57:022 Principles of Design II
Homework Solutions Fall 1996

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1. A telephone exchange contains 6 lines. A line can be busy or available for calls and all lines act independently. Each line is busy 75% of the noon period (so that the probability that a line will be busy at any given time during the noon period is 75%).

   a. What is the probability of there being at least three free lines at any given time during this period?

   Sol'n: The number N_6 of free lines at any given time will have the binomial distibution with parameter n=6 and p=0.25, i.e., each of the six lines corresponds to a "trial", with the line being free corresponding to "success". Therefore

$$P\left\{N_6 \geq 3\right\} = \sum_{i=3}^{6} \binom{6}{i} (0.25)^i (1-0.25)^{6-i} = 0.1694$$

   b. What is the expected number of free lines at any time during this period?

   Sol'n: The mean (expected value) of a random variable having a binomial distribution is np, which in this instance is 6(0.25) =1.5

   c. You need to make three calls to this exchange, and each time you receive a busy signal you try again. What is the probability that you require exactly six tries in order to complete your three calls?

   Sol'n: Consider each call to the exchange to be a Bernouilli "trial", and T_k the number of the call on which you receive a free line for the kth time. Then T_k has a Pascal Distribution (or, for the special case of k=1, a geometric distribution). The required probability is therefore

$$P\left\{T_3 = 6\right\} = \binom{6-1}{3-1} (0.25)^3 (1-0.25)^{6-3}$$

$$= 0.1318 + 0.0326 + 0.0044 + 0.0002$$

$$= 0.0659$$

2. The foreman of a casting section in a certain factory finds that on the average, 1 in every 8 castings made is defective.

   N_{10} = # defects in 10 castings.

   N_{10} has the Binomial Distribution with parameters n=10 and p=1/8.

a. If the section makes 10 castings a day, what is the probability that none of these will be defective?

Sol'n: Applying the formula for the binomial distribution, we obtain

$$P\{N_{10}=0\} = \binom{10}{0}\left(\frac{1}{8}\right)^0\left(1-\frac{1}{8}\right)^{10-0} = 0.2631$$

b. If the section makes 10 castings a day, what is the probability that two of these will be defective?

Sol'n: According to the same formula used in (a),

$$P\{N_{10}=2\} = \binom{10}{2}\left(\frac{1}{8}\right)^2\left(1-\frac{1}{8}\right)^{10-2} = 0.2416$$

c. What is the probability that 3 or more defective castings are made in one day?

Sol'n: $$P\{N_{10}\geq 3\} = \sum_{i=3}^{10} P\{N_{10}=i\} = 1-P\{N_{10}<3\}$$

= 1 - [0.2631+0.3758+0.2416]

= 0.1195

3. A light bulb in an apartment entrance fails randomly, with an expected lifetime of 20 days, and is replaced immediately by the custodian. Assume that this bulb's lifetime has an exponential distribution.

a. What is the probability that a bulb lasts longer than its expected lifetime?

Sol'n: T_1, the lifetime of the first bulb (i.e., the time of the first failure), has exponential distribution with parameters λ =1 failure/20 days, i.e., the failure rate is 0.05 failures/day.

$$P\{T_1>20\} = e^{-\left(1/20\right)(20)} = 0.3679$$

b. If the current bulb was inserted 10 days ago, what is the probability that its lifetime (since it was inserted) will exceed the expected lifetime of 20 days?

Sol'n: Because of the Memoryless Property of the exponential distribution, the time between the 10^{th} day and the failure again has the same exponential distribution as T_1. Therefore, since the probability that the time from day 10 until the failure will exceed ten days is

$$P\{T_1>10\} = e^{-\left(1/20\right)(10)} = 0.6065,$$

the probability that the total lifetime will exceed 20 days, given that it is at least 10 days, is 60.65%.

c. If you were to test 10 of these bulbs, what is the probability that more than half will exceed the expected lifetime?

Sol'n: Consider the testing of each bulb to be a Bernouilli trial, with "success" corresponding to the bulb's exceeding the expected lifetime of 20 days. Then the number of "successes" in the 10 trials will have the binomial Distribution with parameters n=10 and p=0.3679 (from the answer in (a)).

$$P\{N_{10} > 5\} = P\{N_{10} = 6\} + \quad P\{N_{10} = 10\}$$
$$= 0.083126 + 0.02765 + 0.00603 + 0.00078 + 0.00004$$
$$= 0.11763$$

d. If the custodian has 2 spare bulbs, what is the probability that these (including the one currently in use) will be sufficient for the next 60 days?

Sol'n: Consider the Poisson process in which an event is defined to be a failure of a bulb. Then the number of events during a 60-day period will have the Poisson Distribution with rate λ =1 event per 20 days, i.e., $^1/_{20}$ = 0.05 events/day. The two spare bulbs will be sufficient for the next 60 days, provided that the number of bulb failures is not greater than 3. According to the formula for the Poisson distribution with λ = 0.05 events/day and t=60 days is

$$P\{N_{60} \leq 3\} = 0.6472$$

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up a hitchhiker is p=4%, i.e, an average of one in twenty-five drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

1. Each car may be considered as a "trial" in a <u>Bernouilli</u> process, with "success" defined as the car's stopping to pick up the hitchhiker.

2. Given that a hitchhiker has counted 15 cars passing him without stopping, what is the probability that he will be picked up by the 25th car or before?

Soln: Let Z_1 = the number of the Bernouilli trial in which "success" first occurs, where p=0.04. Z_1 has the geometric distribution, and because the Bernouilli process is "memoryless", we want the probability that success occurs on or before the 10th trial, i.e.,

$$P\{Z_1 \leq 10\} = \sum_{n=1}^{10} P\{Z_1 = n\} = \sum_{n=1}^{10} (1-p)^{n-1} p$$
$$= 0.04\left(0.96^0 + 0.96^1 + 0.96^2 + \ldots\ldots + 0.96^9\right) = 0.33517$$

n	$P\{Z_1=n\}$	$P\{Z_1 \le n\}$
1	0.04	0.04
2	0.0384	0.0784
3	0.03686	0.11526
4	0.03539	0.15065
5	0.03397	0.18463
6	0.03261	0.21724
7	0.03131	0.24855
8	0.03006	0.27861
9	0.02886	0.30747
10	0.02770	0.33517

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 15 per minute. Define "success" for the hitchhiker to occur at time t provided that both an arrival occurs at t <u>and</u> that car stops to pick him up. Let Y_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time t =0.

3. What is the arrival rate of "successes"? _____<u>0.6</u>______

   Soln: Arrival rate of "successes" $\lambda = (\lambda')\ (p) = (15)\ (0.04) = 0.6$

   (since the arrivals of the cars form a Poisson process at the average rate of 15/min. and each of these cars have probability p=0.04 that it will pick up a hitchhiker.)

4. What is the name of the probability distribution of Y_1? <u>Exponential</u>

5. What is the value of $E(Y_1)$?___<u>1.6667</u>__

   Soln: The mean of Exponential Distribution is $\frac{1}{\lambda} = \frac{1}{0.6}$

6. What's the value of $Var(Y_1)$?___<u>2.7778</u>___

   Soln: The variance of Exponential Distribution is $\frac{1}{\lambda^2} = \frac{1}{(0.6)^2}$

7. What is the probability that he must wait less than 2 minutes for a ride ($P\{Y_1 < 2\}$? ___<u>0.6988</u>__

   Soln: The CDF (cumulative distribution function) for Y_1, which has the exponential distribution, is

   $$F(t) = P\{Y_1 \le t\} = 1 - e^{-\lambda t} \text{ where } \lambda = 0.6.$$

   Another equivalent approach:

If $P\{T_n < t\}$ is true, then $P\{N_t \geq n\}$ is true, i.e., $P\{T_n < t\} \quad P\{N_t \geq n\}$

$$P\{T_1 < 2\} = P\{N_2 \geq 1\} = 1 - P\{N_2 < 1\} = 1 - P\{N_2 = 0\}$$

$$= 1 - e^{-(0.6)(2)} \frac{\{(0.6)(2)\}^0}{0!} = 0.6988$$

8. What is the probability that he must wait more than 2 minutes for a ride($P\{Y_1 > 2\}$? Soln: ___0.3012___

$$P\{Y_1 > 2\} = 1 - P\{Y_1 \leq 2\} = 1 - F(2) = e^{-(0.6)2} = 0.3012$$

Equivalently,

Using the property "$P\{T_n > t\} \Leftrightarrow P\{N_t < n\}$"

$$P\{T_1 > 2\} = P\{N_2 < 1\} = P\{N_2 = 0\} = e^{-(0.6)(2)} \frac{\{(0.6)(2)\}^0}{0!} = 0.3012$$

9. What is the probability that he must wait exactly 2 minutes for a ride? ($P\{Y_1 = 2\}$? Soln: ____0_____.

The probability that the event occurs in a time interval of length zero is zero!

Note that your answers in 7-9 must have a sum equal to 1!

Suppose that after 2 minutes (during which 28 cars have passed by) the hitchhiker is still there waiting for a ride.

10. What is the conditional expected value of Y_1 (expected total waiting time, given that he has already waited 2 minutes). ____3.6667___

Soln: Because of the memoryless property of the Exponential distribution, the expected additional time until a car stops is identical to the original expected time until the car stops. That is, he now expects to wait a total of $2 + E(T_1) = 2 + \left(\frac{1}{0.6}\right) = 3.6667$

10. What is your ID number? ___485-21-1250__ (Han-Suk's ID #)

Note: Your solutions to #11, 12, 14, and 15 below will vary because of the different random number seed used in the simulation!

11. Using the first four digits of your ID number as the "seed" for the "Midsquare" technique, generate a sequence of 10 pseudo-random numbers $R_1, R_2, \ldots R_{10}$ uniformly distributed in the interval [0,1].

Soln: Starting with the "seed" X_0=4852, we get the "pseudo-random" sequence

i	X_i	X_i^2	R_i
1	4852	23541904	0.5419
2	5419	29365561	0.3655
3	3655	13359025	0.3590
4	3590	12888100	0.8881
5	8881	78872161	0.8721
6	8721	76055841	0.0558
7	0558	00311364	0.3113
8	3113	09690769	0.6907
9	6907	47706649	0.7066
10	7066	49928356	0.9283

12. Using the "Inverse Transformation" technique and the 10 numbers generated in (10.), generate the interarrival times $\tau_1, \tau_2, \ldots \tau_{10}$ for the first ten cars with arrival rate λ = 15/minute, and then the arrival times $T_1, T_2, \ldots T_{10}$ of those ten cars.

Soln:

i	R_i	τ_i	T_i
1	0.5419	0.05204452	0.05204452
2	0.3655	0.03032787	0.08237238
3	0.3590	0.02964839	0.11202077
4	0.8881	0.14600998	0.25803075
5	0.8721	0.13710044	0.39513119
6	0.0558	0.00382782	0.39895901
7	0.3113	0.02486330	0.42382231
8	0.6907	0.07822957	0.50205188
9	0.7066	0.08174789	0.58379978
10	0.9283	0.17568430	0.75948408

(Since $\tau_i = -\frac{\ln\left(\overline{R_i}\right)}{\lambda} = -\frac{\ln\left(1-R_i\right)}{\lambda}$, where τ_i is the <u>inter</u>arrival time, i.e., $\tau_1 = T_1$, $\tau_2 = T_2 - T_1$, $\tau_3 = T_3 - T_2$, etc., so that the arrival times are $T_1=\tau_1$, $T_2=\tau_1+\tau_2$, $T_3=\tau_1+\tau_2+\tau_3$, etc.)

13. What is the expected number of arrivals during the first 20 seconds?

Soln: $E\left(N_{1/3}\right) = (15/\text{min})(\frac{1}{3}\text{ min}) = 5$

14. What is the actual # of arrivals during the first 20 seconds of your simulation?

Soln: 4 arrivals in the first 20 seconds. (Since the 5th arrival occurs at 0.3952 minutes which is later than 0.3333 minutes(= 20 seconds).)

15. What is the probability that you would observe exactly this number of arrivals in this Poisson process? <u>0.1755</u> .

Soln:

$$P\left\{N_{1/3}=4\right\}=e^{-(15)(1/3)}\frac{\left\{(15)(1/3)\right\}^4}{4!}=e^{-5}\frac{5^4}{4!}=0.1755$$

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1. Consider again the proposed drive-up bank teller window in the Hypercard Stack "Intro. to Simulation", but where the arrival rate of customers has increased to 20/hour (one every 3 minutes). The SLAM models for two models (one with a single teller window and the second with two teller windows), together with output, appear below.

• • • • • • • • • • • • • • • Case A: single teller • • • • • • • • • • • • • • •

```
GEN,BRICKER,BANKTELLERS,9/12/1996,,,,,,72;
LIM,2,1,50;
INIT,0,480;
NETWORK;
        CREATE,EXPON(3.0),,1;
        QUE(1),0,4,BALK(OVFLO);
        ACT(1)/1,EXPON(2.0);
        COLCT,INTVL(1),CUSTOMER_TIME,20/.5/.5;
        TERM;
OVFLO   COLCT,ALL,OVERFLOW;
        TERM;
        END;
FIN;
```

S L A M   I I   S U M M A R Y   R E P O R T

```
SIMULATION PROJECT BANKTELLERS              BY BRICKER
DATE  9/12/1996                             RUN NUMBER    1 OF    1

CURRENT TIME    .4800E+03
STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00
```

During the 480 simulated time units, there were a total of 158 cars served by the bank teller. The average time in the system for these cars was 5.86 time units with a standard deviation of 5.20 time units and times ranged from 0.0345 to 23.5 time units. The distribution for time in the system is depicted by the histogram generated by SLAM II. There was a total of 1 observation of overflow.

A histogram of the values collected at a COLCT node can be obtained. This is accomplished by specifying on input the number of interior cells, NCEL; the upper limit of the first cell, HLOW; and a cell width, HWID, for the histogram. The number of cells specified, NCEL, each of which will have a width of HWID. Two additional cells will be added that contain the interval (-∞,HLOW] and (HLOW+NCEL*HWID,∞).

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	**.586E+01**	.520E+01	.887E+00	.345E-01	.235E+02	**158**
OVERFLOW	.182E+03	.000E+00	.000E+00	.182E+03	.182E+03	**1**

The second category of statistics for this example is the "file statistics". The statistics for file 1 correspond to the cars waiting for service at bank teller window was 1.25 cars, with a standard deviation of 1.473 cars, a maximum of 4 cars waited, and at the end of the simulation there were no cars in the queue.

****FILE STATISTICS****

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	1.250	1.473	4	0	**3.797**
2		.000	.000	0	0	.000
3	CALENDAR	1.679	.467	3	1	1.729

The last category of statistics for this example is "statistics on service activities". The first row of service activity statistics corresponds to the server at bank teller window who was busy 67.9 percent of the time. Since the capacity of the server is one, the values 12.84 and 87.10 refer to the maximum length of the server idle period and the server busy period, respectively.

****SERVICE ACTIVITY STATISTICS****

ACT NUM	ACT LABEL OR START NODE	SER CAP	AVERAGE UTIL	STD DEV	CUR UTIL	AVERAGE BLOCK	MAX IDL TME/SER	MAX BSY TME/SER	ENT CNT
1	QUEUE	1	**.679**	.47	0	.00	12.84	87.10	158

****HISTOGRAM NUMBER 1****

CUSTOMER_TIME

```
OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0        20        40        60        80        100
                    +    +    +    +    +    +    +    +    +    +    +
  9  .057  .500E+00 +***                       ^                      +
 11  .070  .100E+01 +***  C                    |                      +
 13  .082  .150E+01 +****     C                |                      +
 14  .089  .200E+01 +****          C           |                      +
  4  .025  .250E+01 +*              C          |                      +
  8  .051  .300E+01 +***               C       |                      +
  9  .057  .350E+01 +***                  C    |                      +
  2  .013  .400E+01 +*                    C    |                      +
  7  .044  .450E+01 +**                     C  |                      +
  9  .057  .500E+01 +***                       C                      +
  4  .025  .550E+01 +*                         |C                     +
  6  .038  .600E+01 +**                        |  C                   +
  4  .025  .650E+01 +*                         |    C                 +
  5  .032  .700E+01 +**                        |     C                +
  5  .032  .750E+01 +**                        |       C              +
  5  .032  .800E+01 +**                        |        C             +
  5  .032  .850E+01 +**                        |          C           +
```

```
  6  .038  .900E+01 +**                       |            C         +
  2  .013  .950E+01 +*                        |             C        +
  4  .025  .100E+02 +*                        |              C       +
  3  .019  .105E+02 +*                        |               C      +
 23  .146    INF    +*******                  ˘                      C
---                 +    +    +    +    +    +    +    +    +    +    +
158                 0         20        40        60        80       100
```

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	.586E+01	.520E+01	.887E+00	.345E-01	.235E+02	158

• • • • • • • • • • • • • • • Case B: two tellers • • • • • • • • • • • • • • •

```
GEN,BRICKER,BANKTELLERS,9/12/1996,,,,,,72;
LIM,2,1,50;
INIT,0,480;
NETWORK;
        CREATE,EXPON(3.0),,1;
        QUE(1),0,4,BALK(OVFLO);
        ACT(2)/1,EXPON(2.0);
        COLCT,INTVL(1),CUSTOMER_TIME,20/.5/.5;
        TERM;
OVFLO   COLCT,ALL,OVERFLOW;
        TERM;
        END;
FIN;
```

S L A M   I I   S U M M A R Y   R E P O R T

```
SIMULATION PROJECT BANKTELLERS               BY BRICKER
DATE  9/12/1996                              RUN NUMBER    1 OF    1

CURRENT TIME    .4800E+03
STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00
```

During the 480 simulated time units, there were a total of 179 cars served by the bank teller. The average time in the system for these cars was 2.37 time units with a standard deviation of 2.60 time units and times ranged from 0.0117 to 23.6 time units. The distribution for time in the system is depicted by the histogram generated by SLAM II.

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	**.237E+01**	.260E+01	.110E+01	.117E-01	.236E+02	**179**
OVERFLOW			**NO** VALUES RECORDED			

The second category of statistics for this example is the “file statistics”. The statistics for file 1 correspond to the cars waiting for service at bank teller window was 0.099 cars, with a standard deviation

of 0.355 cars, a maximum of 4 cars waited, and at the end of the simulation there were no cars in the queue.

****FILE STATISTICS****

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	.099	.355	4	0	0.266
2		.000	.000	0	0	.000
3	CALENDAR	1.785	.794	4	1	2.164

The last category of statistics for this example is "statistics on service activities". The first row of service activity statistics corresponds to the service activity at the bank teller window: the average utilization was 0.785 which is the average number of busy servers. Since there are two servers, the utilization of each individual server was 0.785/2 = 39.25%, which means that each server was busy an average of 39.25% of the day. Note that since this activity has a capacity of 2, the maximum idle and busy values refer to the number of servers, instead of the service times.

