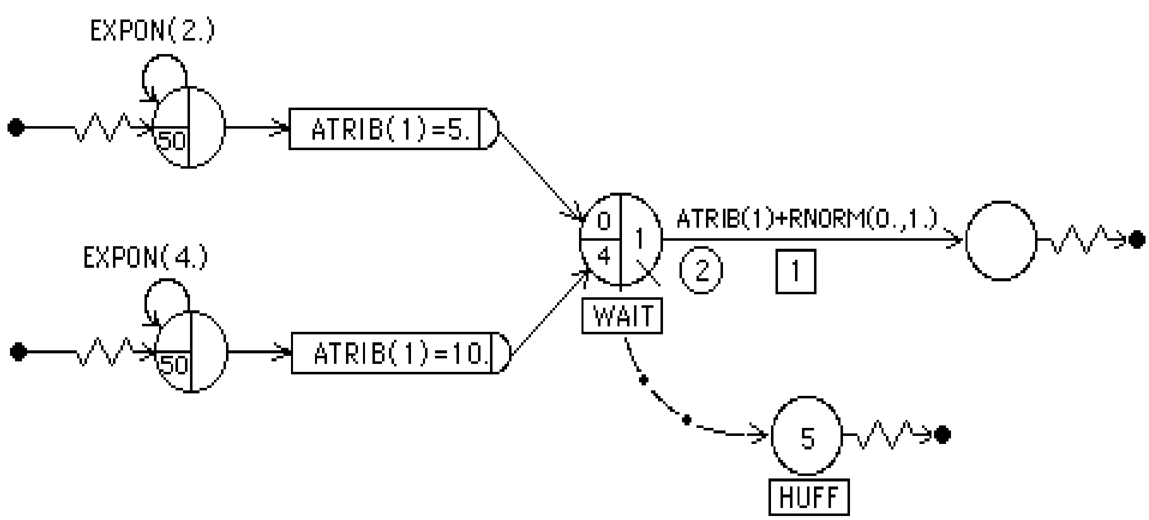
```
NETWORK;
QUE1    QUEUE(1),,2,BALK(OUT);
        ACTIVITY(2)/1,EXPON(2.);
QUE2    QUEUE(2),,2,BLOCK;
        ACTIVITY/2,EXPON(1.),,QUE1;
OUT     TERMINATE;
        END;
```



2.

```
        NETWORK;
                CREATE,EXPON(2.),,,50;
                ASSIGN,ATRIB(1)=5.0;
                    ACTIVITY,,,WAIT;
                CREATE,EXPON(4.),,,50;
                ASSIGN,ATRIB(1)=10.0;
        WAIT    QUEUE(1),,4,BALK(HUFF);
                ACTIVITY(2)/1,ATRIB(1)+RNORM(0.,1.);
                TERM;
        HUFF    TERM,5;
                END;
```

The network model which corresponds to the SLAM statements above is:



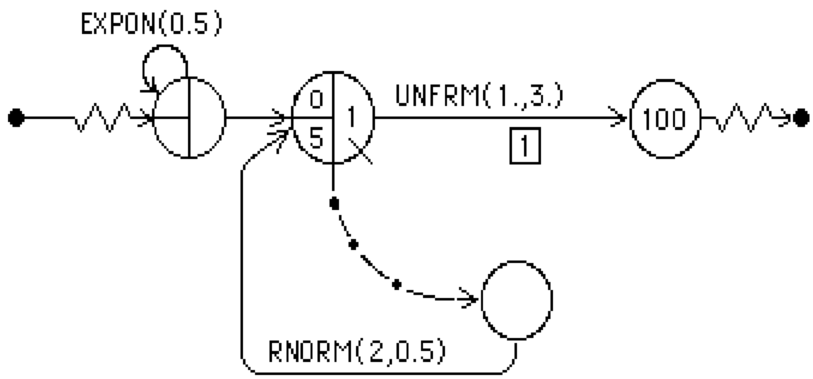
The system has the following features:

- Fifty each of two types of entities (p. 127) are created and enter the queue named WAIT

- WAIT queue has a capacity of 4. If an entity arrives when there are already 4 entities in the queue, it balks to the terminate node HUFF.
- entities are removed from the queue and processed by one of 2 parallel servers
- When 5 entities have balked to HUFF (or all 100 entities have passed through the system), the simulation is ended.

3. At a drive-in bank, with only one teller, there is space for 5 waiting cars. Customers arrive according to a Poisson process at the average rate of 1 every 2 minutes. The time spent by a customer at the teller window is uniformly distributed between 1 minute and 3 minutes. If a customer arrives when the waiting line is full, the customer drives around the block (which requires an amount of time having normal distribution with average 2 minutes and standard deviation 30 seconds) and tries to join the waiting line again. Initially, no customers are waiting, and the teller is idle. The simulation is to be terminated when 100 cars have been served.

The SLAM network model for this system is as follows:

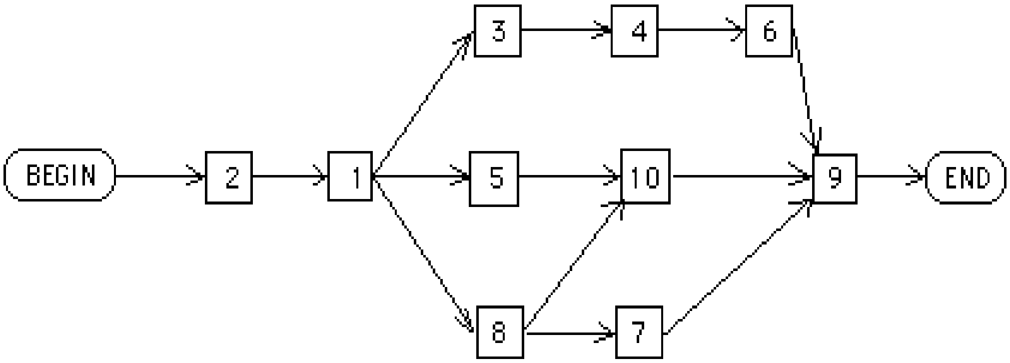


«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #5 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

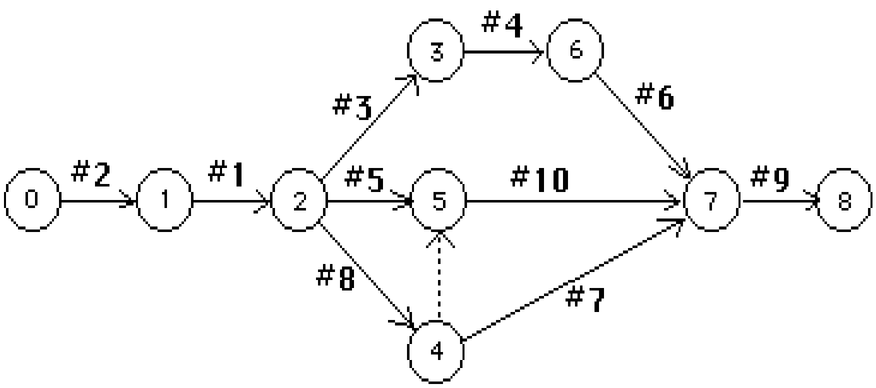
Project Scheduling: Hawkeye Construction Co. has prepared the following table listing the tasks required to complete construction of a house:

Task #	Task Description	Immediate Predecessor(s)	Most likely time (days)	Optimistic time (days)	Pessimistic time (days)
1	Walls & ceilings	2	5	3	7
2	Foundation	none	2	1	4
3	Roof timbers	1	2	1	2.5
4	Roof sheathing	3	3	2.5	3.5
5	Electrical wiring	1	4	2.5	6
6	Roof shingles	4	8	5	10
7	Exterior siding	8	5	2.5	8
8	Windows	1	2	1	3
9	Paint	6,7,10	2	1	5
10	Inside wallboard	8,5	3	2	4

a. A-O-N (activity on node) network representing this project:



b. A-O-A (activity on arrow) network representing this project:

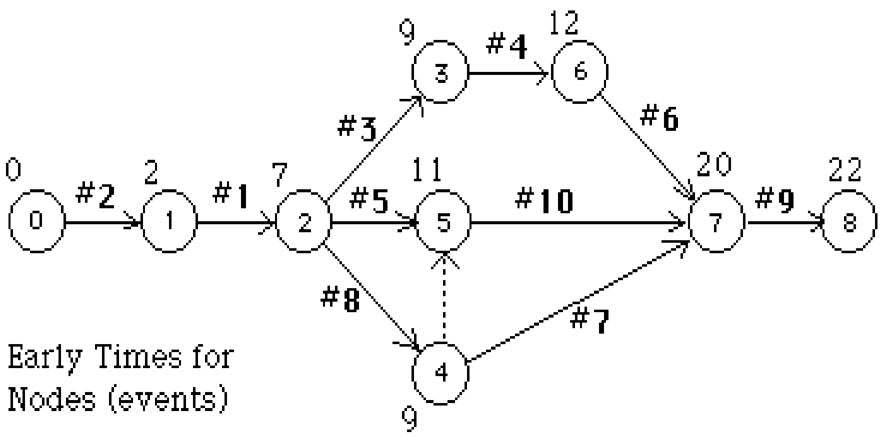


Note that one dummy activity is required!

c. The nodes are numbered above so that if there is an arrow from node i to node j, then i<j. (Other sets of node numbers would also satisfy this property: for example, the node numbers on nodes 3 & 4 could have been interchanged.)

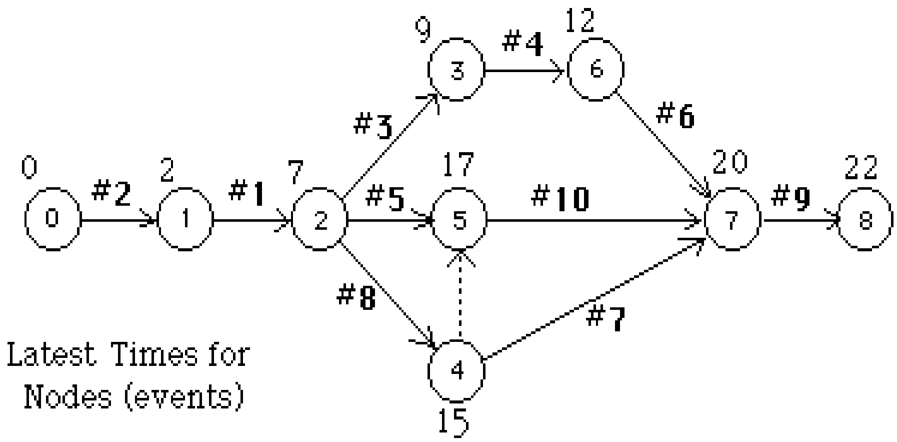
In parts (c)-(g), it is assumed that the most likely completion times will be the actual completion times:

d. The early times for each node:



e. The earliest completion time for the house os 22 days.

f. The latest times for each node in order to complete the house as early as possible:



g.