****SERVICE ACTIVITY STATISTICS****

ACT NUM	ACT LABEL OR START NODE	SER CAP	AVERAGE UTIL	STD DEV	CUR UTIL	AVERAGE BLOCK	MAX IDL TME/SER	MAX BSY TME/SER	ENT CNT
1	QUEUE	2	**.785**	.79	0	.00	2.00	2.00	179

****HISTOGRAM NUMBER 1****
CUSTOMER_TIME

```
OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0        20        40        60        80       100
                    +    +    +    +    +    +    +    +    +    +    +
 27  .151  .500E+00 +********                                    ^    +
 30  .168  .100E+01 +********       C                            |    +
 28  .156  .150E+01 +********               C                    |    +
 16  .089  .200E+01 +****                       C                |    +
 17  .095  .250E+01 +*****                           C           |    +
 11  .061  .300E+01 +***                                C        |    +
 11  .061  .350E+01 +***                                   C     |    +
  9  .050  .400E+01 +***                                      C  |    +
  9  .050  .450E+01 +***                                        C|    +
  4  .022  .500E+01 +*                                           C    +
  3  .017  .550E+01 +*                                           |C   +
  2  .011  .600E+01 +*                                           | C  +
  2  .011  .650E+01 +*                                           | C  +
  2  .011  .700E+01 +*                                           |  C +
  2  .011  .750E+01 +*                                           |  C +
  1  .006  .800E+01 +                                            |   C+
  0  .000  .850E+01 +                                            |   C+
  1  .006  .900E+01 +                                            |   C+
  1  .006  .950E+01 +                                            |   C+
  0  .000  .100E+02 +                                            |   C+
  1  .006  .105E+02 +                                            |   C+
  2  .011    INF    +*                                           ˇ   C
---                 +    +    +    +    +    +    +    +    +    +    +
179                 0        20        40        60        80       100
```

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	.237E+01	.260E+01	.110E+01	.117E-01	.236E+02	179

• •

a. How many cars were unable to enter the queue each day, because the queue was filled to capacity?
Case A: 1 Case B: none

b. What fraction of the time was each teller busy each day?
Case A: 67.9 % Case B: 39.25 %

c. Estimate the mean (average) time in the system for the customers.
Case A: 5.86 min . Case B: 2.37 min.

d. What fraction of the customers spend more than 5 minutes (total of both waiting and being served) at the bank?
Case A: $^{72}/_{158}$ = 45.6% Case B: $^{17}/_{179}$ = 9.5%

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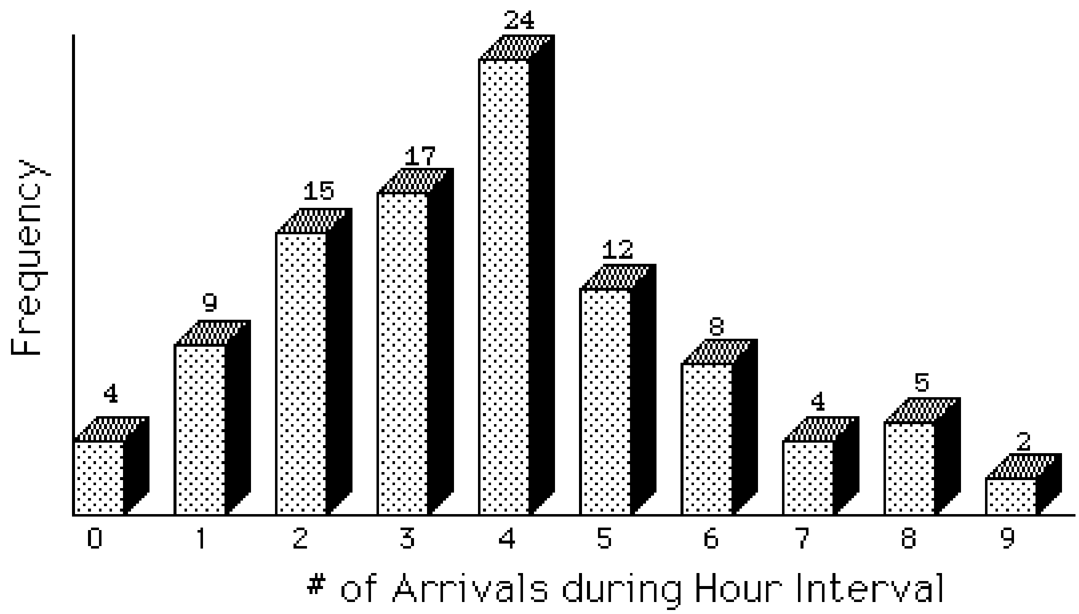
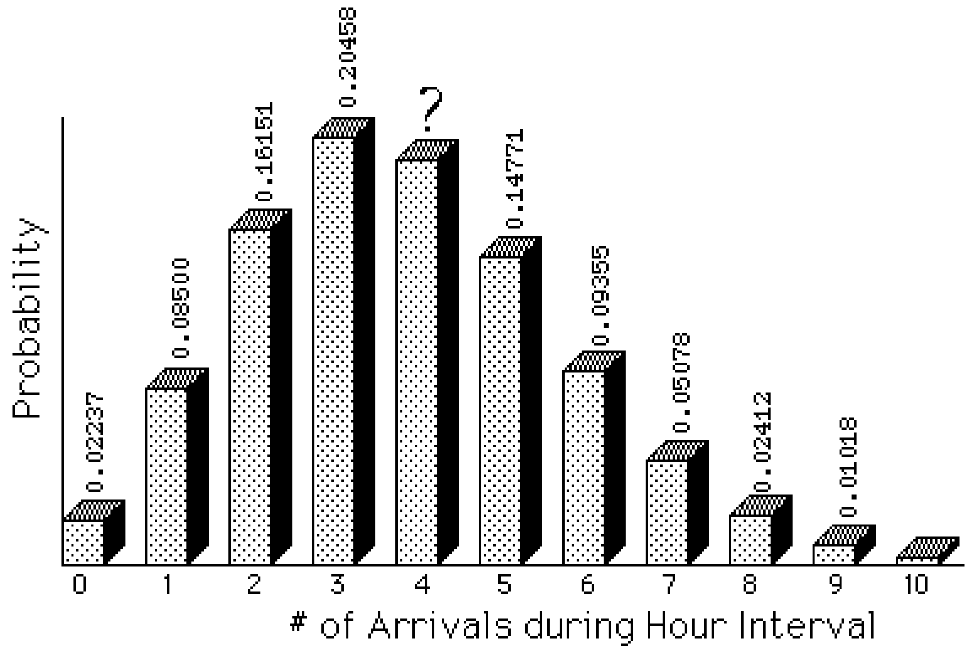
1. The numbers of arrivals during 100 hours of what is believed to be a Poisson process were recorded. The observed numbers ranged from zero to nine, with frequencies O_0 through O_9:



The average number of arrivals was 3.8/hour. We wish to test the "goodness-of-fit" of the assumption that the arrivals correspond to a Poisson process with arrival rate 3.8/hour. The first step is to compute the probability of each observed value, 0 through 9:

i	P{N=i}
0	0.0223708
1	0.0850089
2	0.161517
3	0.204588
4	0.1943588
5	0.147713
6	0.0935513
7	0.050785
8	0.0241229
9	0.0101852



a. What is the value missing above? (That is, the probability that the number of arrivals is exactly 4.)

$$e^{-3.8}\frac{(3.8)^4}{4!} = 0.1943588$$

b. Now, we can compute the expected number of observations of each of the values 0 through 9, which we denote by E_0 through E_9.

• What is the expected number of times in which we would observe four arrivals per hour?

Soln: 19.44

• Did we observe more or fewer than the expected number?

Soln: The observed number (=24) is more than the expected number (=19.44)

c. Complete the table below:

i	P{N=i}	E_i	O_i	$(E_i-O_i)^2$	$\frac{(E_i-O_i)^2}{E_i}$
0	0.0223708	2.23708	4	3.1079	1.38927
1	0.0850089	8.50089	9	0.249107	0.0293037
2	0.161517	16.1517	15	1.32641	0.0821218
3	0.204588	20.4588	17	11.9634	0.584756
4	0.1943588	19.43588	24	20.83119	1.07179
5	0.147713	14.7713	12	7.67991	0.519922
6	0.0935513	9.35513	8	1.83639	0.196298
7	0.050785	5.0785	4	1.16317	0.229037
8	0.0241229	2.41229	5	6.69625	2.77589
9	0.0101852	1.01852	2	0.9633	0.945782

d. Ignoring the suggestion that cells should be aggregated so that they contain at least five observations, what is the observed value of

$$D = \sum_i \frac{(E_i-O_i)^2}{E_i} \quad ?$$

Soln: 7.8241705

e. Keeping in mind that the assumed arrival rate λ=3.8/hour was estimated from the data, what is the number of "degrees of freedom"?
Soln: 8 (=10-1-1) degrees of freedom

f. Using a value of α = 5%, what is the value of the quantity $\chi^2_{5\%}$ such that D exceeds $\chi^2_{5\%}$ with probability 5% (if the assumption is correct that the arrivals form a Poisson process with arrival rate 3.8/hour)?
Soln: 15.507

g. Is the observed value greater than or less than $\chi^2_{5\%}$?

Soln: D (=7.82) < $\chi^2_{5\%}$ (=15.507)

Should we accept or reject the assumption that the arrival process is Poisson with rate 3.8/hour?
Soln: Accept the assumption that the arrival process is Poisson with rate 3.8/hour.

~~~~~~

2. Consider again the SLAM II example of the garment factory with 50 sewing machine operators and machines, 54 machines (including 4 backup machines) and 3 repairmen. The original system was modeled by the following SLAM statement. Note that the time units are hours, and that the system is simulated for 2000 hours, i.e., 50 weeks of 40 hours each.

```
GEN,BRICKER,GARMENTFACTORY,9/18/1996,,,,,,,72;
LIM,2,,55;
INIT,0,2000;
NETWORK;
Q1    QUE(1),4;
      ACTIVITY(50)/1,EXPON(157);
Q2    QUE(2);
      ACTIVITY(3)/2,UNFRM(4,10),,Q1;
      END;
FIN;
```

The output of this SLAM model is

S L A M   I I   S U M M A R Y   R E P O R T

```
SIMULATION PROJECT GARMENTFACTORY           BY BRICKER
DATE  9/18/1996                             RUN NUMBER    1 OF    1
CURRENT TIME    .2000E+04
STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00
```

FILE STATISTICS

FILE NUMBER	LABEL/TYPE		AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	Q1	QUEUE	1.420	1.311	4	0	4.396
2	Q2	QUEUE	.802	1.578	11	4	2.473
3		CALENDAR	51.778	1.390	54	50	48.778

SERVICE ACTIVITY STATISTICS
~~~~~~

```
ACT ACT LABEL OR  SER AVERAGE    STD  CUR AVERAGE MAX IDL MAX BSY  ENT
NUM START NODE    CAP   UTIL     DEV  UTIL BLOCK  TME/SER TME/SER  CNT
  1 Q1   QUEUE     50  49.542   1.29   47     .00   10.00   50.00  649
  2 Q2   QUEUE      3   2.236    .98    3     .00    3.00    3.00  642
```

~~~~~~

The modified example, in which the repair of the failed machines is subcontracted if there are already two machines in the repair queue, is modeled by the SLAM statements:

```
GEN,BRICKER,GARMENTFACTORY,9/18/1996,,,,,,,72;
LIM,2,,55;
INIT,0,2000;
NETWORK;
Q1     QUE(1),4;
       ACTIVITY(50)/1,EXPON(157);
Q2     QUE(2),,2,BALK(SUB);
       ACTIVITY(3)/2,UNFRM(4,10),,Q1;
SUB    COLCT,BETWEEN,SUBCONTRACT;
       ACTIVITY,UNFRM(20,30),,Q1;
       END;
FIN;
```

The output of this SLAM model is

```
                  S L A M   I I   S U M M A R Y   R E P O R T

  SIMULATION PROJECT GARMENTFACTORY            BY BRICKER
  DATE  9/18/1996                              RUN NUMBER    1 OF    1

  CURRENT TIME    .2000E+04
  STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00

               **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                   MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                   VALUE   DEVIATION VARIATION   VALUE     VALUE    OBS

 SUBCONTRACT      .527E+02  .729E+02  .138E+01  .881E-01  .262E+03    35

                              **FILE STATISTICS**


 FILE                     AVERAGE  STANDARD   MAXIMUM  CURRENT AVERAGE
 NUMBER  LABEL/TYPE       LENGTH   DEVIATION  LENGTH   LENGTH  WAIT TIME

    1    Q1   QUEUE          1.488     1.270        4        1      4.694
    2    Q2   QUEUE           .277      .577        2        0       .926
    3         CALENDAR      52.235     1.060       54       53     51.086

                              **SERVICE ACTIVITY STATISTICS**

 ACT ACT LABEL OR  SER AVERAGE    STD  CUR AVERAGE MAX IDL MAX BSY  ENT
 NUM START NODE    CAP   UTIL     DEV  UTIL BLOCK  TME/SER TME/SER  CNT

   1 Q1   QUEUE     50  49.706    .83   50     .00    7.00   50.00  633
   2 Q2   QUEUE      3   2.082    .98    3     .00    3.00    3.00  594
```
~~~~~~

~~~~~~

Note: When "between" statistics are collected, the number of observations is the number of intervals between arrival of entities, i.e., one less than the number of arrivals!

Suppose that

- the hourly wage of a sewing machine operator (including benefits, etc.) is $15.
- the hourly wage of a machine repairman (including benefits, etc.) is $25.
- each machine operator (when busy) generates $27.50 per hour of revenue for the company,
- the cost of subcontracting the repair of a machine is $250.

a. What is the annual payroll for machine operators?
Soln: $ 30,000 (=2000 × 15) per operator; total is 50($30,000) = $1.5 million

b. What is the annual payroll for repairmen?
Soln: $ 50,000 (=2000 × 25) per repairman; total is 3($50,000) = $150,000

c. What is the average direct cost of repairing a machine in-house?
Soln: $ 77.88 / machine =$25(2000/642)

d. How many repairs were subcontracted during the year?
Soln: 36 (Since there were 35 observations of the interval <u>between</u> such repairs, the number of the subcontracted repairs is 36.)

e. How frequently were repairs subcontracted?
Soln: once every 52.7 hours (from SLAM II output).

f. What is the average length of time that machines wait in the repair queue?
Soln: Without subcontracting: 2.473 hours
Soln: With subcontracting: 0.926 hours

g. What is the maximum number of idle machine operators during the year?
Soln: Without subcontracting: 10 operators
Soln: With subcontracting: 7 operators

h. What is the utilization of each repairman, i.e. the % of time busy?
Soln: Without subcontracting: 74.5%
Soln: With subcontracting: 69.4 %

i.. How many more man-hours of machine operator time per year are utilized when the repairs are subcontracted?
Soln: 328 hours more (= 49.706-49.542)× 2000)

j. How much more revenue per year is earned by the company when repairs are subcontracted?
Soln: With subcontracting : (49.706× 27.5× 2000)-(250× 36) = 2,724,830
Without subcontracting, revenue is (49.542× 27.5× 2000) = 2,724,810
Difference is $20

k. Is the decision to subcontract the repairs cost-effective? That is, will the increase in the company 's revenues compensate for the cost of subcontracting?
Soln: Yes, but the increase is so negligible that the company should decide based upon other non-quantifiable factors.

•••••••••••••••••••••••••• Homework # 5 •••••••••••••••••••••••••••••

Regression Analysis. Tests on the fuel consumption of a vehicle traveling at different speeds yeilded the following results:

Speed s (mph)	20	30	40	50	60	70	80	90
Consumption C (mile/gal.)	11.4	17.9	22.1	25.5	26.1	27.6	29.2	29.8

(Note: the above data is complete fictitious!)
~~~~~~

Solutions

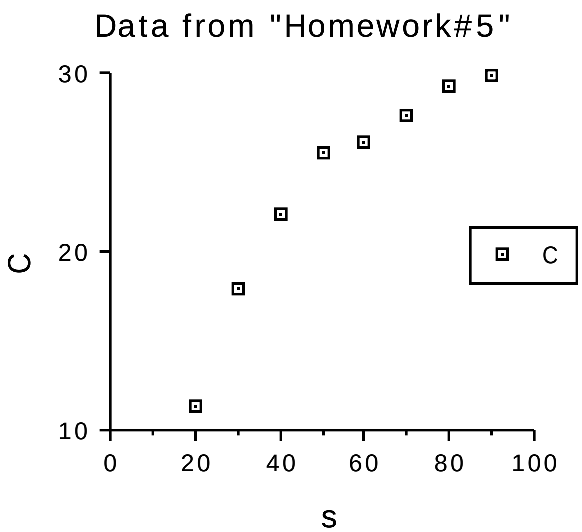
It is suggested that the relationship between the two variables is of the form $C = a + b/s$.

a. Run the "Cricket Graph" software package (on the Macintosh), and enter the above observed values of s and C (columns 1 and 2).

Untitled Data

	1	2	3
	C	s	
1	11.4	20	
2	17.9	30	
3	22.1	40	
4	25.5	50	
5	26.1	60	
6	27.6	70	
7	29.2	80	
8	29.8	90	

b. Plot the "scatter plot" of C versus s by choosing "scatter" on the Graph menu, and specifying s on the horizontal axis and C on the vertical axis. Does the plot appear to be linear? ___NO____

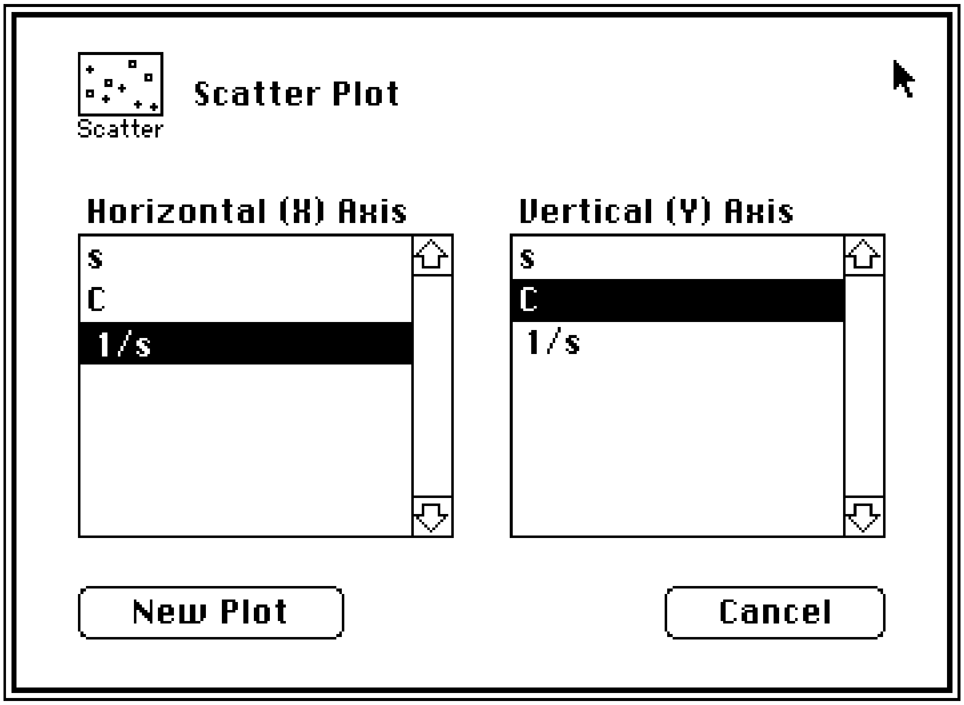


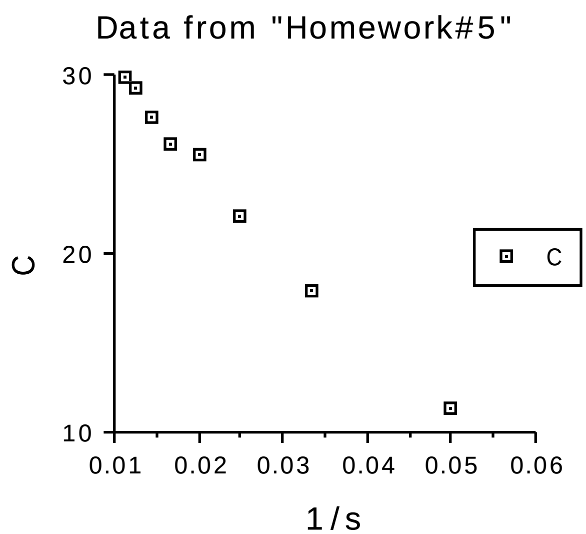
c. Choose "transform" from the "data" menu to create a new variable 1/s which is the reciprocal of s. (Put this new variable into Column 3.)

Untitled Data

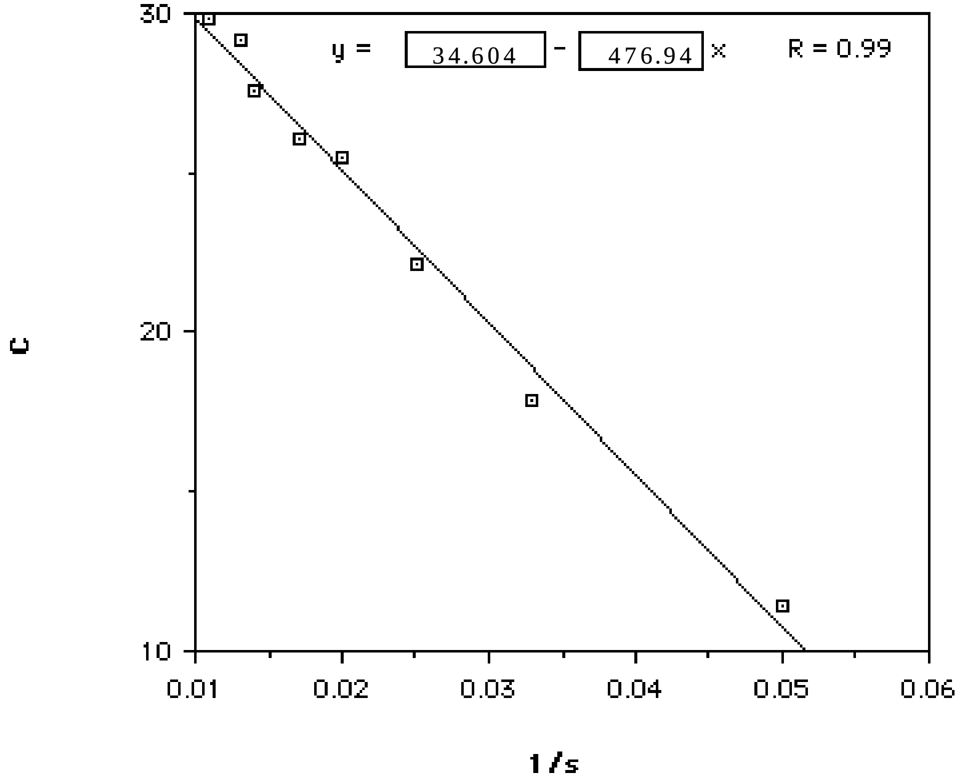
	1	2	3
	C	s	1/s
1	11.4	20	0.050
2	17.9	30	0.033
3	22.1	40	0.025
4	25.5	50	0.020
5	26.1	60	0.017
6	27.6	70	0.014
7	29.2	80	0.013
8	29.8	90	0.011

d. Plot the "scatter plot" of 1/s (horizontal axis) versus C (vertical axis). Does the plot appear to be linear? ___Yes ____

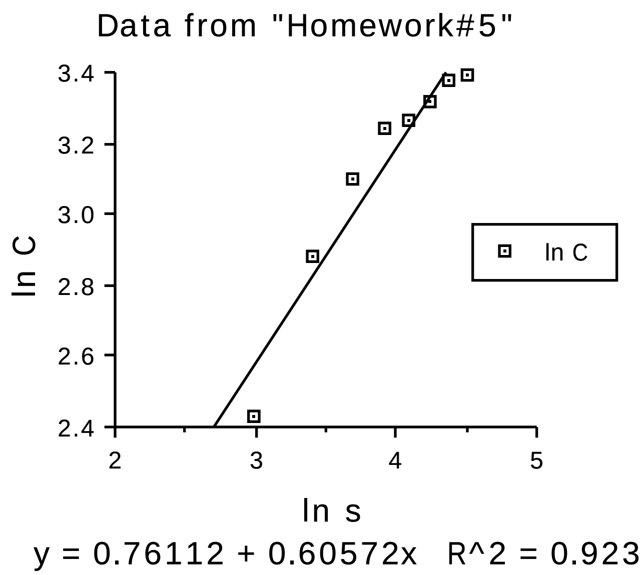




e. After plotting C versus 1/s, select "Simple" from the "Curve Fit" menu, in order to fit a simple <u>linear</u> relationship between C and 1/s, i.e., to determine a and b such that C ≈ a + b($^1/_s$). What is the value of a? ___34.604 ___ of b? ___-476.94 ___


y = 34.604 - 476.94 x
R = 0.99
30
20
10
C
0.01
0.02
0.03
0.04
0.05
0.06
1/s

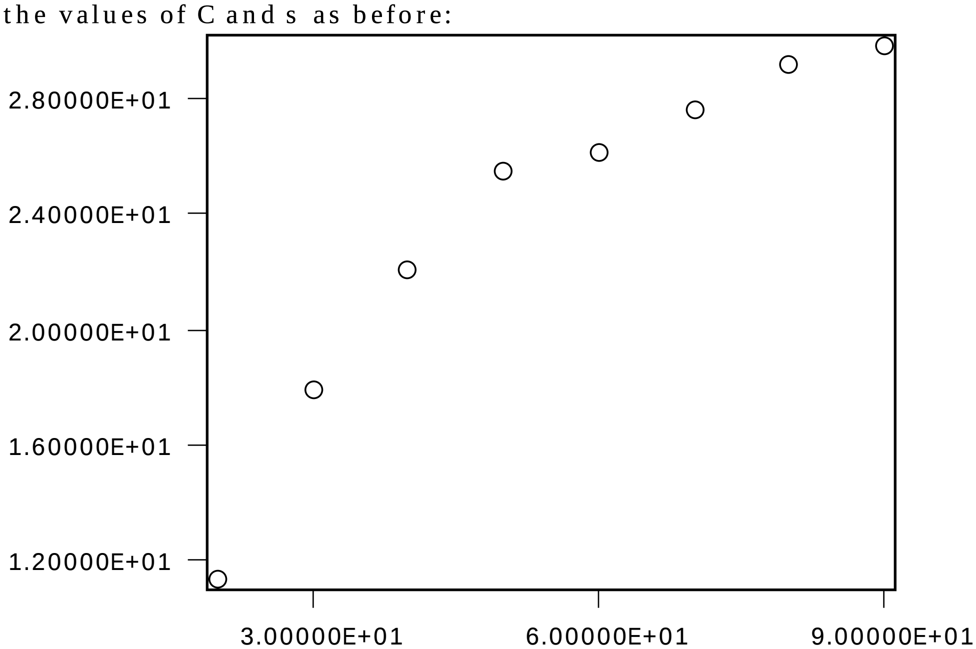
f. Another suggestion is that the relationship is of the form $C = ax^b$. Perform the appropriate linear regression to fit a curve of this type. What is the value of a? _2.14067(=e $^{0.76112}$)_ of b? __0.60572 _

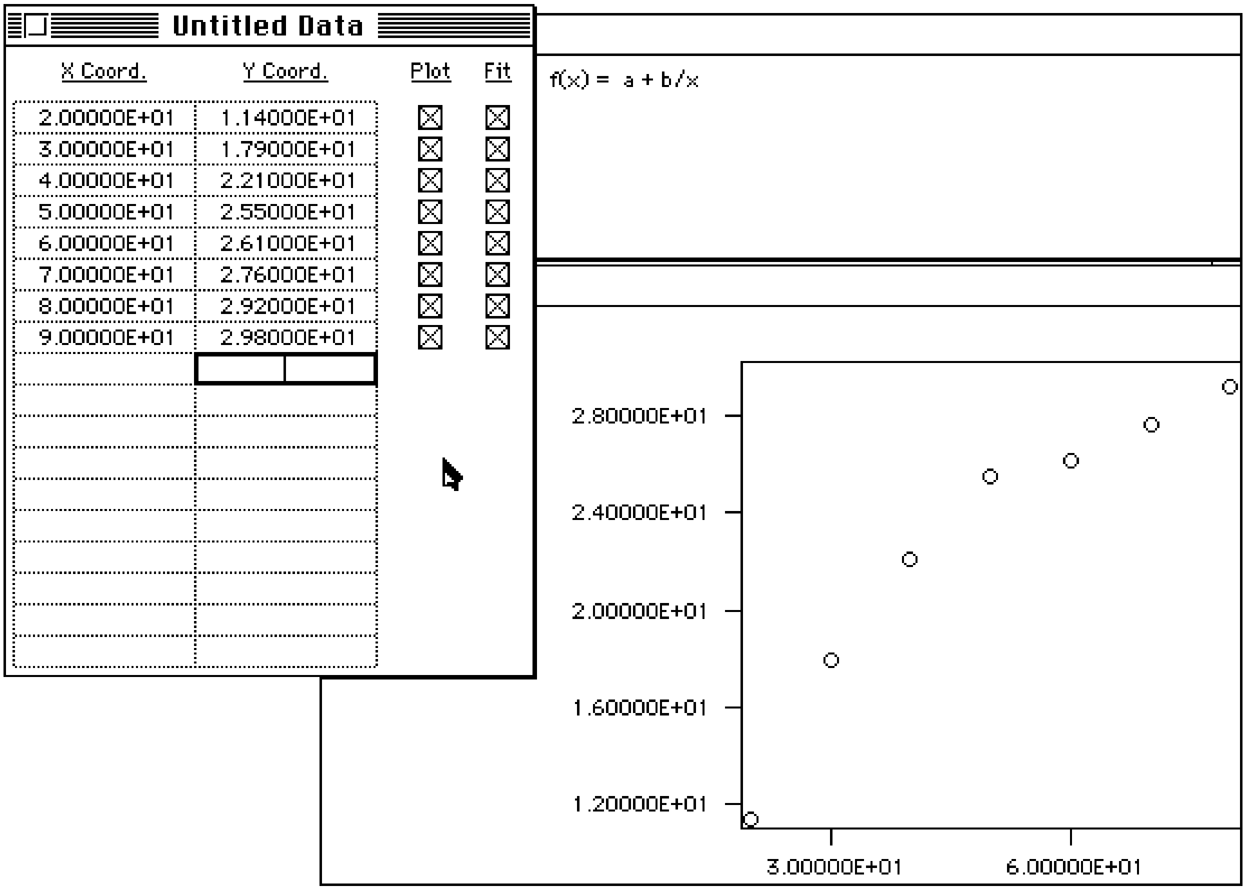


◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊

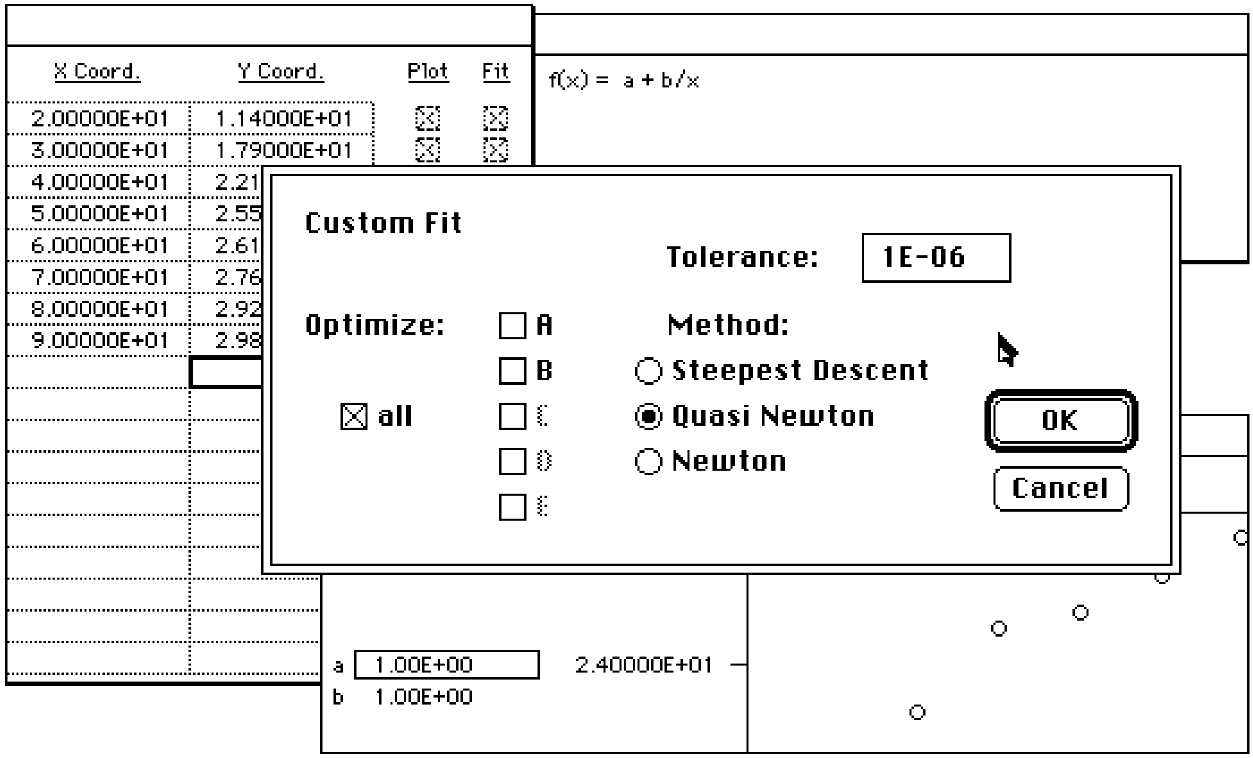
Next, start the shareware program "Curve Fit 0.7e " (author: Kevin Raner) which can be found on the "Public Software" fileserver of ICAEN. This program can perform nonlinear regression directly, without first linearizing the function.

g. Enter the values of C and s as before:

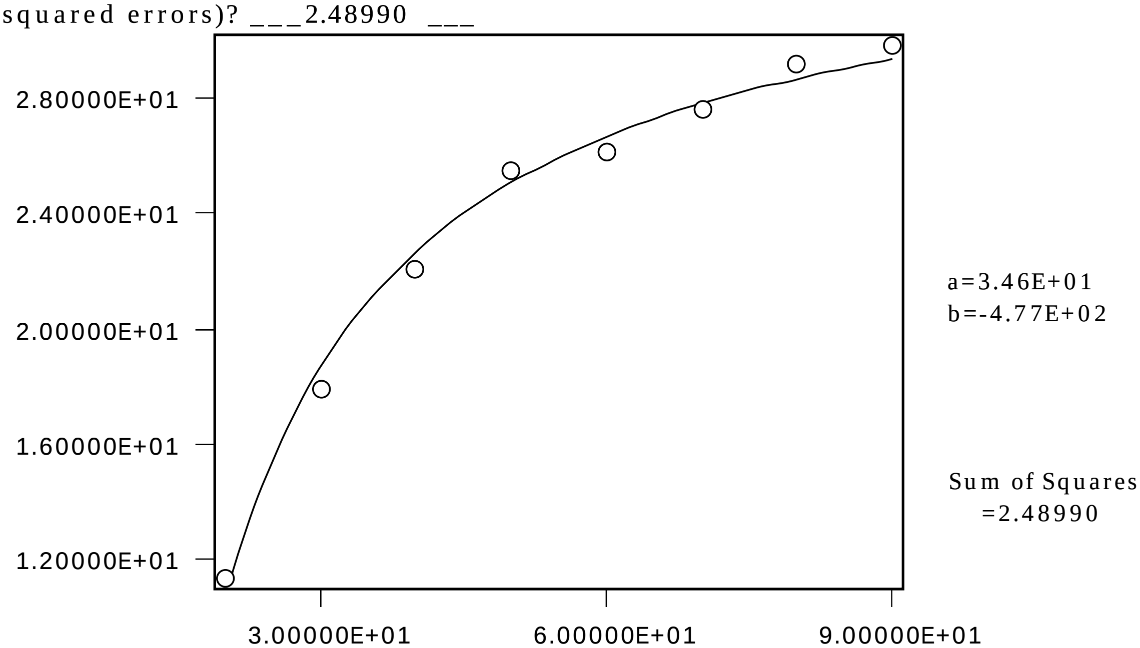




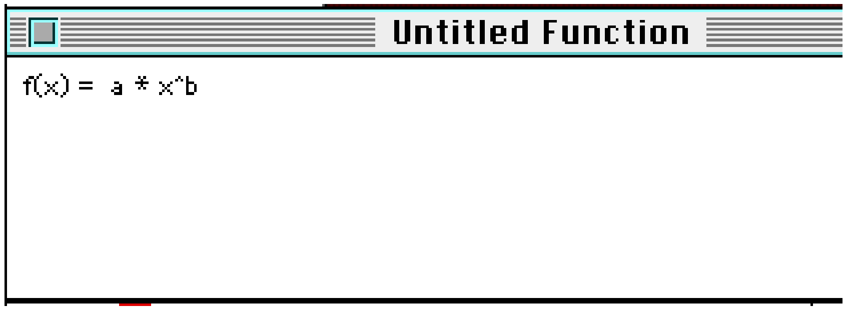
Next, enter the function "f(x) = a + b/x" and give initial values of 1 to the parameters a & b. Then choose "Custom Fn" from the menu in order to find the values of a & b which will minimize the sum of the squared errors, using a nonlinear optimizing algorithm, e.g. a Quasi-Newton algorithm. (You may have encountered this type of algorithm, e.g. Fletcher-Powell, in 57:021 "Principles of Design I".) Be sure to indicate that both a & b are to be selected:



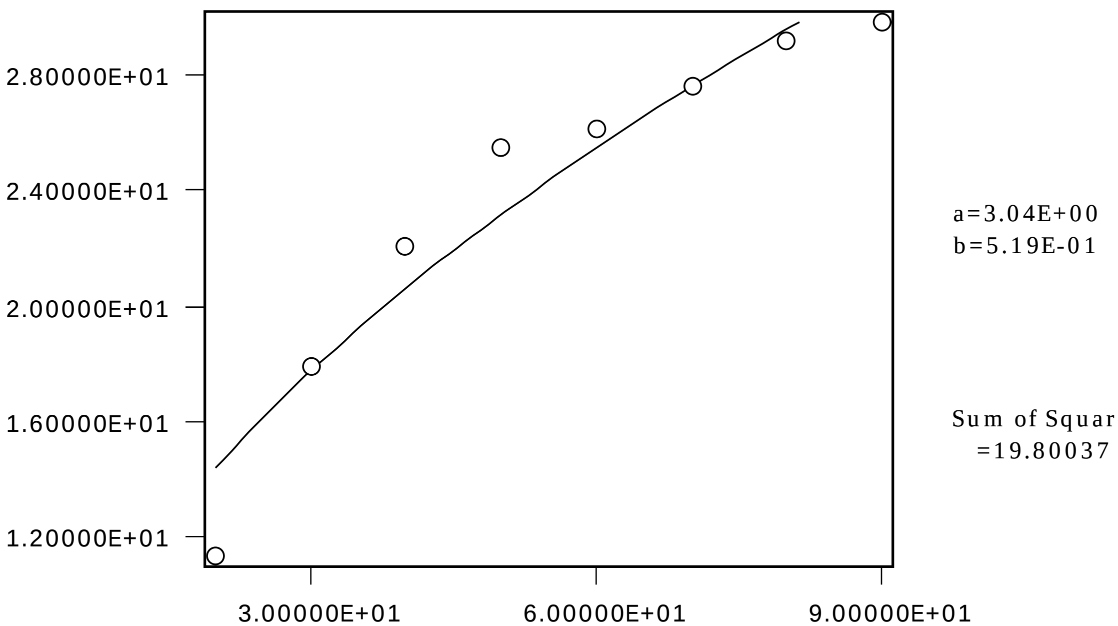
h. What is the optimal fitted curve? C = (34.6) + (-477) / s. What is the SSE (sum of the squared errors)? ___2.48990 ___



i. Next, enter the power function



and find the optimal curve of this type: $C = 3.04\ s^{0.519}$. What is the SSE for this fitted curve? 19.80037



j. You now have four candidate curves. Write them below, together with their SSE's. (You may need to compute the values of SSE for the curves fit by Cricket Graph manually.)