Task	i	j	ES	EF	LS	LF	TF
1	1	2	2	7	2	7	0
2	0	1	0	2	0	2	0
3	2	3	7	9	7	9	0
4	3	6	9	12	9	12	0
5	2	5	7	11	13	17	6
6	6	7	12	20	12	20	0
7	4	7	9	14	15	20	6
8	2	4	7	9	13	15	6
9	7	8	20	22	20	22	0
10	5	7	11	14	17	20	6

h. The tasks # 2,1,3,4,6, & 9 form a critical path.

i. According to the assumptions of PERT (i.e., the Central Limit Theorem), the probability distribution of the completion time of the house has the Normal distribution.

j. The mean and standard deviation of the activities on the critical path, assuming that each has the beta distribution. are found by

$$\mu = \frac{a + 4m + b}{6}, \quad \sigma = \frac{b-a}{6}$$

and are as follows:

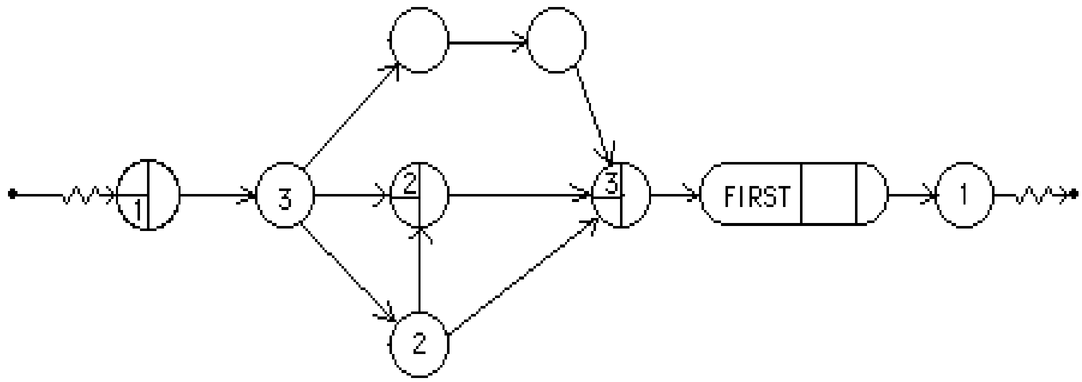
Critical task	Mean	Std. deviation
1	5.0	0.667
2	2.167	0.5
3	1.9167	0.25
4	3.0	0.167
6	7.833	0.833
9	2.333	0.667

Sum of means= 22.25; sum of variances (squares of std. deviations)= 1.9235.

k. The length of the critical path, under the assumptions of PERT, has the Normal distribution with mean and standard deviation 22.25 and 1.387 (square root of 1.9235), respectively.

l. Suppose that the company contracts to build the house in a length of time equal to the expected completion time plus 5 days, i.e., 27.25 days. Based on the assumption that the project completion time is N(22.25,1.378), the probability that the house can be completed within this time (the expected value plus 3.42 standard deviations) is 99.97%.

m. TheSLAM network corresponding to the A-O-A network in (b):



The numbers within the GOON nodes might be omitted, since the default is infinity. It will be assumed that activity durations have the triangular distribution (rather than beta), for simplicity.

n. ***SLAM code:***

```
GEN,BRICKER,SCHEDULE,3/30/92,500,,NO,,NO,YES/500,72;
LIM,,1,100;
INIT,,,NO;
NETWORK;
N0   CREATE;
     ACT,TRIAG(1,2,4);        ACTIVITY 2
N1   GOON;
     ACT,TRIAG(3,5,7);        ACTIVITY 1
N2   GOON,3;
     ACT,TRIAG(1,2,2.5),,N3;  ACTIVITY 3
     ACT,TRIAG(2.5,4,6),,N5;  ACTIVITY 5
     ACT,TRIAG(1,2,3),,N4;    ACTIVITY 8
N3   GOON;
     ACT,TRIAG(2.5,3,3.5),,N6; ACTIVITY 4
N4   GOON,2;
     ACT,,,N5;                DUMMY ACTIVITY
     ACT,TRIAG(2.5,5,8),,N7;  ACTIVITY 7
N5   ACCUM,2;
     ACT,TRIAG(2,3,4),,N7;    ACTIVITY 10
N6   GOON;
     ACT,TRIAG(5,8,10),,N7;   ACTIVITY 6
N7   ACCUM,3;
     ACT,TRIAG(1,2,5);        ACTIVITY 9
N8   COLCT,FIRST,FINISHED,15/19/0.5;
     TERM,1;
     END;
FIN;
```