	$C = as^b$	SSE	$C = a + b/s$	SSE
Cricket Graph:	$C = 2.14067s^{0.60572}$	25.7847687	C = 34.604 - 476.94 / s	2.48989893
Curve Fit	$C = 3.04s^{0.519}$	19.80037	C = 34.6 - 477 / s	2.48990

Which one has the smallest SSE?

Soln : The question was perhaps vague, in that "SSE" did not specify in which equation the error was to be computed, i.e., $\ln C_i = \left[\ln a + b \times \ln s_i\right]$ or $C_i = as_i^b$. My intention was the latter, i.e., the error in the original curve rather than the linearized curve.

There is neglible difference between the SSE in the fitted curves of the form C = a + b/s found by Cricket Graph and Curve Fit 0.7e. (This is because the objective functions for the optimization are identical.)

However, this SSE is considerably smaller than that of the two fitted curves of the form $C = as^b$. Note that for the form $C = as^b$, the SSE in the curve found by Curve Fit 0.7e is less (19.8 compared to 25.78) than that in the curve found by Cricket Graph. (In these two cases, the objective functions were different, namely

$$\text{Minimize} \sum_{i=1}^{8} \left(\ln C_i - \left[\ln a + b \times \ln s_i\right]\right)^2 \text{ in the case of Cricket Graph,}$$

$$\text{Minimize} \sum_{i=1}^{8} \left(C_i - a s_i^b\right)^2 \text{ in the case of Curve Fit 0.7e.}$$

••••••••••••••••••••••••••• Homework # 6 ••••••••••••••••••••••••••••••

Reasoning that the failure time of a mechanical device is the minimum of the failure times of its individual elements (all nonnegative random variables), we will therefore assume that the failure time of the device has approximately a Weibull distribution. Suppose that 500 units of this device are operated simultaneously for 280 days, at which time 100 have failed, with the failure times:

23.585	43.639	55.393	83.733	96.471	99.379	102.81	111.04	118.25	118.38
119.35	121.04	123.12	132.9	134.05	134.98	141.16	144.6	144.93	147.61
148.41	150.33	152.3	155.09	155.9	157.25	158.73	162.14	162.78	163.68
171.49	172.19	172.81	173.62	174.02	175.25	179.03	184.85	185.7	190.47
192.28	193.93	195.73	196.12	196.81	197.58	198.3	206	206.2	208.76
211.4	212.86	212.95	213.53	213.72	214.08	219.81	221.37	221.63	225.57
225.94	228.11	230.14	230.51	231.86	233.59	233.98	234.18	234.29	238.37
238.54	238.84	242.49	244.23	246.23	246.92	248.19	248.75	249.05	250.08
250.4	250.9	251.04	251.11	252.55	252.57	252.9	253.98	257.93	260.59
261.85	262.93	263.53	265.73	266.77	267.64	269.72	272.86	274.08	277.21

To estimate the expected lifetime (and its standard deviation) by recording the failure times of all 500 units would require perhaps several years, an excessive amount of time. Hence, we wish to estimate the Weibull parameters u & k from only the data above, and use these to estimate μ and σ.

a. Group the observations as follows:

Interval	# failures	Cumulative # failures	% surviving
0- 50	2	2	99.6
50-100	4	6	98.8
100-150	15	21	95.8
150-200	26	47	90.6
200-250	32	79	84.2
250-280	21	100	80.0

b. Enter the observed values of t and R (failure time & reliability, i.e., the fraction surviving) into the Cricket Graph program.

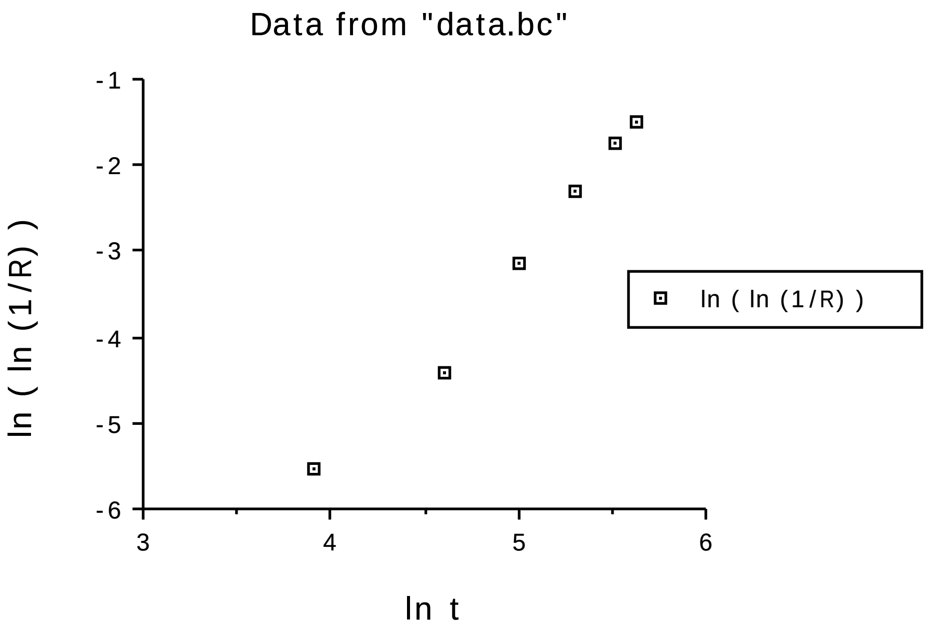
c. Using "transform" on the menu:
- compute ln t and place it into column 3 of the data matrix.
- compute R-1 and place it into column 4 of the data matrix.
- compute ln R-1 and place it into column 5 of the data matrix.
- compute ln (ln R-1) and place it into column 6 of the data matrix.

t	# of failures (= NF)	NS	FS	% Surviving (= % FS)
5.00E+01	2.00E+00	4.98E+02	9.96E-01	9.96E+01
1.00E+02	6.00E+00	4.94E+02	9.88E-01	9.88E+01
1.50E+02	2.10E+01	4.79E+02	9.58E-01	9.58E+01
2.00E+02	4.70E+01	4.53E+02	9.06E-01	9.06E+01
2.50E+02	7.90E+01	4.21E+02	8.42E-01	8.42E+01
2.80E+02	1.00E+02	4.00E+02	8.00E-01	8.00E+01

t	R	In t	1/R	In (1/R)	In (In(1/R))
5.00E+01	9.96E-01	3.91E+00	1.00E+00	4.01E-03	-5.52E+00
1.00E+02	9.88E-01	4.61E+00	1.01E+00	1.21E-02	-4.42E+00
1.50E+02	9.58E-01	5.01E+00	1.04E+00	4.29E-02	-3.15E+00
2.00E+02	9.06E-01	5.30E+00	1.10E+00	9.87E-02	-2.32E+00
2.50E+02	8.42E-01	5.52E+00	1.19E+00	1.72E-01	-1.76E+00
2.80E+02	8.00E-01	5.63E+00	1.25E+00	2.23E-01	-1.50E+00

d. Plot the scatter graph of the data, with ln t (column 3) on the horizontal axis, and ln (ln R-1) (column 6) on the vertical axis. Do the points appear to lie on a straight line?

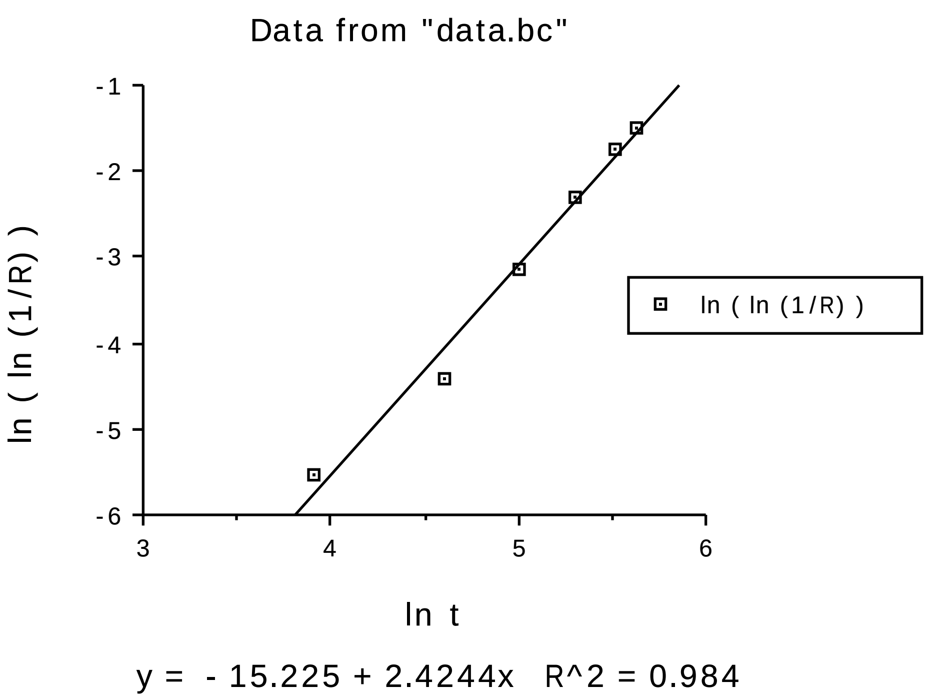
Yes, the points appear to lie on a straight line.



e. Fit a line to the points, using the Cricket Graph program. What is the equation of the line? What is its slope and y-intercept?

The Equation is " y = -15.225 + 2.4244 x "

with slope 2.4244 and y-intercept -15.225



f. Based upon the fitted line, what are the parameters u & k of the Weibull distribution? u = _______________ , k= _______________

k = slope = 2.4244 ≈ 2.4

y-intercept = -k In u = -15.225 In u = 6.2799 u = $e^{6.2799}$ = 533.7353

g. According to these values of u & k, what is
... the expected lifetime of the device? 473.1563 days
... the standard deviation of the device's lifetime? 210.0341 days
... the probability that the device will fail during the first 400 hours is 0.3937

Soln: $\mu_Y = u\,\Gamma\left(1+\frac{1}{k}\right)$ = (533.7353)(0.8865) = 473.1563 = expected lifetime

$$\frac{\sigma_Y}{\mu_Y} = \sqrt{\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1} \qquad \frac{\sigma_Y}{473.1563} = \sqrt{\frac{0.9407}{(0.8865)^2} - 1} = 0.4439 \qquad \sigma_Y = 210.0341$$

Note: The above evaluations of the Gamma function were done using the tables which were e-mailed to the class (& put onto the web site).

F(400) = P{t ≤ 400} = $1 - e^{-\left(\frac{400}{533.7353}\right)^{2.4}}$ = 0.3937 39.37 %

or using the "ProbLib" APL workspace on the macintosh, we can find that the expected lifetime of the device is 473.1468,
the standard deviation of the device's lifetime is 209.9998, and
the probability that the device will fail during the first 400 hours is 0.3937

h. Using the parameters u & k, compute the expected number of failures (of the 100 units tested) in each of the intervals:

Interval	Observed # failures	Expected # failures
0- 50	___2___	__1.7__
50-100	___4___	__7.2__
100-150	__15___	_14.3__
150-200	__26____	_22.0__
200-250	__32____	_29.55_
250-280	__21____	_21.0__

Note: The Weibull cumulative distribution function (CDF) may be evaluated using the "ProbLib" APL workspace on the Macintosh, and used to calculate the expected # of failures in the table above.

Soln: $p_i = F(t_i) - F(t_{i-1}) \quad i \geq 1$, where $F(t) = \left(1 - e^{-(t/u)^k}\right)$. Thus,

$p_1 = F(t_1) - F(t_0)$ = F(50) - F(0) = $\left(1 - e^{-(50/u)^k}\right) - \left(1 - e^{-(0/u)^k}\right)$ = 0.0034

$E_1 = 500p_1$.= 1.7

$p_2 = F(t_2) - F(t_1)$ = F(100) - F(50) = $\left(1 - e^{-(100/u)^k}\right) - \left(1 - e^{-(50/u)^k}\right)$ = 0.0144

$E_1 = 500p_1$.= 7.2

and so on ...

Following this procedure, we may obtain the following table:

Interval	Pi	Oi	Ei	$\frac{(O_i - E_i)^2}{E_i}$
0 - 50	0.0034	2	1.7	0.05294118
50-100	0.0144	4	7.2	1.42222222
100-150	0.0286	15	14.3	0.03426573
150-200	0.0440	26	22.0	0.72727273
200-250	0.0591	32	29.55	0.20313029
250-280	0.0420	21	21.0	0

D = 2.43983215

i. Perform a Chi-Square Goodness of Fit test to determine whether this distribution provides an acceptable fit to the observed data, with a = 5%.

Soln: Since (from above) " D (2.4398) < $\chi^2_{5\%}$ (= 7.815) ", we can conclude that this distribution provides an acceptable fit to the observed data with α =5%.

j. According to these estimates of u & k, at what time should
... 5% of the units have failed? ___156.77__ hours
... 50% of the units? ___458.85__ hours
... 90% of the units? ___752.87__ hours

Soln: Exact estimates require solving the following equation for t:

$F(t) = 1 - e^{-\left(t/u\right)^k}$ where F(t) is 5%, 50%, and 90%, respectively. The inverse of F(t) was derived in the notes: $F^{-1}(p) = u\left(-\ln\left[1 - p\right]\right)^{1/k}$, where k = 2.4244, u= 533.7353

Thus we obtain

$$F^{-1}(0.05) = 533.7353\left(-\ln 0.95\right)^{1/2.4244} = 156.77 \text{ hours}$$

$$F^{-1}(0.5) = 533.7353\left(-\ln 0.5\right)^{1/2.4244} = 458.85 \text{ hours}$$

$$F^{-1}(0.9) = 533.7353\left(-\ln 0.1\right)^{1/2.4244} = 752.87 \text{ hours}$$

k. The manufacturer wishes to state a warranty period such that 99% of the units produced are expected to survive the warranty period. According to these estimates of u & k, what should be the length of the warranty period ?

Soln: Again, we solve $F^{-1}(p) = u\left(-\ln\left[1 - p\right]\right)^{1/k}$ where u & k are as above, and p = 1%. This gives us a warranty period of 80 days (more exactly, 80.0352 days)

l. According to the value of k, is the failure rate increasing or decreasing with time? ________________

Soln: k is greater than 1, which indicates that the failure rate is increasing with time.

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Use the Curve Fit 0.7e software to try fitting a Weibull distribution in its original (nonlinear) form. If x = upper limit of the intervals and y = fraction failed, then you should enter the function f(x) = 1 - 2.718281828^(-(x/a)^b)
where a=u and b=k. (Note that Curve Fit allows only variables a, b, c, d, & e.)

m. Using as starting values for a & b the values which you found in (f), try fitting the curve to the data. What are the values of a & b?

a = _______________ +/- __________________

b = _______________ +/- __________________

Was Curve Fit able to find values with small error estimates (e.g. with error less than about 5 or 10% of the parameter)? ________

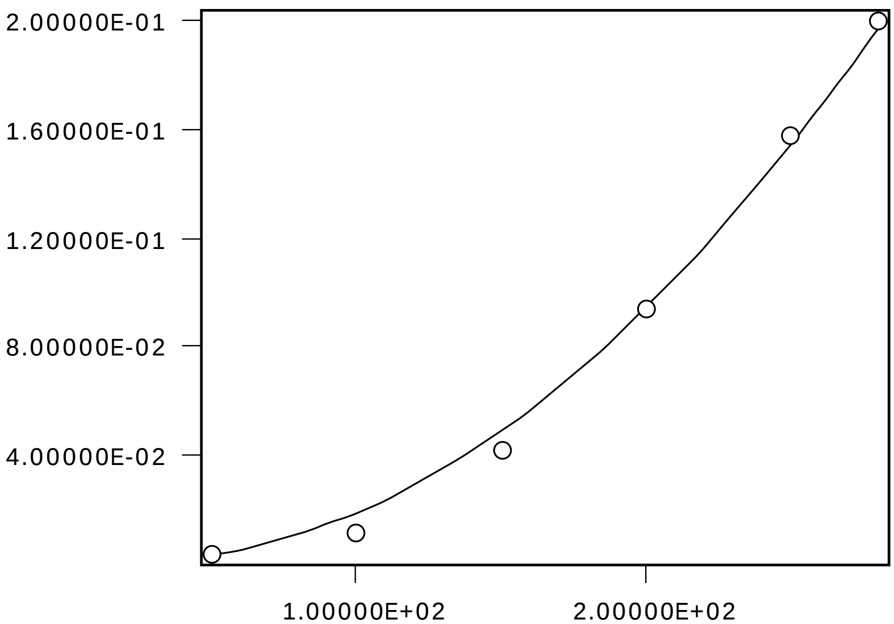
If yes, proceed to question (n); otherwise, stop.

Soln: According to the Curve Fit output, the parameters are

a = 533.73486 +/- 23.74058

b = 2.35061 +/- 0.13497

Yes, the Curve fit can find values with small error estimates.



Corr. Coeff. (R2) = 0.99607

Sum of Squares = 0.00013

n. What are the values of the parameters for this new fitted distribution?

u = 534 , k = 2.35

o. According to these values of u & k, what is

... the expected lifetime of the device? 473.2137

... the standard deviation of the device's lifetime? 214.0274

... the probability that the device will fail during the first 400 hours? 0.3978

Soln: See solution of (g) above.

p. Using these new parameters u & k, compute the expected number of failures (of the 100 units tested) in each of the intervals:

Interval	Observed # failures	Expected # failures
0- 50	________	_______
50-100	________	_______
100-150	________	_______

150-200	________	_______
200-250	________	_______
250-280	________	_______

Soln: $p_i = F(t_i) - F(t_{i-1}) \quad i \geq 1$, where $F(t) = \left(1 - e^{-(t/u)^k}\right)$. Thus,

$p_1 = F(t_1) - F(t_0)$ = F(50) - F(0) = $\left(1 - e^{-(50/u)^k}\right) - \left(1 - e^{-(0/u)^k}\right)$ = 0.0038

$E_1 = 500 p_1$ = 1.90

$p_2 = F(t_2) - F(t_1)$ = F(100) - F(50) = $\left(1 - e^{-(100/u)^k}\right) - \left(1 - e^{-(50/u)^k}\right)$ = 0.0155

$E_1 = 500 p_1$ = 7.75

and so on ...

Following this procedure, we may obtain the following table:

Interval	Pi	Oi	Ei	$\frac{(O_i - E_i)^2}{E_i}$
0 - 50	0.0038	2	1.90	0.00526316
50-100	0.0155	4	7.75	1.81451613
100-150	0.0300	15	15.00	0
150-200	0.0454	26	22.70	0.47973568
200-250	0.0600	32	30.00	0.13333333
250-280	0.0423	21	21.15	0.00106383

D = 2.43391213

q. According to these new estimates of u & k, at what time should
... 5% of the units have failed? 156.84 days
... 50% of the units? 459.09 days
... 90% of the units? 753.26 days

Soln: See solution of (j) above.

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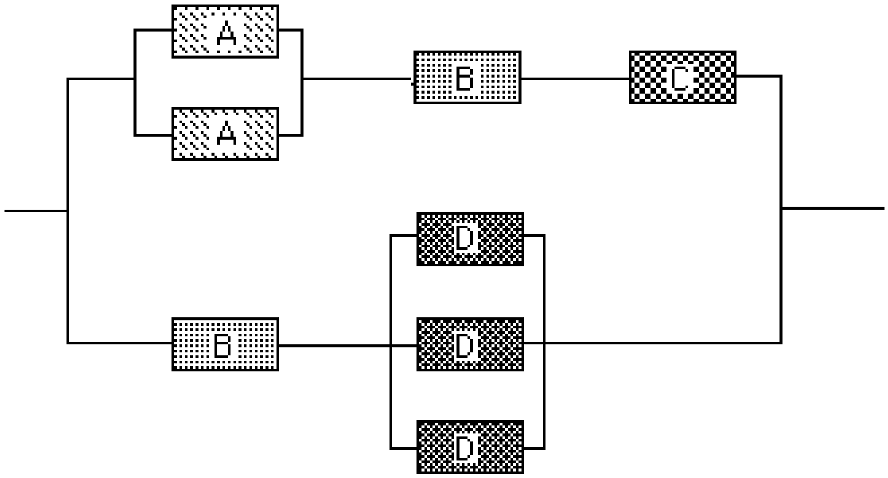
r. You needn't perform a "goodness-of-fit" test to confirm it, but state your opinion about which Weibull distribution seems to be a better fit.

Soln: If we were to perform the "goodness-of-fit" test, then since " D (=2.4339) $< \chi^2_{5\%}$ (= 7.815) ", we would conclude that this distribution also provides an acceptable fit to the observed data with α =5%. The results are so similar that there is probably no significant difference in the qualities of the fits. If each program found the exact optimum of its objective function, then clearly the result of Curve Fit would be a better fit. However, because Curve Fit is doing computation with nonlinear functions while Cricket Graph is using linear functions, the latter will perform the computations more accurately.

•••••••••••••••••••••••••••• Homework # 7 ••••••••••••••••••••••••••••••

1. A system contains 4 types of devices, with the system reliability represented schematically by

Solutions



It has been estimated that the lifetime probability distributions of the devices are as follows:

A: Weibull, with mean 1000 days and standard deviation 1200 days
B: Exponential, with mean 4000 days
C: Normal, with mean 3000 days and standard deviation 750 days
D: Exponential, with mean 1000 days

a.) Compute the reliabity of a unit of each device for a designed system lifetime of 1000 days:

Sol'n:

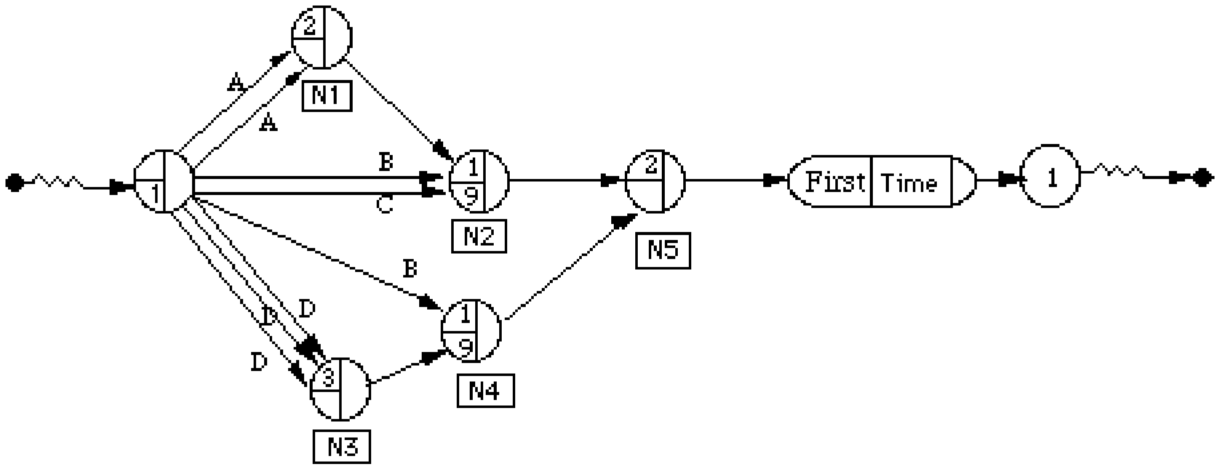
Device	Reliability R(1000)	
A	33.91%	$R_A(1000) = 1 - F_A(1000) = e^{-\left(\frac{1000}{910.6269}\right)^{0.8376}}$
B	77.88%	$R_B(1000) = 1 - F_B(1000) = e^{-\frac{1000}{4000}}$
C	99.62%	From CDF table of Normal Distribution
D	36.79%	$R_D(1000) = 1 - F_D(1000) = e^{-\frac{1000}{1000}}$

b.) Using the reliabilities in (a), compute the system reliability:

Sol'n:

Subsystem	Reliability R(1000)	
AA	0.5632 %	Since $1-(1-R_A)^2 = 1-(1-0.3391)^2$
AA+B+C	0.4370 %	$R_{AA} \times R_B \times R_C$
B+DDD	0.5821 %	$R_B \times R_{DDD} = 0.7788 \times \left\{1-\left(1-0.3679\right)^3\right\}$
Total system:	0.7647 %	$1-(1-R_{AA} \times R_B \times R_C)(1-R_B \times R_{DDD})$

c.) Draw a SLAM network which can simulate the lifetime of this system. (You need not perform the simulation.)



Notes:

- The CREATE node indicates that only 1 entity is initially created.
- "A", "B", "C", and "D" above should be replaced by the lifetime distribution of the corresponding devices, i.e., WEIBL(,), EXPON(4000), RNORM(3000,750), and EXPON(1000), respectively.
- The parameters of the Weibull distribution as defined by SLAM are named ALPHA and BETA, and are not (μ,σ), the mean & standard deviation. ALPHA is identical to the parameter k in the class notes, and BETA is u^k. To determine u and k from the mean and standard deviation, one may roughly estimate k given the coefficient of variation σ/μ =1.2 from the table below (taken from the class notes):

k	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	——	——	15.8	5.41	3.14	2.24	1.76	1.46	1.26	1.11
1	1	0.91	0.837	0.776	0.724	0.679	0.64	0.605	0.575	0.547
2	0.523	0.5	0.48	0.461	0.444	0.428	0.413	0.399	0.387	0.375
3	0.363	0.353	0.343	0.334	0.325	0.316	0.309	0.301	0.294	0.287
4	0.281	0.274	0.268	0.263	0.257	0.252	0.247	0.242	0.238	0.233
5	0.229	0.225	0.221	0.217	0.213	0.21	0.206	0.203	0.2	0.197
6	0.194	0.191	0.188	0.185	0.183	0.18	0.177	0.175	0.173	0.17
7	0.168	0.166	0.164	0.162	0.16	0.158	0.156	0.154	0.152	0.15
8	0.148	0.147	0.145	0.143	0.142	0.14	0.139	0.137	0.136	0.134
9	0.133	0.131	0.13	0.129	0.128	0.126	0.125	0.124	0.123	0.121

Coefficient of Variation $\frac{\sigma}{\mu}$

The value of k is between 0.8 and 0.9; interpolating would give k=0.8 + .1(0.06/0.15) = 0.84. Next, the parameter u may be found from

$$\mu = u\,\Gamma\left(1+\frac{1}{k}\right) \qquad u = \frac{\mu}{\Gamma\left(1+\frac{1}{k}\right)} = \frac{1000}{\Gamma\left(1+\frac{1}{0.84}\right)} \approx \frac{1000}{1.1} = 909.09$$

where the Gamma function is evaluated by interpolating in the table below (also found in the notes):

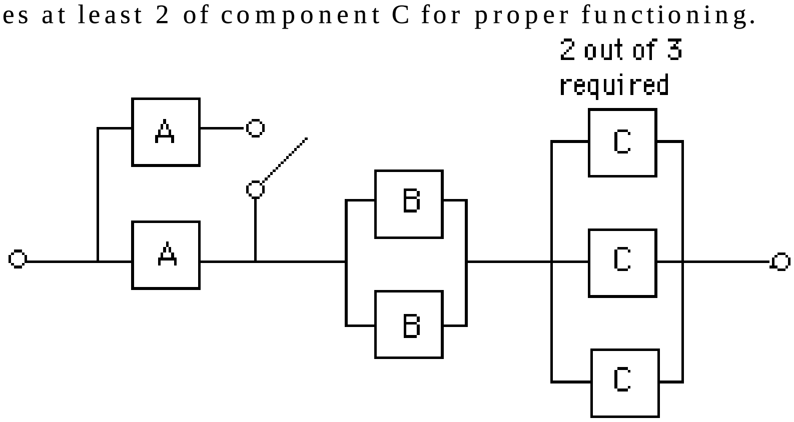
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	362880	120.0000	9.2605	3.3234	2.0000	1.5046	1.2658	1.1330	1.0522
1	1.0000	0.9649	0.9407	0.9236	0.9114	0.9027	0.8966	0.8922	0.8893	0.8874
2	0.8862	0.8857	0.8856	0.8859	0.8865	0.8873	0.8882	0.8893	0.8905	0.8917
3	0.8930	0.8943	0.8957	0.8970	0.8984	0.8997	0.9011	0.9025	0.9038	0.9051
4	0.9064	0.9077	0.9089	0.9102	0.9114	0.9126	0.9137	0.9149	0.9160	0.9171
5	0.9182	0.9192	0.9202	0.9213	0.9222	0.9232	0.9241	0.9251	0.9260	0.9269
6	0.9277	0.9286	0.9294	0.9302	0.9310	0.9318	0.9325	0.9333	0.9340	0.9347
7	0.9354	0.9361	0.9368	0.9375	0.9381	0.9387	0.9394	0.9400	0.9406	0.9412
8	0.9417	0.9423	0.9429	0.9434	0.9439	0.9445	0.9450	0.9455	0.9460	0.9465
9	0.9470	0.9474	0.9479	0.9484	0.9488	0.9492	0.9497	0.9501	0.9505	0.9509

Values of $\Gamma\left(1+\frac{1}{k}\right)$ for k=0.1 through 9.9

Thus, “A” in the diagram above should be WEIBL(0.84, 909).

- The numbers in the ACCUMULATE nodes are: FR = number of arriving entities required to cause the first entity to be “released” from the node; SR = number of arriving entities required for subsequent entities to be “released” from the node.
- In the ACCUMULATE nodes N1, N3, and N5, the value of SR is irrelevant, since FR is assigned a value equal to the total number of arrivals at the node, and no subsequent entities will ever be released from the node. (The default value will be 1.)
- In the ACCUMULATE nodes N2 and N4, additional entities may arrive after the first entity is released from the node, and therefore a value of SR (larger than the maximum number of entities which might arrive) must be specified. In the diagram above, 9 was specified, but any larger and some smaller values are possible.
- The COLCT (COLLECT) node will collect statistics on the time of the FIRST arrival; in this case, the “1” on the TERMINATE node cause the simulation to then end, and so no further arrivals can occur. For this reason, it would be equivalent to specify INTVL(1) instead of FIRST if you also indicate on the CREATE statement that the creation time (0 in this case) is to be recorded in attribute #1 (i.e., the parameter MA = 1).

2. A system has 3 types of components (A, B, & C) which are subject to failure. As shown below, the system requires at least one of components A and B, and at least two of component C. One of component A is in “stand-by”. That is, when the first component A has failed, the second is to be switched on, and until then does not begin to "age" or fail. Assume that the sensor/switch has 95% reliability. In the case of components B & C, on the other hand, all units of these components are in operation simultaneously, so that each unit is subject to failure from the beginning of the system’s operation. Note that the system requires at least 2 of component C for proper functioning.

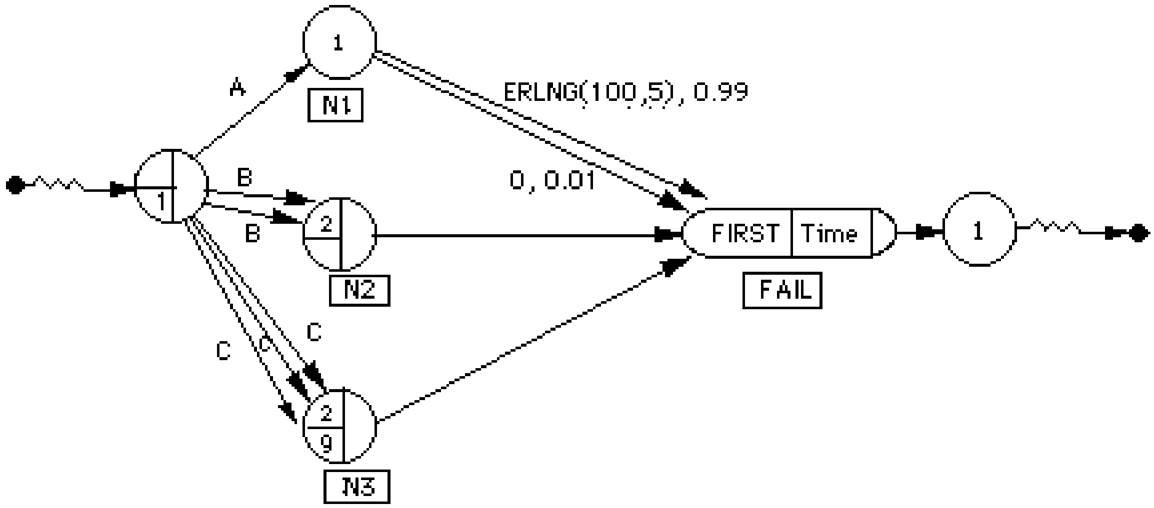


The lifetime distributions of the three component types are:

Component A: Erlang, being the sum of five random variables, each having exponential distribution with mean 100 days.

Component B: Exponential, with expected lifetime 200 days.
Component C: Exponential, with expected lifetime 300 days.

a. Draw a SLAM network which can simulate the lifetime of this system.
Sol'n:



Notes:

- Again, the values "A", "B", and "C" on the diagram above should be replaced by the lifetime distributions, namely ERLNG(100,5), EXPON(200), and EXPON(300), respectively.
- The parameters of the Erlang distribution in SLAM are not the mean and standard deviation (which would be in this case 500 and $\sqrt{5\times 100^2} = 100\sqrt{5} = 223.61$), but the mean of each of the component exponential distributions and the number of such distributions.
- The "1" in the GOON node (labelled N1) indicates that the entity which arrives there (at the time of the failure of the primary unit of device A) must leave node N1 via a single activity only.
- The two activities leaving the GOON node N1 represent (1) the lifetime of the standby unit of device A, which receives the entity with 99% probability, and (2) the failure of the switch, requiring time = 0, which receives the entity with probability 1%.
- See the note in problem 1 about the values of the parameters FR and SR of the ACCUMULATE nodes.
- See the note in problem 1 about the FIRST statistics collected by the COLCT node.

b. Enter the network into the computer, and simulate the system 1000 times, collecting statistics on the time of system failure. Request that a histogram be printed. Specify about 15-20 cells, with HLOW and HWID parameters which will give you a "nice" histogram with the mean approximately in the center and with small tails. (This may require a second simulation, with the histogram parameters selected after observing the results of a preliminary simulation.)

Note: Be sure to specify on the GEN statement that you do NOT want intermediate results, and that the SUMMARY report is to be printed only after the 1000th run. Also specify on the INITIALIZE statement that the statistical arrays should not be cleared between runs. The following should work:

```
GEN,yourname,RELIABILITY,10/16/96,1000,,N,,N,Y/1000,72;
LIM,,,8;
INIT,,,NO;
NETWORK;
```

(SLAM II network statements go here)
END;
FIN;

Sol'n: The (edited) SLAM output appears below:

```
GEN,Hansuk Sohn,Reliability,10/9/1996,1000,,N,,N,Y/1000,72;
LIM,,,8;
INIT,,,NO;
NETWORK;
      CREATE;
      ACT/1,ERLNG(100,5),,Na;  Component A
      ACT/2,EXPON(200),,Nb;    Component B
      ACT/3,EXPON(200),,Nb;    Component B
      ACT/4,EXPON(300),,Nc;    Component C
      ACT/5,EXPON(300),,Nc;    Component C
      ACT/6,EXPON(300),,Nc;    Component C
Na    GOON,1;
      ACT,ERLNG(100,5),0.95,FAIL;
      ACT,0,0.05,FAIL;
Nb    ACCUM,2;    Node "b" will be released when both Bs fail.
      ACT,,,FAIL;
Nc    ACCUM,2;    Node "c" will be released when 2 out of 3 Cs fail.
      ACT,,,FAIL;
FAIL  COLCT,FIRST,TIME_OF_FAILURE,20/0/20;
      TERM,1;
      END;
FIN;
```

```
                  S L A M   I I   S U M M A R Y   R E P O R T

            SIMULATION PROJECT RELIABILITY       BY HANSUK SOHN
       DATE 10/ 9/1996                       RUN NUMBER 1000 OF 1000

                          CURRENT TIME    .1267E+03
  STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00


              **STATISTICS FOR VARIABLES BASED ON OBSERVATION**
                 MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                 VALUE   DEVIATION VARIATION   VALUE     VALUE     OBS
TIME_OF_FAILURE .174E+03  .116E+03  .669E+00  .224E+01  .666E+03   1000

            **REGULAR ACTIVITY STATISTICS**
 ACTIVITY          AVERAGE      STANDARD     MAXIMUM CURRENT   ENTITY
 INDEX/LABEL       UTILIZATION  DEVIATION    UTIL    UTIL      COUNT
    1   COMPONENT A  .1315        .3380        1        1        0
    2   COMPONENT B  .4178        .4932        1        1        0
    3   COMPONENT B  .3887        .4874        1        0        1
    4   COMPONENT C  .3057        .4607        1        1        0
    5   COMPONENT C  .3012        .4588        1        0        1
    6   COMPONENT C  .3357        .4722        1        0        1

                           **HISTOGRAM NUMBER 1**
                              TIME_OF_FAILURE

 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0        20        40        60        80       100
                     +    +    +    +    +    +    +    +    +    +    +
   0  .000  .000E+00 +                                                 +
  22  .022  .200E+02 +*                                                +
```

```
  53  .053  .400E+02 +***C                                             +
  73  .073  .600E+02 +****  C                                          +
  75  .075  .800E+02 +****      C                                      +
  85  .085  .100E+03 +****          C                                  +
  84  .084  .120E+03 +****               C                             +
  77  .077  .140E+03 +****                  C                          +
  70  .070  .160E+03 +****                      C                      +
  62  .062  .180E+03 +***                          C                   +
  58  .058  .200E+03 +***                             C                +
  41  .041  .220E+03 +**                                C              +
  53  .053  .240E+03 +***                                  C           +
  51  .051  .260E+03 +***                                    C         +
  35  .035  .280E+03 +**                                       C       +
  25  .025  .300E+03 +*                                         C      +
  24  .024  .320E+03 +*                                          C     +
  22  .022  .340E+03 +*                                           C    +
  15  .015  .360E+03 +*                                            C   +
  12  .012  .380E+03 +*                                             C  +
  11  .011  .400E+03 +*                                             C  +
  52  .052    INF    +***                                              C
 ---                 +    +    +    +    +    +    +    +    +    +    +
 ***                 0         20        40        60        80      100

                 **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                   MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                   VALUE   DEVIATION VARIATION   VALUE      VALUE    OBS
TIME_OF_FAILURE  .174E+03  .116E+03  .669E+00  .224E+01  .666E+03  1000
```

c. What is your estimate of the average lifetime of this system, based upon the simulation results?

Sol'n: 174 days (from SLAM output)

d. Suppose that the system is required to survive for a 100-day mission. What is the estimated reliability of the system, i.e., the probability that the system survives 100 days?

Sol'n: From the histogram above, we see that 308 (the sum of the first six cells) failures occurred during the first 100 days (and 1000-308=692 survived at least 100 days). Hence,

$$R(100) = 1-\frac{308}{1000} = 0.692$$

e. Suppose your company will offer a warranty on this system, specifying the length of the warranty period such that 98% of the systems will survive past the warranty period. What should be the length of the warranty?