SLAM output:

```
        S L A M   I I   S U M M A R Y   R E P O R T


 SIMULATION PROJECT SCHEDULE              BY BRICKER

 DATE  3/30/1992                   RUN NUMBER  500 OF  500


 CURRENT TIME   0.2397E+02
 STATISTICAL ARRAYS CLEARED AT TIME  0.0000E+00
```

```
          **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

               MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
               VALUE   DEVIATION VARIATION   VALUE     VALUE    OBS

 FINISHED      0.225E+02 0.171E+01 0.759E-01 0.171E+02 0.271E+02   500

                   **HISTOGRAM NUMBER 1**
                        FINISHED

 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0       20       40       60       80      100
                    +    +    +    +    +    +    +    +    +    +    +
   7 0.014 0.190E+02 +*                                              +
  12 0.024 0.195E+02 +*C                                             +
  18 0.036 0.200E+02 +** C                                           +
  22 0.044 0.205E+02 +**   C                                         +
  39 0.078 0.210E+02 +****     C                                     +
  42 0.084 0.215E+02 +****        C                                  +
  51 0.102 0.220E+02 +*****           C                              +
  52 0.104 0.225E+02 +*****               C                          +
  62 0.124 0.230E+02 +******                    C                    +
  48 0.096 0.235E+02 +*****                       C                  +
  42 0.084 0.240E+02 +****                           C               +
  39 0.078 0.245E+02 +****                              C            +
  27 0.054 0.250E+02 +***                                C   +
  15 0.030 0.255E+02 +**                                  C +
  16 0.032 0.260E+02 +**                                   C+
   6 0.012 0.265E+02 +*                                     C
   2 0.004    INF    +                                   C
 ---                +    +    +    +    +    +    +    +    +    +    +
 500                 0        20       40       60       80      100


          **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

               MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
               VALUE   DEVIATION VARIATION   VALUE     VALUE    OBS

 FINISHED      0.225E+02 0.171E+01 0.759E-01 0.171E+02 0.271E+02   500
```

o. Unfortunately, I didn't specify a sufficient number of cells of the histogram above to answer the question. Based on the histogram, the probability that the completion time exceeds 26.5 days is 1 - 2/500, i.e., 99.6%, and we see that the maximum value for the 500 simulation runs was 27.1 days, so that probably only 1 run had a completion time greater than 27 days, in which case the required estimate would be 1- 1/500, i.e., 99.8%.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #6 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

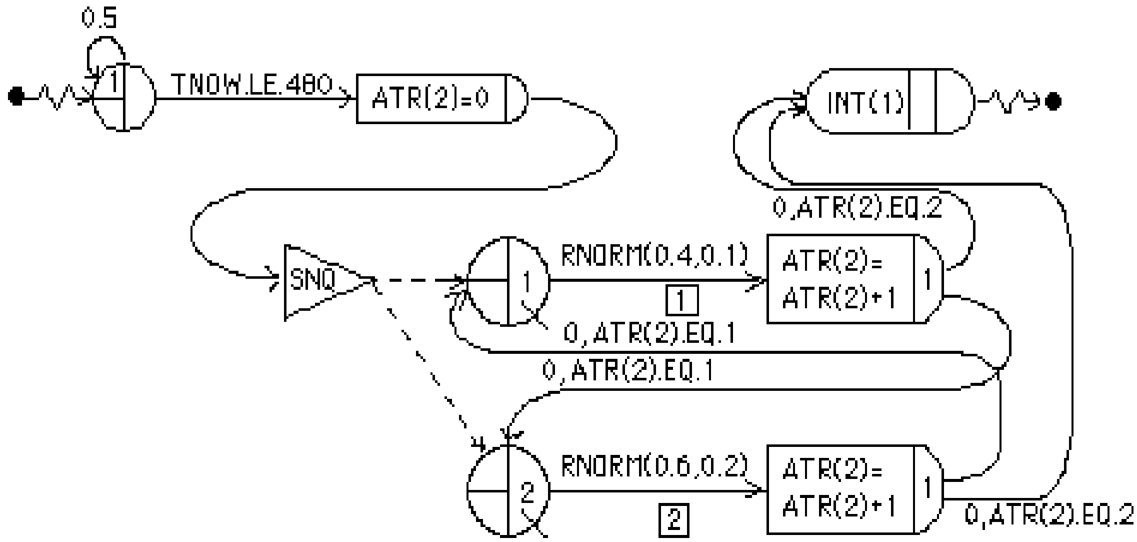
System Descriptions: a manufacturing department which processes items at 2 stations, or machines:

- Items may visit the stations in any order, but must visit each station exactly once before it leaves the department.
- Each station processes only one item at a time, and has unlimited space for items awaiting service.
- When items arrive (at a constant rate of 2/minute), they are directed to the station with the smallest number of waiting items.

- At each station, items which have already visited the other station have priority for service.
- Processing time at station #1 is normally distributed with mean 0.4 minute and standard deviation 0.1 minute, while processing time at station #2 is normally distributed with mean 0.6 minute and standard deviation 0.2 minute.
- The arrival process stops after 8 hours, but the department continues working until all items currently in the department have been processed.

Statistics are desired on the number of items waiting at each station as well as the time spent by items in the department.

SOLUTION: SLAM network: In this network model, entities represent jobs, and have 2 attributes: (1) time of creation, and (2) number of machines visited.



You must also include the SLAM control statements

PRIORITY/1,HVF(2);
PRIORITY/2,HVF(2);

in order that jobs which have already been processed on one machine have priority on the other machine.