Sol'n: The above histogram doesn't provide sufficient detail at the low end to allow an accurate estimate, but if we use a linear interpolation we would estimate that the 20th failure occurred at time 20(20/22) = 18.2 days, leaving at that time 980 survivors (98% of the systems simulated.) Therefore, according to this estimate, 18.2 days should be the length of the warranty (which undoubtedly would be rounded to 18 days for the sake of convenience). Interpolating again to find the reliability of the system for an 18-day lifetime, we would estimate that 22(18/20) = 19.8 systems, or 1.98% of the total, failed during the first 18 days, leaving 98.02% surviving.

To get a more accurate estimate of the correct warranty period, we should

- increase the number of runs from 1000, e.g., to as many as 10000
- change the specifications for the histogram so as to get a count of failures each day, e.g.,

```
FAIL  COLCT,FIRST,TIME_OF_FAILURE,30/0/1;
```

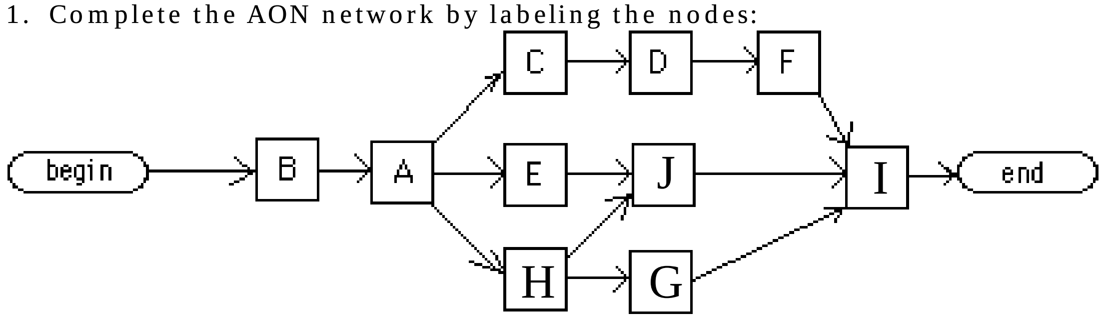
which would give 30 cells, each of length 1 day.

••••••••••••••••••••••••••• Homework # 8 ••••••••••••••••••••••••••••••

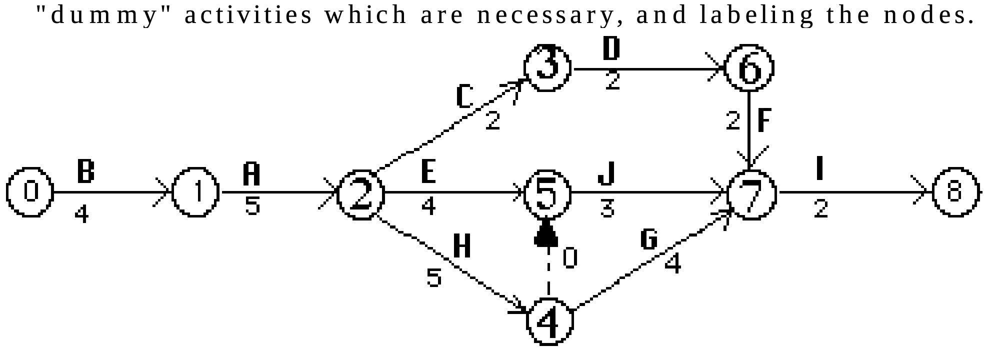
1. Project Scheduling. Consider the home-building project:

Activity	Description	Predecessor Activities	Duration (days) Mean	Std Dev
A	Walls & ceiling	B	5	2
B	Foundation	none	4	1
C	Roof timbers	A	2	1
D	Roof sheathing	C	2	1
E	Electrical wiring	A	4	2
F	Roof shingles	D	2	1
G	Exterior siding	H	5	1
H	Windows	A	4	1
I	Paint	F,G,J	3	1
J	Inside wall board	E,H	3	1

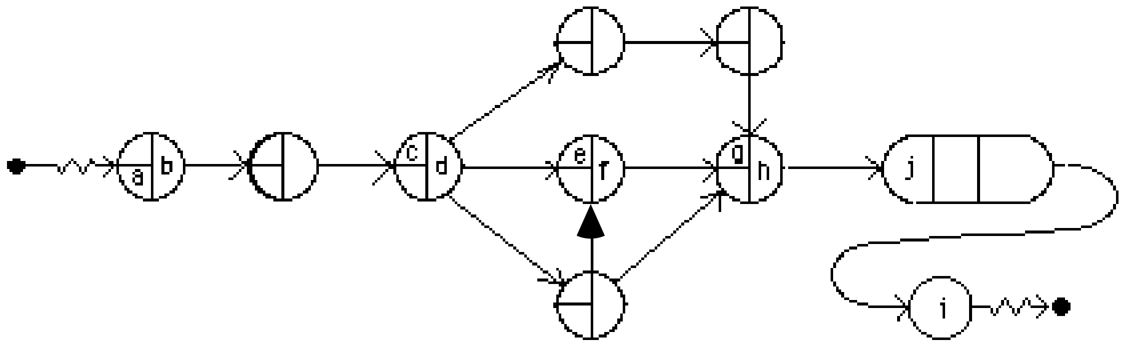
1. Complete the AON network by labeling the nodes:



2. Complete the AOA & the corresponding SLAM networks below by inserting any "dummy" activities which are necessary, and labeling the nodes.

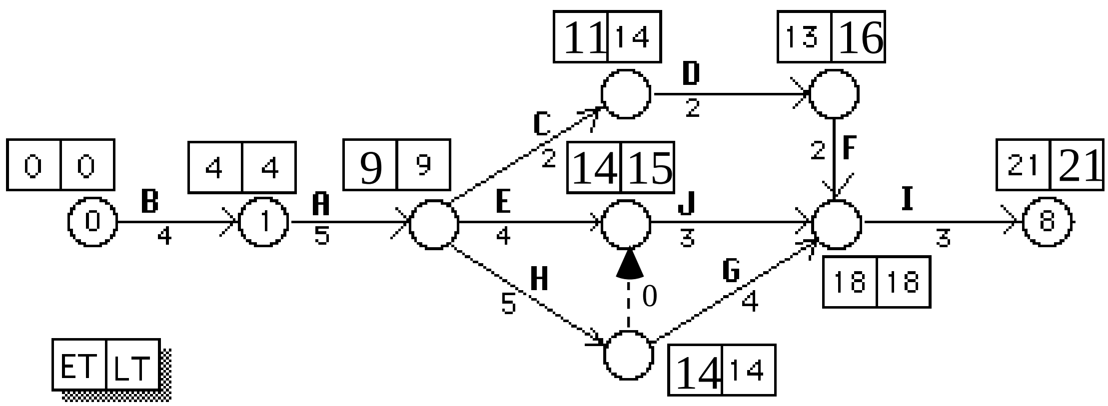


3. Give numerical values (0, 1, 2, 3, 4, or ∞) for parameters "a" - "i" and a statistic type for “j” on the SLAM network below which would simulate the project and collect statistics on the completion time. (You needn’t insert the probability distributions for the durations!)



a	b	c	d	e	f	g	h	i	j
1	1	1	3	2	1	3	1	1	FIRST

5. Complete the ETs (earliest times) & LTs (latest times) in the network below, using the expected activity durations, as indicated. Don't forget any "dummy" activities which you entered above!



6. What are the critical activities?
Soln : B - A - H - G - I

7. What is the "total slack" or "total float" in activity D? 3 days

8. What is the expected completion time of the project? 21 days
...the standard deviation of the project's completion time? 2.8284 days

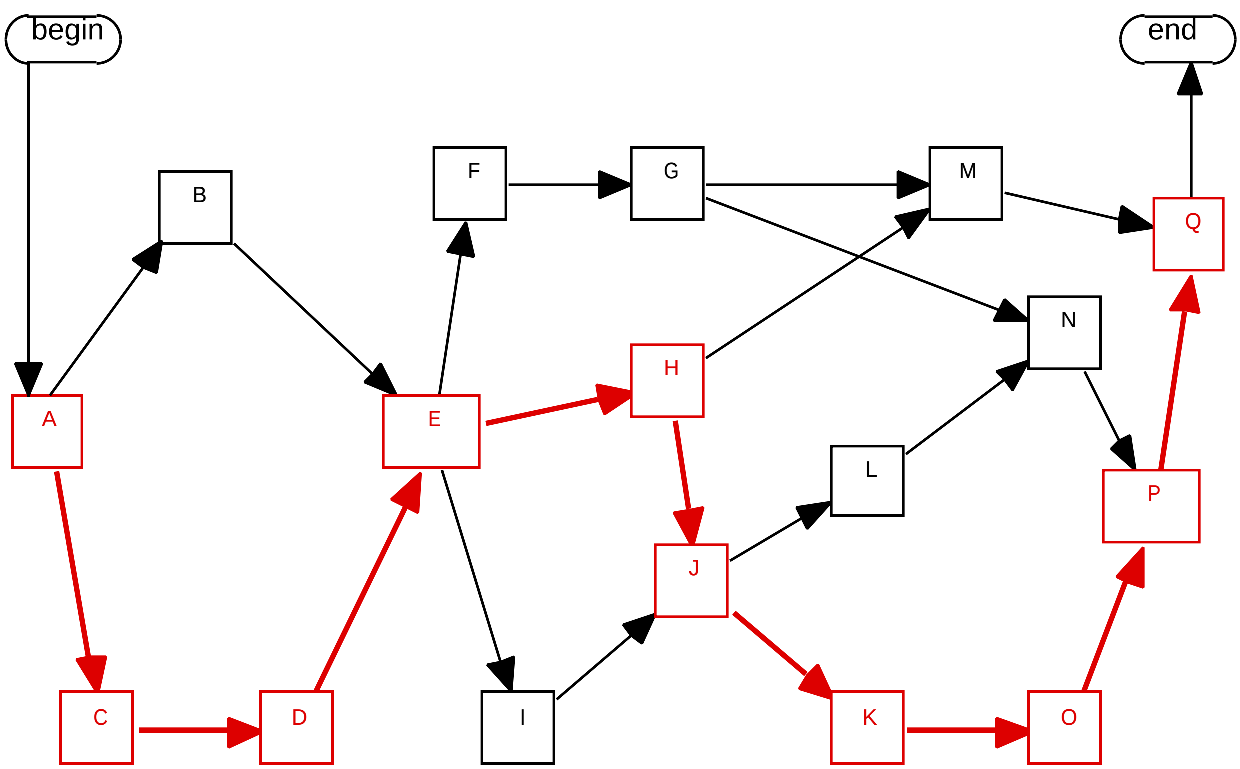
Activity	i	j	μ	σ^2
B	0	1	4	1
A	1	2	5	2^2
H	2	5	4	1
G	5	7	5	1
I	7	8	3	1
		sum	21	8

$$\sigma_T = \sqrt{8} = 2.8284$$

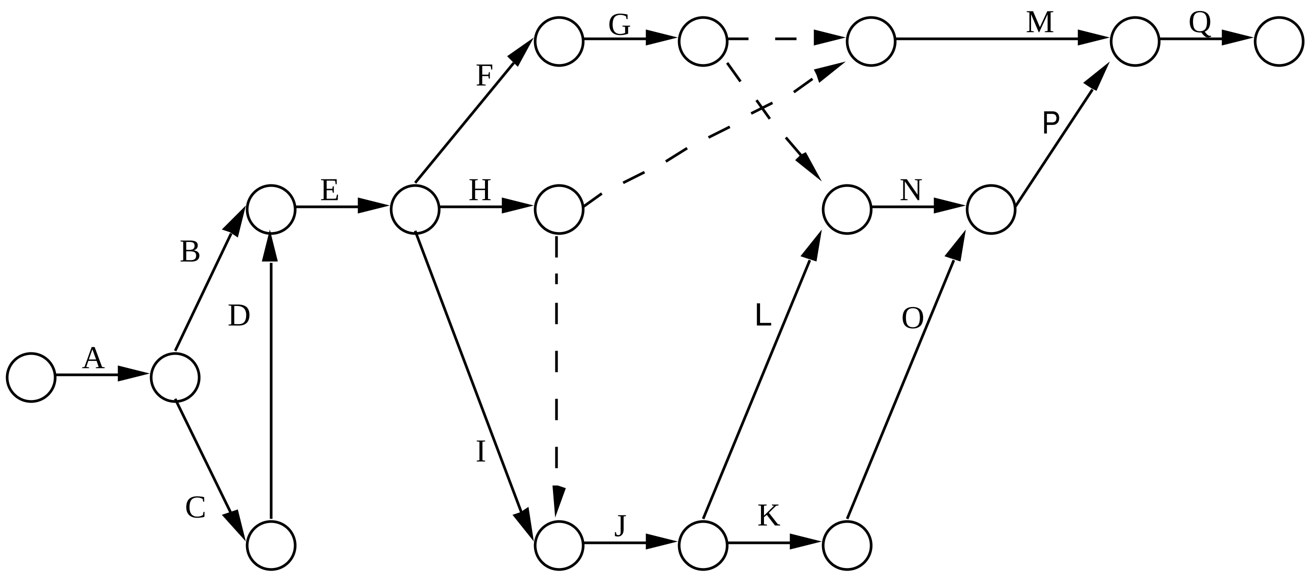
2. Building a Hydroelectric Power Station. Below is a rough breakdown of a building project into tasks or activities. (In practice, each of the tasks below, e.g. #11 “Dam building”, would itself be divided into many individual jobs.) The task descriptions and estimated durations (in days) are:

Activity	Description	Predecessor(s)	Duration
A	Ecological survey of dam site	none	137
B	File environmental impact report and get EPA approval	A	201
C	Economic feasibility study	A	161
D	Preliminary design & cost estimation	C	93
E	Project approval & commitment of funds	B,D	225
F	Call quotations for electrical equipment (turbines, generators, etc.)	E	95
G	Select suppliers for electrical equipment	F	69
H	Final design of project	E	143
I	Select construction contractors	E	60
J	Arrange construction mat’ls supply	H,I	115
K	Dam building	J	546
L	Power station building	J	405
M	Power lines erection	G,H	447
N	Electrical equipment installation	G,L	150
O	Build up reservoir water level	K	47
P	Commission the generators	N,O	27
Q	Start supplying power	M,P	25

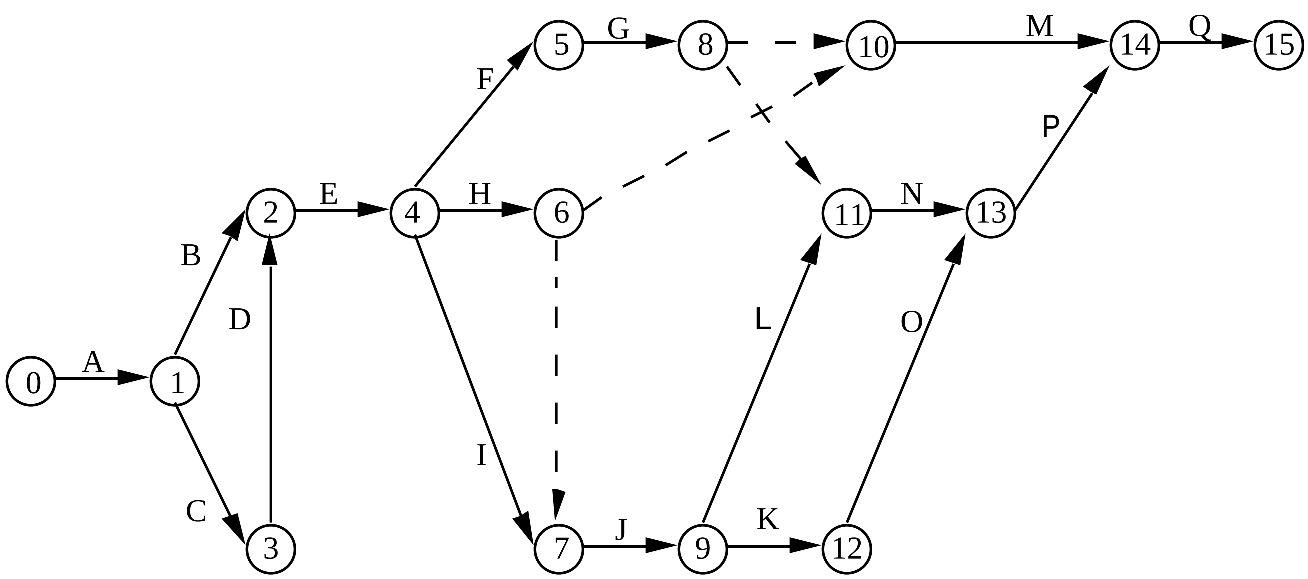
a. Draw the AON (activity-on-node) network representing this project.



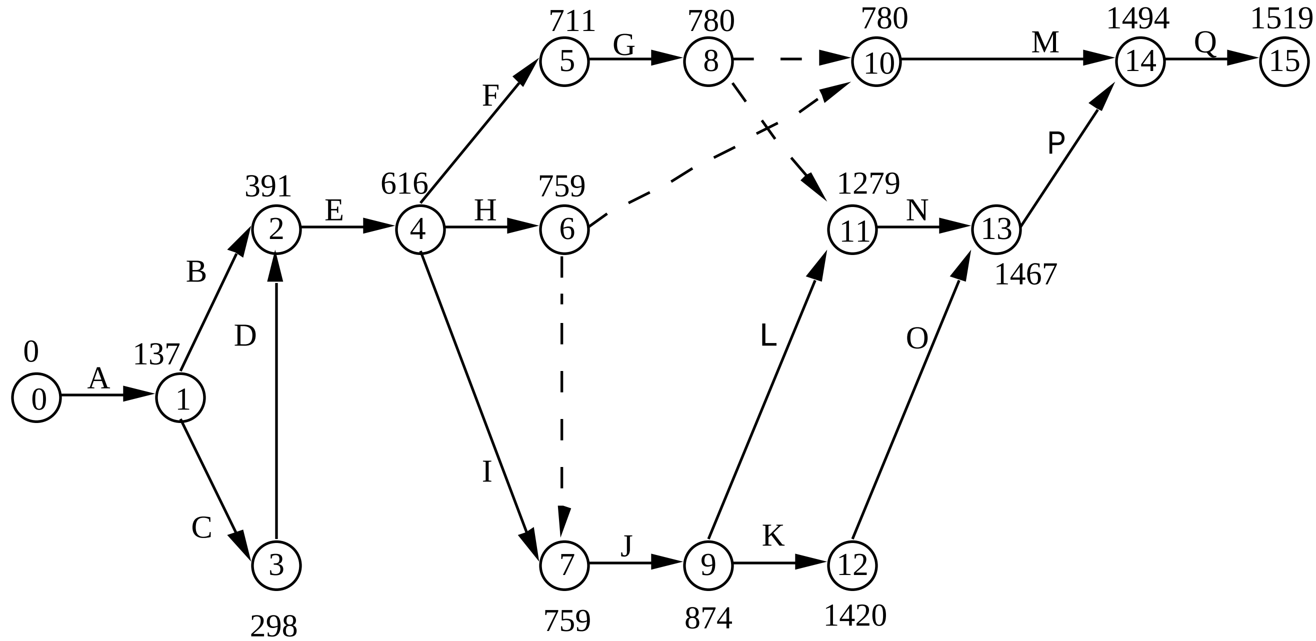
b. Draw the AOA (activity-on-arrow) network representing this project. Explain the necessity for any "dummy" activities which you have included.



c. Label the nodes of the AOA network, so that i<j if there is an activity with node i as its start and node j as its end node.

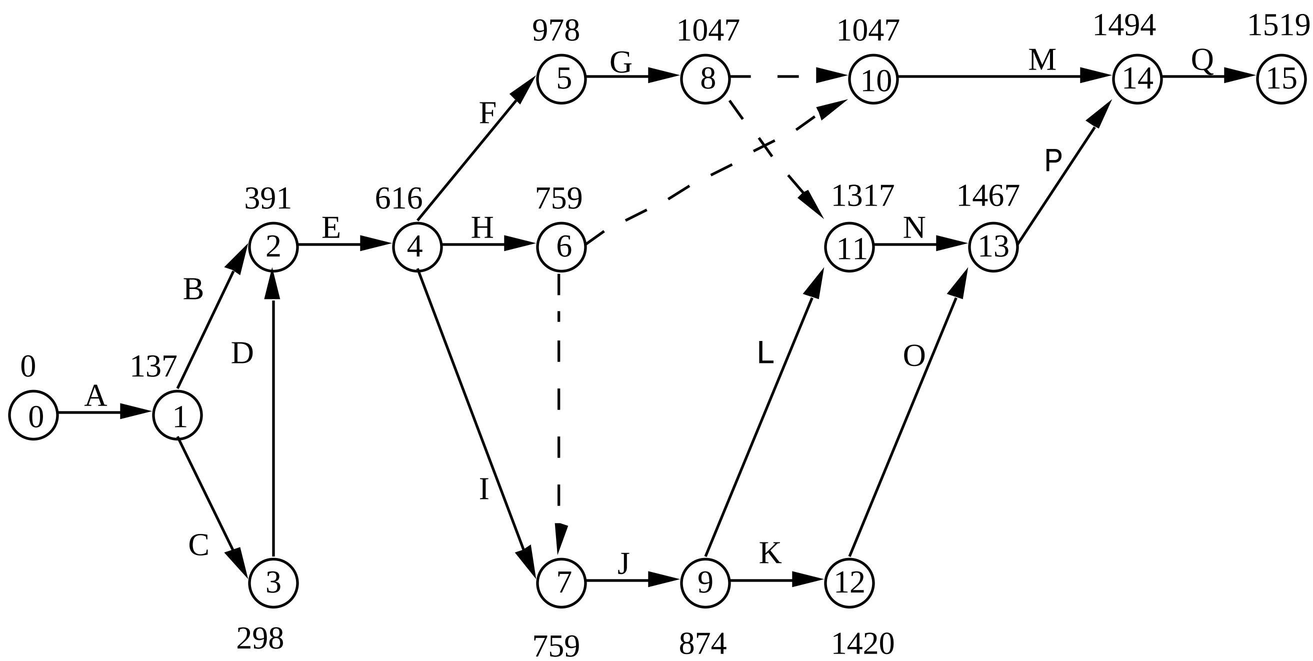


d. Perform the forward pass through the AOA network to obtain for each node i, ET(i) = earliest possible time for event i.



e. What is the earliest completion time for this project? <u>1519 days</u>

f. Perform the backward pass through the AOA network to obtain, for each node i, LT(i) = latest possible time for event i (assuming the project is to be completed in the time which you have specified in (e).)



g. For each activity, compute and record below:

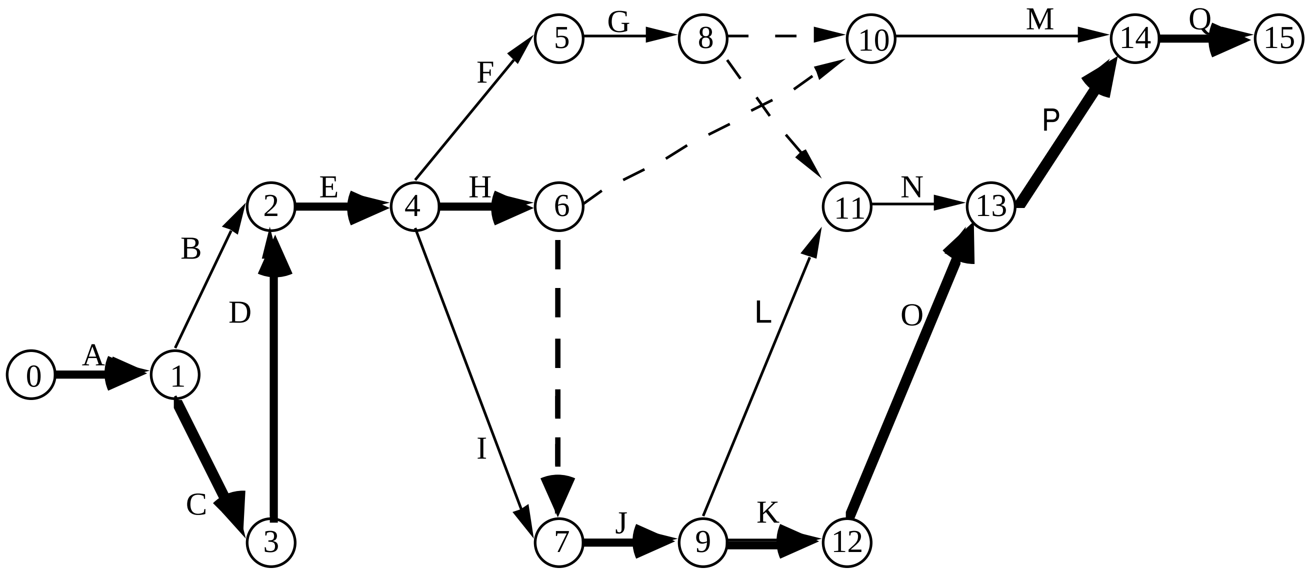
ES = earliest start time
LS = latest start time
EF = earliest finish time
LF = latest finish time
TF = total float (slack)

Sol'n:

Critical Activity	Activity	ES	LS	EF	LF	TF
Yes	A	0	0	137	137	0
No	B	137	190	338	391	53
Yes	C	137	137	298	298	0
Yes	D	298	298	391	391	0
Yes	E	391	391	616	616	0
No	F	616	883	711	978	267
No	G	711	978	780	1047	267
Yes	H	616	616	759	759	0
No	I	616	699	676	759	83
Yes	J	759	759	874	874	0
Yes	K	874	874	1420	1420	0
No	L	874	912	1279	1317	38
No	M	780	1047	1227	1494	267
No	N	1279	1317	1429	1467	38
Yes	O	1420	1420	1467	1467	0
Yes	P	1467	1467	1494	1494	0
Yes	Q	1494	1494	1519	1519	0

h. Which activities are "critical", i.e., have zero float? (Circle the labels A-Q of the critical activities above.) Indicate the critical path in your AOA network in part (b).

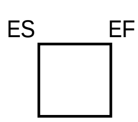
Soln : A - C - D - E - H - J - K - O - P - Q



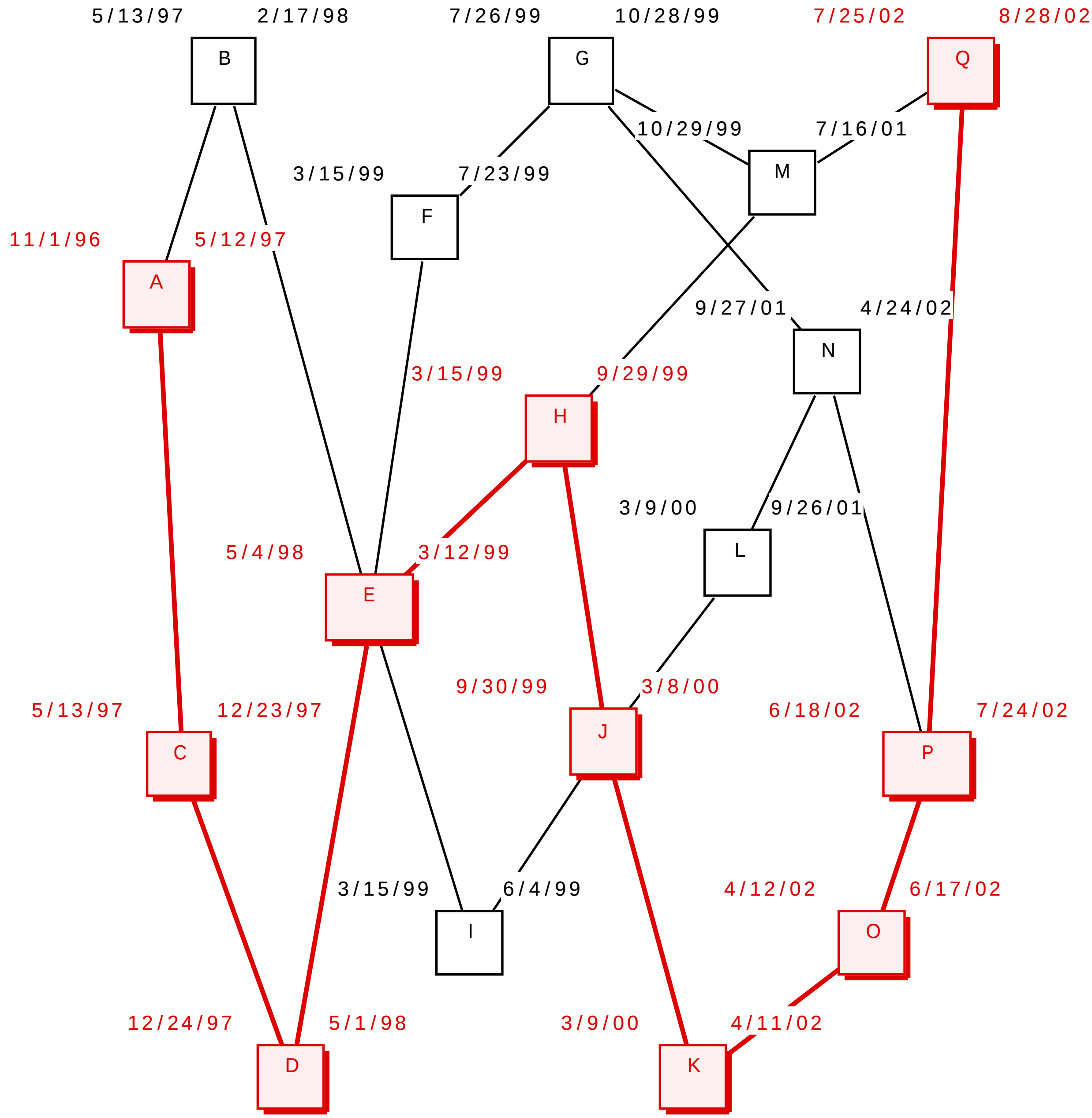
i. Schedule this project by entering the AON network into the MacProject PRO software Specify that the start time for the project will be November 1, 1996. What is the earliest completion date for the project? August 28, 2002

(Note that

- stated to the left and right of each node, i.e., acitivity, are the earliest start and latest finish dates, respectively.
- MacProject PRO assumes 5-day work weeks by default, so that the time between beginning & completion date isn't the same as your answer in (e).)



ES : Earliest Start Time
EF : Earliest Finish Time



•••••••••••••••••••••••••••• Homework # 9 •••••••••••••••••••••••••••••••

Building a Hydroelectric Power Station. Below is a rough breakdown of a building project into tasks or activities. (In practice, each of the tasks below, e.g. K (#11) "Dam building", would itself be divided into many individual jobs.) The task descriptions and estimated durations (in days) are:

Activity	Description	Predecessor(s)	Duration
A	Ecological survey of dam site	none	137 ± 12
B	File environmental impact report and get EPA approval	A	201 ± 18
C	Economic feasibility study	A	161 ± 14
D	Preliminary design & cost estimation	C	93 ± 5

E	Project approval & commitment of funds	B,D	225 ± 20
F	Call quotations for electrical equipment (turbines, generators, etc.)	E	95 ± 10
G	Select suppliers for electrical equipment	F	69 ± 5
H	Final design of project	E	143 ± 12
I	Select construction contractors	E	60 ± 5
J	Arrange construction mat'ls supply	H,I	115 ± 15
K	Dam building	J	546 ± 30
L	Power station building	J	405 ± 25
M	Power lines erection	G,H	447 ± 15
N	Electrical equipment installation	G,L	150 ± 10
O	Build up reservoir water level	K	47 ± 15
P	Commission the generators	N,O	27 ± 2
Q	Start supplying power	M,P	25 ± 3

(This is the same data as given in Homework #8, except that the durations are now given as intervals centered around the originally-given duration.)

The AOA network representing this project is shown on the next page.

a. Use SLAM to simulate the project 1000 times, with a triangular distribution for the activity durations, e.g., TRIAG(125,137,149) for activity A. (Note that the mean value for each duration is the central value which was used in Homework #8, e.g., 137 days for activity A.)

Warning! In the case of a "dummy" activity with duration 0, or any other constant, don't specify a probability distribution such as TRIAG(0,0,0), which would lead to an error message!

Solutions :

```
l-ecn015% rslam 1a-1

GEN,Hansuk Sohn,Hydroelectric Power Station,11/5/1996,1000,,N,,N,Y/1000,72;
LIM,20,50,500;
INIT,,,NO;
NETWORK;
      CREATE,,,,1;
      ACT/1,TRIAG(125,137,149),,N1;    Activity A      +/- 12
N1    ACCUM,,,,2;
      ACT/2,TRIAG(147,161,175),,N2;    Activity C      +/- 14
      ACT/3,TRIAG(183,201,219),,N3;    Activity B      +/- 18
N2    GOON;
      ACT/4,TRIAG(88,93,98),,N3;       Activity D      +/-  5
N3    ACCUM,2;
      ACT/5,TRIAG(205,225,245),,N4;    Activity E      +/- 20
N4    ACCUM,,,,3;
      ACT/6,TRIAG(85,95,105),,N5;      Activity F      +/- 10
      ACT/7,TRIAG(131,143,155),,N6;    Activity H      +/- 12
      ACT/8,TRIAG(55,60,65),,N7;       Activity I      +/-  5
N5    GOON;
      ACT/9,TRIAG(64,69,74),,N8;       Activity G      +/-  5
N6    ACCUM,,,,2;
      ACT/10,,,N7;             DummyAct 1
      ACT/11,,,N10;            DummyAct 2
N7    ACCUM,2;
      ACT/12,TRIAG(100,115,130),,N9;   Activity J      +/- 15
N8    GOON,2;
      ACT/13,,,N10;            DummyAct 3
      ACT/14,,,N11;            DummyAct 4
N9    GOON,2;
      ACT/15,TRIAG(516,546,576),,N12;  Activity K      +/- 30
      ACT/16,TRIAG(380,405,430),,N11;  Activity L      +/- 25
N10   ACCUM,2;
      ACT/17,TRIAG(432,447,462),,N14;  Activity M      +/- 15
N11   ACCUM,2;
      ACT/18,TRIAG(140,150,160),,N13;  Activity N      +/- 10
N12   GOON;
      ACT/19,TRIAG(32,47,62),,N13;     Activity O      +/- 15
N13   ACCUM,2;
      ACT/20,TRIAG(25,27,29),,N14;     Activity P      +/-  2
N14   ACCUM,2;
      ACT/21,TRIAG(22,25,28),,EFT;     Activity Q      +/-  3
EFT   COLCT,FIRST,Earliest_Finish_Time,50/1450/3
      TERM,1;
      END;
FIN;

              S L A M   I I   S U M M A R Y   R E P O R T


  SIMULATION PROJECT HYDROELECTRIC POWER       BY HANSUK SOHN
  DATE 11/ 5/1996                              RUN NUMBER 1000 OF 1000


  CURRENT TIME    .1488E+04
  STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00

                   **STATISTICS FOR VARIABLES BASED ON OBSERVATION**
```

	MEAN VALUE	**STANDARD DEVIATION**	COEFF. OF VARIATION	**MINIMUM VALUE**	**MAXIMUM VALUE**	NO.OF OBS
EARLIEST_FINISH_	**.152E+04**	**.190E+02**	.125E-01	**.147E+04**	**.157E+04**	1000

REGULAR ACTIVITY STATISTICS

ACTIVITY INDEX/LABEL		AVERAGE UTILIZATION	STANDARD DEVIATION	MAXIMUM UTIL	CURRENT UTIL	ENTITY COUNT
1	ACTIVITY A	.0903	.2866	1	0	1
2	ACTIVITY C	.1062	.3080	1	0	1
3	ACTIVITY B	.1323	.3388	1	0	1
4	ACTIVITY D	.0611	.2395	1	0	1
5	ACTIVITY E	.1481	.3552	1	0	1
6	ACTIVITY F	.0625	.2420	1	0	1
7	ACTIVITY H	.0940	.2919	1	0	1
8	ACTIVITY I	.0395	.1948	1	0	1
9	ACTIVITY G	.0454	.2081	1	0	1
10	DUMMYACT 1	.0000	.0000	1	0	1
11	DUMMYACT 2	.0000	.0000	1	0	1
12	ACTIVITY J	.0756	.2644	1	0	1
13	DUMMYACT 3	.0000	.0000	1	0	1
14	DUMMYACT 4	.0000	.0000	1	0	1
15	ACTIVITY K	.3596	.4799	1	0	1
16	ACTIVITY L	.2665	.4421	1	0	1
17	ACTIVITY M	.2940	.4556	1	0	1
18	ACTIVITY N	.0986	.2981	1	0	1
19	ACTIVITY O	.0309	.1732	1	0	1
20	ACTIVITY P	.0178	.1321	1	0	1
21	ACTIVITY Q	.0165	.1272	1	0	1

HISTOGRAM NUMBER 1
EARLIEST_FINISH_

```
OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0        20        40        60        80       100
                    +    +    +    +    +    +    +    +    +    +    +
  0  .000  .1455+04 +                                                 +
  0  .000  .1459+04 +                                                 +
  0  .000  .1462+04 +                                                 +
  0  .000  .1465+04 +                                                 +
  0  .000  .1468+04 +                                                 +
  0  .000  .1471+04 +                                                 +
  0  .000  .1474+04 +                                                 +
  3  .003  .1477+04 +                                                 +
  1  .001  .1470+04 +                                                 +
  3  .003  .1483+04 +                                                 +
  1  .001  .1486+04 +                                                 +
  6  .006  .1489+04 +C                                                +
  9  .009  .1492+04 +C                                                +
 15  .015  .1495+04 +*C                                               +
 24  .024  .1498+04 +* C                                              +
 38  .038  .1501+04 +**  C                                            +
 40  .040  .1504+04 +**    C                                          +
 36  .036  .1507+04 +**      C                                        +
 37  .037  .1500+04 +**        C                                      +
 49  .049  .1513+04 +**          C                                    +
 57  .057  .1516+04 +***            C                                 +
```

```
 57  .057  .1519+04 +***               C                              +
 47  .047  .1522+04 +**                  C                            +
 53  .053  .1525+04 +***                    C                         +
 78  .078  .1528+04 +****                      C                     +
 60  .060  .1531+04 +***                           C                  +
 35  .035  .1534+04 +**                             C                 +
 48  .048  .1537+04 +**                                C              +
 55  .055  .1530+04 +***                                  C           +
 48  .048  .1543+04 +**                                     C         +
 39  .039  .1546+04 +**                                       C       +
 36  .036  .1549+04 +**                                         C     +
 29  .029  .1552+04 +*                                           C    +
 31  .031  .155E+04 +**                                            C  +
 16  .016  .155E+04 +*                                              C +
 16  .016  .156E+04 +*                                              C +
 13  .013  .156E+04 +*                                               C+
  5  .005  .156E+04 +                                                C+
  6  .006  .156E+04 +                                                 C
  4  .004  .157E+04 +                                                 C
  1  .001  .157E+04 +                                                 C
  2  .002  .157E+04 +                                                 C
  2  .002  .158E+04 +                                                 C
  0  .000  .158E+04 +                                                 C
  0  .000  .158E+04 +                                                 C
  0  .000  .159E+04 +                                                 C
  0  .000  .159E+04 +                                                 C
  0  .000  .159E+04 +                                                 C
  0  .000  .159E+04 +                                                 C
  0  .000  .160E+04 +                                                 C
  0  .000  .160E+04 +                                                 C
  0  .000    INF    +                                                 C
---                 +    +    +    +    +    +    +    +    +    +    +
***                 0         20       40        60        80       100
```

b. In your simulation, what are the

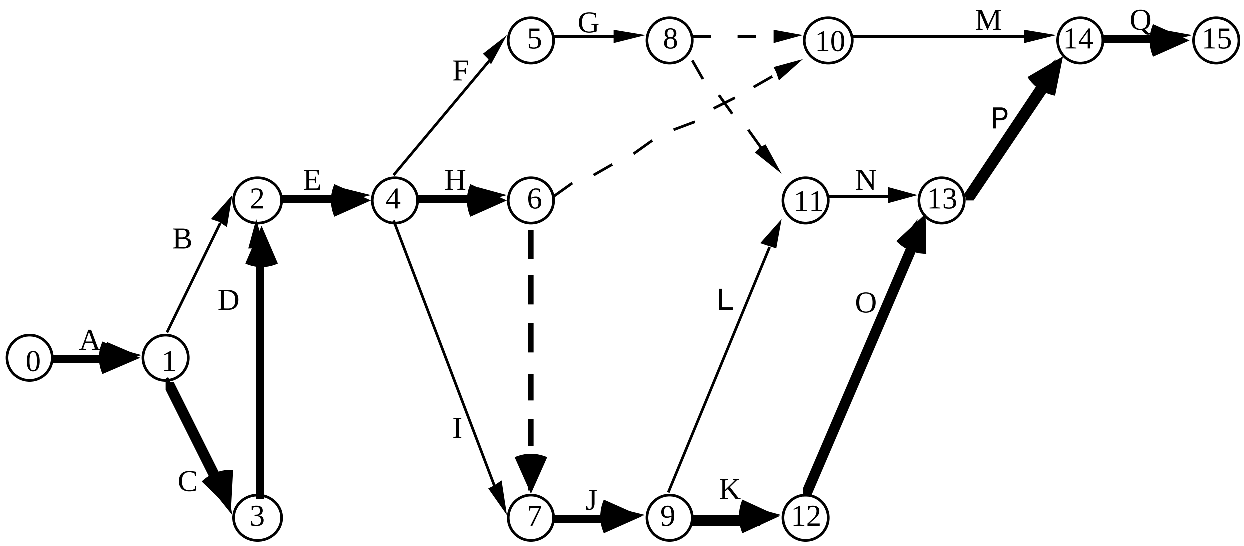
- average completion time? __1520_ days
- standard deviation? _19_ days
- minimum completion time? _1470_ days
- maximum completion time? _1570_ days

(Recall that in Homework #8, the duration was determined to be 1519 days.)

c. According to PERT, the critical path which you found in Homework #8, assuming the durations to be equal to their expected values, namely

[A - C - D - E - H - J - K - O - P - Q],

is <u>always</u> the critical path. Determine whether that is true for the 1000 simulations you have performed. (Because of the relatively large "float" of the noncritical activities found in Homework #8, it might be true. If necessary, modify your SLAM model to determine this!) If not true, in how many simulations was this path <u>not</u> the critical path?