Using the given time between arrivals, the queue for machine #2 will grow indefinitely, because the average service time (0.6 minutes) exceeds the average time between arrivals (0.5 minutes). The SLAM input & output below have the time between arrivals increased to 0.75 minutes (i.e., 45 sec.) so that the system approaches a "steady state". Note that the simulation is run for 600 minutes (10 hrs) to allow all jobs to be processed through the machines after arrivals stop at 8 hrs.

```
GEN,BRICKER,EXAMPLE,3/25/92,1,Y,Y,Y/N,Y,Y,72;
LIMITS,2,2,100;
INITIALIZE,0,600;
PRIORITY/1,HVF(2);
PRIORITY/2,HVF(2);
NETWORK;
        CREATE,0.75,,1;          ARRIVAL RATE 1/45 SEC.
        ACT,0,TNOW.LE.480;
        ASSIGN,ATR(2)=0;
        SELECT,SNQ,,,Q1,Q2;
Q1      QUEUE(1);              QUEUE OF JOBS FOR MACHINE 1
        ACTIVITY/1,RNORM(0.4,0.1);
        ASSIGN,ATR(2)=ATR(2)+1,1;
        ACTIVITY,0,ATR(2).EQ.1,Q2;
        ACTIVITY,0,ATR(2).EQ.2,DONE;
Q2      QUEUE(2);              QUEUE OF JOBS FOR MACHINE 2
```

```
          ACTIVITY/2,RNORM(0.6,0.2);
          ASSIGN,ATR(2)=ATR(2)+1,1;
          ACTIVITY,0,ATR(2).EQ.1,Q1;
          ACTIVITY,0,ATR(2).EQ.2,DONE;
DONE      COLCT,INT(1),TIME IN SYSTEM,15/.5/.2;
          TERM;
          END;
FIN;
```

OUTPUT:

```
     **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                MEAN     STANDARD  COEFF. OF  MINIMUM    MAXIMUM  NO.OF
                VALUE    DEVIATION VARIATION   VALUE      VALUE    OBS

TIME IN SYSTEM  0.141E+01 0.356E+00 0.252E+00 0.582E+00 0.254E+01   641



     **FILE STATISTICS**

FILE                AVERAGE  STANDARD   MAXIMUM  CURRENT AVERAGE
NUMBER  LABEL/TYPE   LENGTH   DEVIATION  LENGTH   LENGTH  WAIT TIME

   1    Q1   QUEUE     0.143     0.365      2        0      0.134
   2    Q2   QUEUE     0.312     0.520      2        0      0.292
   3       CALENDAR    2.055     0.670      4        1      0.256



     **SERVICE ACTIVITY STATISTICS**

ACT ACT LABEL OR  SER AVERAGE    STD  CUR AVERAGE MAX IDL MAX BSY  ENT
NUM START NODE    CAP   UTIL     DEV  UTIL BLOCK  TME/SER TME/SER  CNT

  1 Q1   QUEUE     1  0.428  0.49   0   0.00  119.51    3.31  641
  2 Q2   QUEUE     1  0.627  0.48   0   0.00  118.61    8.20  641


               **HISTOGRAM NUMBER 1**
                  TIME IN SYSTEM

OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0       20       40       60       80       100
                   +    +    +    +    +    +    +    +    +    +    +
  0 0.000 0.500E+00 +                                        +
  6 0.009 0.700E+00 +                                        +
 27 0.042 0.900E+00 +**C                                        +
111 0.173 0.110E+01 +********* C                                  +
110 0.172 0.130E+01 +*********         C                          +
137 0.214 0.150E+01 +***********              C                    +
114 0.178 0.170E+01 +*********                    C           +
 72 0.112 0.190E+01 +******                         C    +
 43 0.067 0.210E+01 +***                             C +
 16 0.025 0.230E+01 +*                                C
  4 0.006 0.250E+01 +                                C
  1 0.002 0.270E+01 +                                C
  0 0.000 0.290E+01 +                                C
  0 0.000 0.310E+01 +                                C
```

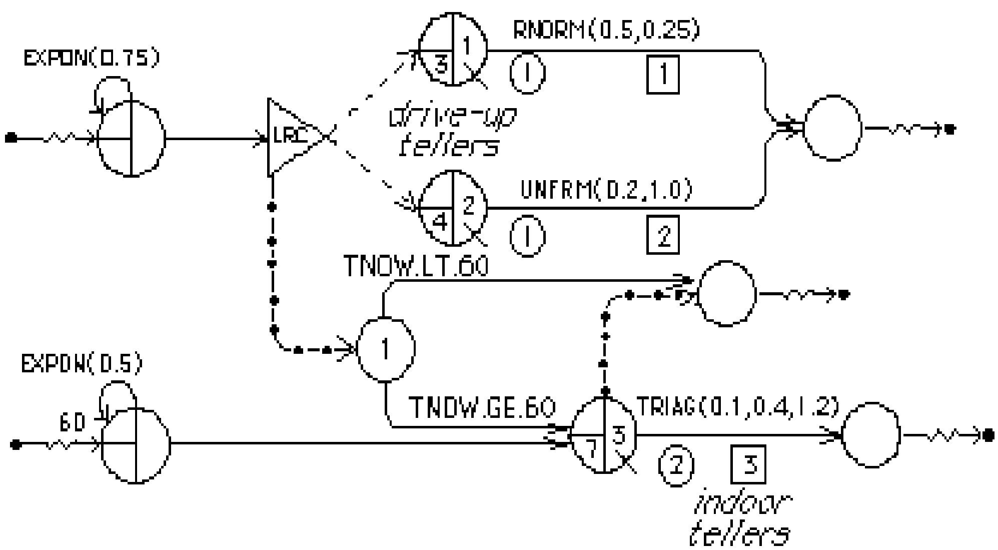
```
  0 0.000 0.330E+01 +                                    C
  0 0.000 0.350E+01 +                                    C
  0 0.000     INF    +                                 C
---              +    +    +    +    +    +    +    +    +    +    +
641                0        20        40        60        80       100
```

STATISTICS FOR VARIABLES BASED ON OBSERVATION

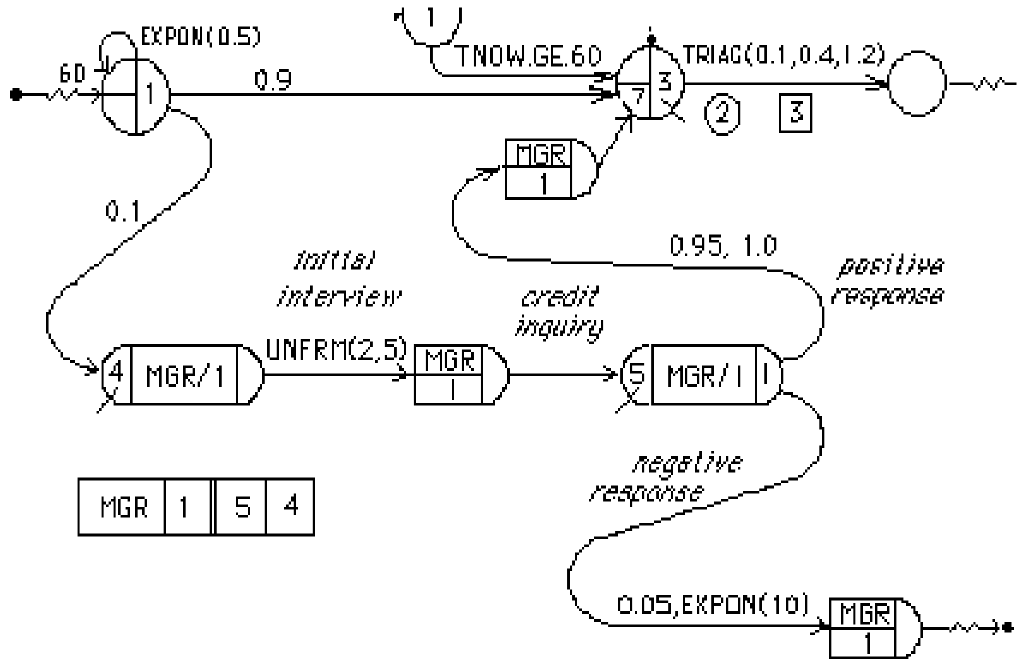
	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
TIME IN SYSTEM	0.141E+01	0.356E+00	0.252E+00	0.582E+00	0.254E+01	641

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #7 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

(Exercises 7-11 & 7-12 from Chapter 7 of SLAM book by Pritsker.)



To model the credit inquiries, we add the following to the SLAM network:



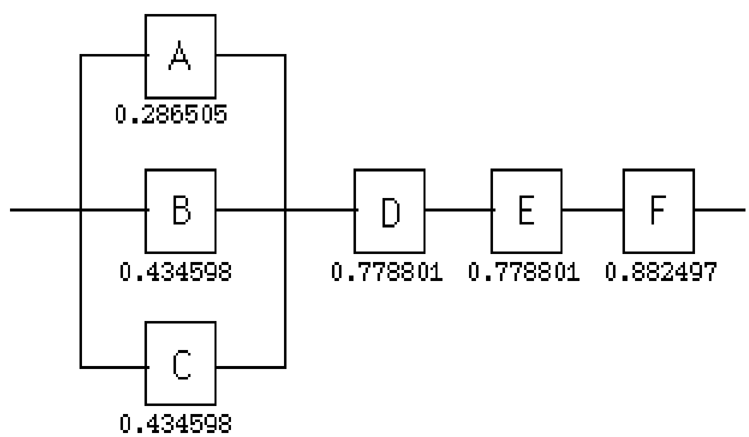
The problem statement in the book was rather vague. The above network assumes that:

- the customers requiring the credit inquiry do not first visit the teller
- after the credit inquiry is complete, then whether the result is positive or negative the customer must return to the manager for the result.
- if the result is negative, the manager spends an amount of time having exponential distribution with mean 10; if positive, the manager spends one minute with the customer.

Note that the credit inquiries could also be modeled using a queue node and server, but at the expense of additional complexity, using attributes and conditional branching.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #8 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

(1.) & (2.) A, B, and C are in parallel, followed by D, E, and F in series:



The reliabilities are shown above, and are found by first evaluating $F_i(500)$ for each component, i=A, B, ... For example,

$$F_A(t) = 1 - e^{-\lambda_A t}, \text{ where } \lambda_A = \frac{1}{400\text{days}}$$

and so

$$F_A(500) = 1 - e^{-1.2}$$

Then $R_i(500) = 1 - F_i(500)$.

(3.) The reliability of the subsystem composed of components A, B, and C is

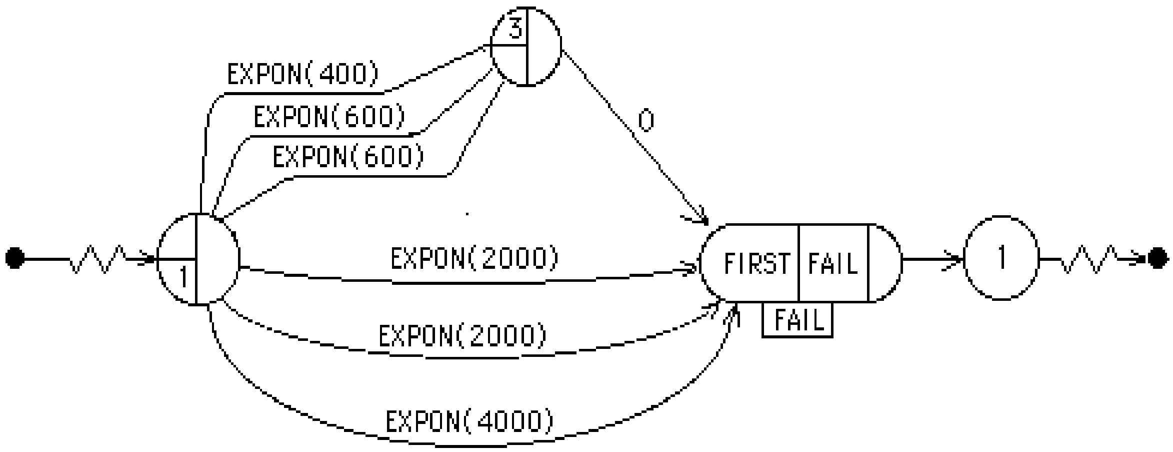
$$R_{ABC}(500) = 1- (1-R_A(500)(1-R_B(500)(1-R_C(500)$$
$$= 0.77191$$

The system reliability is then $R_{sys}(500)=R_{ABC}(500)R_D(500)R_E(500)R_F(500)$

$$= 0.413174$$

i.e., 41.3% probability that the system survives 500 days.

(4.)



5. The SLAM code to simulate this network model is:

```
GEN,BRICKER,HW8,4/15/92,500,,NO,,NO,YES/500,72;
LIM,,1,10;
INIT,,,NO;
NETWORK;
    CREATE;
    ACTIVITY/1,EXPON(400),,SUB1;
    ACTIVITY/2,EXPON(600),,SUB1;
    ACTIVITY/3,EXPON(600),,SUB1;
    ACTIVITY/4,EXPON(2000),,FAIL;
    ACTIVITY/5,EXPON(2000),,FAIL;
    ACTIVITY/6,EXPON(4000),,FAIL;
SUB1  ACCUM,3;
FAIL  COLCT,FIRST,FAIL TIME,20/50/75;
    TERM,1;
    END;
FIN;
```

The SLAM output is as follows:

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
FAIL TIME	0.489E+03	0.381E+03	0.780E+00	0.209E+00	0.217E+04	500

REGULAR ACTIVITY STATISTICS

ACTIVITY INDEX/LABEL	AVERAGE UTILIZATION	STANDARD DEVIATION	MAXIMUM UTIL	CURRENT UTIL	ENTITY COUNT
1	0.3567	0.4790	1	1	0

2	0.4002	0.4899	1	1	0
3	0.4031	0.4905	1	1	0
4	0.1974	0.3980	1	1	0
5	0.1993	0.3995	1	1	0
6	0.1090	0.3117	1	0	1

```
                  **HISTOGRAM NUMBER 1**
                       FAIL TIME

 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0       20       40       60       80       100
                +   +   +   +   +   +   +   +   +   +   +
  32 0.064 0.500E+02 +***                                     +
  34 0.068 0.125E+03 +***   C                                  +
  65 0.130 0.200E+03 +*******     C                              +
  44 0.088 0.275E+03 +****          C                          +
  41 0.082 0.350E+03 +****             C                       +
  44 0.088 0.425E+03 +****                C                    +
  49 0.098 0.500E+03 +*****                   C                +
  30 0.060 0.575E+03 +***                       C             +
  30 0.060 0.650E+03 +***                         C           +
  20 0.040 0.725E+03 +**                           C         +
  19 0.038 0.800E+03 +**                             C       +
  12 0.024 0.875E+03 +*                              C      +
  15 0.030 0.950E+03 +**                              C      +
  14 0.028 0.103E+04 +*                                C    +
   7 0.014 0.110E+04 +*                                C   +
   8 0.016 0.118E+04 +*                                C   +
   7 0.014 0.125E+04 +*                                 C  +
   9 0.018 0.133E+04 +*                                  C +
   5 0.010 0.140E+04 +*                                  C +
   4 0.008 0.148E+04 +                                   C+
   3 0.006 0.155E+04 +                                   C+
   8 0.016    INF    +*                                C
 ---               +   +   +   +   +   +   +   +   +   +   +
 500                0       20       40       60       80      100
```

(The histogram parameters were adjusted after a first run to get a somewhat more even distribution of observations in the cells.)

6. According to the histogram, 93.6% of the systems (1- the relative frequency of the first cell, 0.064) survive past 50 days. The histogram parameters could be adjusted still again to get a value nearer to 95%, but if we interpolate, we will estimate that 5% of the systems will have failed after 39 days.

7. Using the mean (489) and standard deviation (381) from the simulation output, the program in the PROBLIB workspace computes the parameters U=528.961 and k=1.29394:

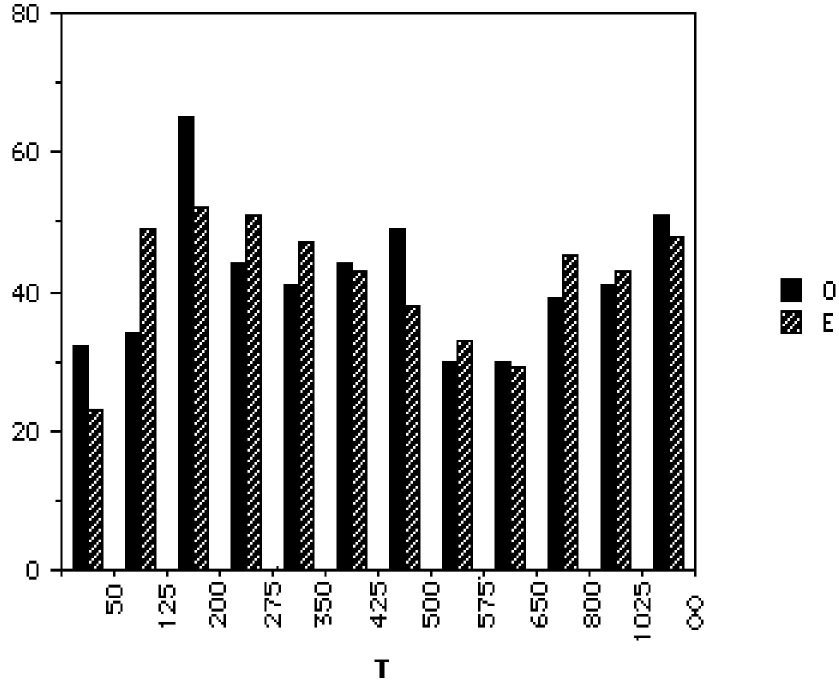
```
                          Weibull CDF
                          -----------

Mean: 489, Standard deviation: 381
Parameters of distribution:
  u=528.961 (scale), k=1.29394 (shape)
                         t      F(t)      1-F(t)
                       ---- --------- ---------

                         50 0.0461529 0.953847
                        125 0.143279  0.856721
                        200 0.247297  0.752703
                        275 0.348806  0.651194
                        350 0.443471  0.556529
                        425 0.529241  0.470759
                        500 0.605341  0.394659
                        575 0.671767  0.328233
                        650 0.728977  0.271023
                        725 0.777692  0.222308
                        800 0.818764  0.181236
                        875 0.853091  0.146909
                        950 0.881552  0.118448
                       1025 0.904982  0.0950181
                       1100 0.924142  0.0758583
```

8. Based upon the cumulative probabilities F(t), the following probabilities and expected values for each cell were calculated, where t_i is the upper limit of the cell:

i	t_i	p_i	E_i	O_i
1	50	0.04615	23.07647	32
2	125	0.09713	48.56294	34
3	200	0.10402	52.00927	65
4	275	0.10151	50.75408	44
5	350	0.09467	47.33290	41
6	425	0.08577	42.88484	44
7	500	0.07610	38.05003	49
8	575	0.06643	33.21305	30
9	650	0.05721	28.60495	30
10	725	0.04871	24.35746	20
11	800	0.04107	20.53625	19
12	875	0.03433	17.16303	12
13	950	0.02846	14.23084	15
14	1025	0.02343	11.71482	14
15	1100	0.01916	9.57989	7
16	1175	0.01557	7.78599	8
17	1250	0.01258	6.29178	7
18	1325	0.01011	5.05699	9
19	1400	0.00809	4.04392	5
20	1475	0.00644	3.21825	4
21	1550	0.00510	2.54946	3

For example, the probability of an observation in the interval 50-125 days is F(125) - F(50) = 0.143729-0.0461529 =0.09713, where F(t) is the CDF from the table in (7).

9. The cells at the upper end have been grouped, as indicated by the horizontal lines, so as to obtain a more even distribution of observations.



Now we calculate for each cell (or group of cells) the square of the deviation of O from E, and divide by E, and then sum to obtain the chi-square statistic:

t	E	O	D
50	23.07647	32	3.45068
125	48.56294	34	4.36710
200	52.00927	65	3.24479
275	50.75408	44	0.89880
350	47.33290	41	0.84731
425	42.88484	44	0.02900
500	38.05003	49	3.15116
575	33.21305	30	0.31083
650	28.60495	30	0.06804
800	44.89371	39	0.77373
1025	43.10869	41	0.10315
∞	47.50906	51	0.25651
SUM	500	500	17.5011

There are 12 cells, and 2 parameters (U & k) were estimated from the data, so that the number of degrees of freedom is 12-1-2=9. The table indicates that with 9 degrees of freedom , if the system lifetime does have the Weibull distribution with the parameters above, P{D>16.919} is only a=5%. The observed value of D (17.5011) exceeds 16.919, and so this leads us to reject the Weibull distribution model with the parameters U=528.961 and k=1.29394.

Suppose that we do not group so many cells together, so that we have 18 cells rather than 12:

t	E	O	D
50	23.076467	32	3.450678
125	48.562942	34	4.367101
200	52.009273	65	3.244787
275	50.754083	44	0.898798
350	47.332899	41	0.847309
425	42.884837	44	0.028998
500	38.050028	49	3.151164
575	33.213055	30	0.310833
650	28.604953	30	0.068036
725	24.357459	20	0.779533
800	20.536251	19	0.114922
875	17.163032	12	1.553158
950	14.230838	15	0.041572
1025	11.714824	14	0.445763
1175	17.365878	15	0.322321
1325	11.348774	16	1.906277
1550	9.811637	12	0.488087
∞	8.982771	8	0.107521
SUM	500	500	22.1269

The number of degrees of freedom is now 15, and according to the table, if the system lifetime does have the Weibull distribution with the parameters above, P{D>24.996} is only a=5%. Since D does not exceed 24.996, now we would ***not*** be able to justify rejecting the Weibull distribution model. (Note that this goodness-of-fit test is used to reject models, but if it fails to reject, this does not mean that the model is indeed correct! Since P{D>22.307} is a=10%, the probability of observing a value of D greater than or equal to that which was observed, if our Weibull model were correct, is only somewhat larger than 10%, which should not convince us that the model ***is*** valid.)

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #9 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

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