Soln: For the 1000 simulations performed 'A-C-D-E-H-J-K-O-P-Q' is “nearly always” the crit path. This was determined by making the adjustments below to the previous SLAM II model

```
l-ecn008% rslam 1c-ok

GEN,Hansuk Sohn,Hydroelectric Power Station,11/5/1996,1000,,N,,N,Y/1000,72;
LIM,20,50,500;
INIT,,,NO;
NETWORK;
       CREATE,,,1,1;
       ACT/1,137,,A1;            Activity A
A1     ASSIGN,ATRIB(1)=1;
N1     ACCUM,,,LAST,2;
       ACT/2,161,,N2;            Activity C
       ACT/3,201,,A3;            Activity B
N2     GOON;
       ACT/4,93,,N3;             Activity D
A3     ASSIGN,ATRIB(1)=0;
N3     ACCUM,2,,LAST;
       ACT/5,225,,N4;            Activity E
N4     ACCUM,,,LAST,3;
       ACT/6,95,,A5;             Activity F
       ACT/7,143,,N6;            Activity H
       ACT/8,60,,A7;             Activity I
A5     ASSIGN,ATRIB(1)=0;
N5     GOON;
       ACT/9,69,,N8;             Activity G
N6     ACCUM,,,LAST,2;
       ACT/10,,,N7;              DummyAct 1
       ACT/11,,,A10;             DummyAct 2
A7     ASSIGN,ATRIB(1)=0;
N7     ACCUM,2,,LAST;
       ACT/12,115,,N9;           Activity J
N8     GOON,2;
       ACT/13,,,A10;             DummyAct 3
       ACT/14,,,A11;             DummyAct 4
N9     GOON,2;
       ACT/15,546,,N12;          Activity K
       ACT/16,405,,A11;          Activity L
A10    ASSIGN,ATRIB(1)=0;
```

```
N10    ACCUM,2,,LAST;
       ACT/17,447,,N14;         Activity M
A11    ASSIGN,ATRIB(1)=0;
N11    ACCUM,2,,LAST;
       ACT/18,150,,N13;         Activity N
N12    GOON;
       ACT/19,47,,N13;          Activity O
N13    ACCUM,2,,LAST;
       ACT/20,27,,N14;          Activity P
N14    ACCUM,2,,LAST;
       ACT/21,25,,CHECK;        Activity Q
CHECK  COLCT,ATRIB(1),Number_Same_CP
EFT    COLCT,FIRST,Earliest_Finish_Time
       TERM,1;
       END;
FIN;
```

S L A M   I I   S U M M A R Y   R E P O R T

SIMULATION PROJECT HYDROELECTRIC POWER   BY HANSUK SOHN

DATE 11/ 5/1996   RUN NUMBER 1000 OF 1000

CURRENT TIME .1519E+04
STATISTICAL ARRAYS CLEARED AT TIME .0000E+00

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	**NO.OF OBS**
NUMBER_SAME_CP	100E+01	.000E+00	.000E+00	.100E+01	.100E+01	**1000**
EARLIEST_FINISH_	.152E+04	.000E+00	.000E+00	.152E+04	.152E+04	1000

A new COLCT node was inserted to collect statistics on the value of ATTRIBUTE #1, which is equal to 1 if the path of interest [A - C - D - E - H - J - K - O - P - Q] is critical, and zero otherwise. We see that the minimum value and maximum value of attribute #1 are both equal to 1, indicating that the path was always the longest path in the network. Warning: this isn't true in general! In this particular example, all noncritical activities had rather large values of slack ("float"), as found in the previous homework assignment!

Based upon your simulation, if you wish to set a deadline for completion of the project so that you are 90% certain of meeting this deadline, what should this deadline be?

Sol'n: See the histogram of the project completion time.

```
36  .036  .1549+04 +**                                        C     +
29  .029  .1552+04 +*                                          C    +
```

By summing the counts of the cells, we find that at time=1549 days, 875 (i.e., 87.5%) of the simulated projects had been completed, while at time=1552 days, 904 (i.e. 90.4%) of the projects had been completed. Using a linear interpolation, we estimate that 90% of the simulated projects should be completed at time= 1549+ 3(25/29) = <u>1551.6</u> days.

To get a more accurate estimate of the correct warranty period, we should

- increase the number of runs from 1000, e.g., to as many as 10000

- change the specifications for the histogram so as to get a count of failures each day, e.g.,

  EFT COLCT,FIRST,Earliest_Finish_Time,50/1545/0.05

which would give 50 cells, each of length 0.05 days.

e. Suppose that, by using improved procedures, the variability in the duration of activity J, “Arrange construction mat’ls supply”, which presently is ± 15 days, might be reduced to ± 5 days. How would this effect the deadline which you set in (d)?

Sol’n: Modify the SLAM statement for activity J, i.e.,

```
24        ACT/12,TRIAG(110,115,120),,N9;   Activity J       +/- 5
```

and re-run the simulation. In the new histogram, we find that at time=1548 days 895 projects had been completed, and at time=1551, 918 projects had been completed:

```
38  .038  .1548+04 +**                                            C    +
23  .023  .1551+04 +*                                              C   +
```

Using linear interpolation as before, we estimate that 900 projects would be complete at time = 1548 + 3(5/23) = 1548.7 days (2.9 days earlier than estimated in part (d).)

To get a more accurate estimate of the correct warranty period, we should

- increase the number of runs from 1000, e.g., to as many as 10000
- change the specifications for the histogram so as to get a count of failures each day, e.g.,

  EFT COLCT,FIRST,Earliest_Finish_Time,50/1543/0.1

which would give 50 cells, each of length 0.1 days.

f. According to PERT, the project duration should have approximately a normal distribution. Use the chi-square goodness of fit test to determine whether this assumption is valid for this project.

Solution: To reduce the computational burden somewhat, the below table was computed with each interval of 6 days length, rather than the 3 days in the SLAM histogram. Using normal distribution tables with mean $\mu = 1520$ and standard deviation $\sigma = 19$, we compute the probability p_i that one of the 1000 observations falls into cell #i, and then multiply this by 1000 to get E_i, the expected number of observations in cell #i:

#	Interval	Oi	Ei	Pi	$\frac{(O_i - E_i)^2}{E_i}$
1	1470 - 1475	2	4.682	0.004682	1.53633575
2	1475 - 1480	3	8.648	0.008648	3.68870305
3	1480 - 1485	3	14.980	0.014980	9.58080107
4	1485 - 1490	12	24.354	0.024354	6.26678640
5	1490 - 1495	25	37.068	0.037068	3.92890428
6	1495 - 1500	55	52.590	0.052590	0.11044115
7	1500 - 1505	59	69.306	0.069306	1.53253161
8	1505 - 1510	68	84.775	0.084775	3.31938219
9	1510 - 1515	92	96.525	0.096525	0.21212769
10	1515 - 1520	84	102.802	0.102802	3.43879695

11	1520 - 1525	99	102.802	0.102802	0.14061209
12	1525 - 1530	112	96.525	0.096525	2.48096996
13	1530 - 1535	66	84.775	0.084775	4.15807284
14	1535 - 1540	85	69.306	0.069306	3.55382847
15	1540 - 1545	74	52.590	0.052590	8.71625975
16	1545 - 1550	53	37.068	0.037068	6.84764821
17	1550 - 1555	50	24.354	0.024354	27.00654170
18	1555 - 1560	25	14.980	0.014980	6.70229640
19	1560 - 1565	17	8.648	0.008648	8.06613136
20	1565 - 1570	9	4.682	0.004682	3.98229902
21	1570 - 1575	3	2.368	0.002368	0.16867568
22	1575 - 1580	4	1.110	0.001110	7.52441441
	sum =	1000		D =	112.96256000

Note : $p_i = F(t_i) - F(t_{i-1}) \quad i \geq 1$, where $t \sim N(1520,19)$ Thus, for example,

$p_1 = F(t_1) - F(t_0)$ = F(1475) - F(1470) = 0.004682

$E_1 = 1000 \times P_1 = 4.682$

$p_1 = F(t_1) - F(t_0)$ = F(1480) - F(1475) = 0.008648

$E_2 = 1000 \times P_2 = 8.648$

We perform a Chi-Square Goodness of Fit test to determine whether this normal distribution provides an acceptable fit to the observed data, with α = 5%.
Since " D (112.96) > $\chi^2_{5\%}$ (= 30.15) ", we can conclude that this distribution does not provide an acceptable fit to the observed data with α =5%.

••••••••••••••••••••••••••••• Homework # 10 •••••••••••••••••••••••••••••••••

1. Consider an inventory system in which the number of items on the shelf is checked at the end of each day. The maximum number on the shelf is 8. If 3 or fewer units are on the shelf, the shelf is refilled overnight. The demand distribution is as follows:

x	0	1	2	3	4	5	6
P{D=x}	.1	.15	.25	.25	.15	.05	.05

The system is modeled as a Markov chain, with the state defined as the number of units on the shelf at the end of each day. The probability transition matrix is:

Transition Probabilities

from \ to	1	2	3	4	5	6	7	8	9
1	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
2	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
3	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
4	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
5	0.25	0.25	0.25	0.15	0.1	0	0	0	0
6	0.1	0.15	0.25	0.25	0.15	0.1	0	0	0
7	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0	0
8	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0
9	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1

a. Explain the derivation of the values P_{19}, P_{35}, P_{51}, P_{83} above. (Note that state 1=inventory level 0, etc.)

State	Definition
1	SOH = 0
2	SOH = 1
3	SOH = 2
4	SOH = 3
5	SOH = 4
6	SOH = 5
7	SOH = 6
8	SOH = 7
9	SOH = 8

Solution: $P_{ij} = P\{X_n = j | X_{n-1} = i\}$

if i > 4 (SOH >3), no replenishment occurs :

$$P_{ij} = \begin{cases} P\{D = (i-j)\} & \text{for } j > 1 \quad (\text{SOH} > 0) \\ P\{D \geq (i-j)\} & \text{for } j = 1 \quad (\text{SOH} = 0) \end{cases}$$

$$P_{83} = P\{D = (8\text{-}3) = 5\} = 0.05$$

$$P_{51} = P\{D \geq (5\text{-}1) = 4\} = 0.15 + 0.05 + 0.05 = 0.25$$

if i ≤ 5 (SOH ≤ 4), the SOH at the beginning of the next day is 8 :

$$P_{ij} = P\{D = (8\text{-}(j\text{-}1))\}$$

$$P_{19} = P\{D = (8\text{-}(9\text{-}1))\} = P\{D = 0\} = 0.1$$

$$P_{35} = P\{D = (8\text{-}(5\text{-}1))\} = P\{D = 4\} = 0.15$$

The steady-state distribution of the above Markov chain is:

Steady State Distribution

i	π_i
1	0.06471513457
2	0.07698357218
3	0.1304613771
4	0.1355295351
5	0.16322964
6	0.1698706746
7	0.1384131423
8	0.0754980776
9	0.04529884656

b. Write two of the equations which define this steady-state distribution. How many equations must be solved to yield the solution above?

Solution: 9 equations must be solved to compute the values of the 9 probabilities. One equation must be

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 + \pi_9 = 1$$

while the remaining 8 are obtained by setting π_i equal to the inner product of column i and π. For example, using column i=1 we would obtain the equation

$$\pi_1 = 0.25\pi_5 + 0.1\pi_6 + 0.05\pi_7$$

c. What is the average number on the shelf at the end of each day?

Solution: Since in state i the stock on hand is (i-1),

$$\text{Average Stock-on-Hand} = \sum_{i=1}^{9} (i-1)\pi_i = 4.968123$$

The mean first passage matrix is:

Mean First Passage Times

from \ to	1	2	3	4	5	6	7	8	9
1	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
2	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
3	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
4	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
5	12.2711	10.4926	6.64702	7.25983	6.12634	6.35574	7.50281	13.7757	23.1867
6	14.3163	11.5197	6.47734	6.42023	5.85772	5.88683	7.68799	13.9609	23.3719
7	14.632	12.4406	7.01794	6.24734	5.2518	5.79028	7.22475	14.3004	23.7114
8	15.2275	12.1857	7.62978	6.77218	5.19576	5.29848	7.10625	13.2454	24.1281
9	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756

d. If the shelf is full Monday morning, what is the expected number of days until the shelf is first emptied ("stockout")?

Solution: m_{91} = 15.4523 (days)

e. What is the expected time between stockouts?

Solution: m_{00} = 15.4523 (days)

f. How frequently will the shelf be restocked? (i.e. what is the average number of days between restocking?)

Solution: Since the shelf is restocked whenever the state of the system is 1,2,3, or 4, the steadystate probability that the shelf is restocked on a day is

$$\sum_{i=1}^{4} \pi = 0.4076 = 1/2.4534,$$

i.e., we expect the shelf to restocked with frequency once each 2.4534 days.

2. Consider a manufacturing process in which raw parts (blanks) are machined on three machines, and inspected after each machining operation. The relevant data is as follows:

Manufacturing System Parameters

Station	Machine Operation			Inspection			
i	T	C	S	T	C	S	R
1	0.5	20	10	0.1	15	10	5
2	0.75	20	5	0.2	15	10	3
3	0.25	20	2	0.25	15	5	2

Pack & Ship: 0.1 hrs at 10 $/hr
Cost per blank: $50; Scrap Value: $10

T = time (hrs) per operation
C = cost ($/hr) of operation
S = scrap rate (%)
R = rework rate (%)

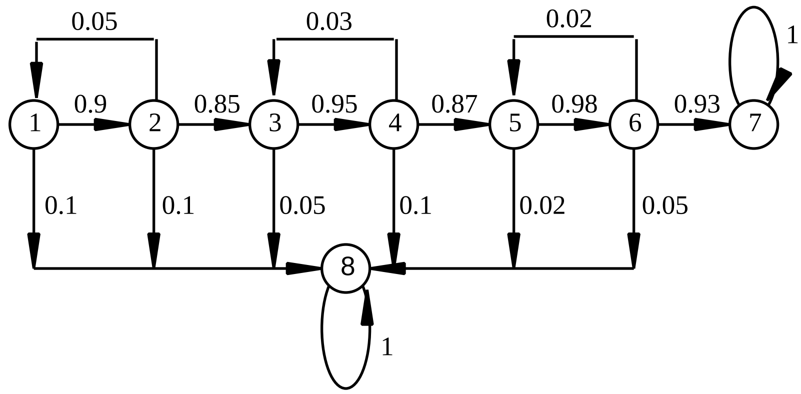
For example, machine #1 requires 0.5 hrs, at $20/hr., and has a 10% scrap rate. Those parts completing this operation are inspected, requiring 0.1 hr. at $15/hr. The inspector scraps 10%, and sends 5% back to machine #1 for rework (after which it is again inspected, etc.)

The Markov chain model of a part moving through this system has transition probability matrix:

Transition Probabilities

from \ to	1	2	3	4	5	6	7	8
1	0	0.9	0	0	0	0	0	0.1
2	0.05	0	0.85	0	0	0	0	0.1
3	0	0	0	0.95	0	0	0	0.05
4	0	0	0.03	0	0.87	0	0	0.1
5	0	0	0	0	0	0.98	0	0.02
6	0	0	0	0	0.02	0	0.93	0.05
7	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	1

a. Draw the diagram for this Markov chain and describe each state.
Solution:



State	Location
1	Machine1
2	Inspection station1
3	Machine2
4	Inspection station2
5	Machine3
6	Inspection station3
7	Pack-&-Ship Dept.
8	Scrap bin

b. Which states are transient? which are absorbing?

Solution: Transient States : 1, 2, 3, 4, 5, and 6

Absorbing States : 7 and 8

The absorption probabilities are:

A = Absorption Probability Matrix

	OK	Scrap
1	0.6335	0.3665
2	0.7039	0.2961
3	0.7909	0.2091
4	0.8325	0.1675
5	0.9296	0.07038
6	0.9486	0.05141

The matrix E is as follows:

E = Expected No. Visits

	1	2	3	4	5	6
1	1.047	0.9424	0.8245	0.7833	0.6951	0.6812
2	0.05236	1.047	0.9162	0.8704	0.7723	0.7569
3	0	0	1.029	0.9779	0.8678	0.8504
4	0	0	0.03088	1.029	0.9134	0.8952
5	0	0	0	0	1.02	0.9996
6	0	0	0	0	0.0204	1.02

c. Explain how matrix E was computed. Explain how matrix A was computed, given E.

Solution:

$$\left[\begin{array}{cccccc|cc} 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0.05 & 0 & 0.85 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0.95 & 0 & 0 & 0 & 0.05 \\ 0 & 0 & 0.03 & 0 & 0.87 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.98 & 0 & 0.02 \\ 0 & 0 & 0 & 0 & 0.02 & 0 & 0.93 & 0.05 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} Q & R \\ \hline 0 & I \end{array}\right]$$

matrices E and A could be computed by equations $E = (I - Q)^{-1}$ and $A = E \times R$, respectively.

d. What percent of the parts which are started are successfully completed?

Solution: $A_{1,OK} = 0.6335 = 63.35\ \%$

e. What is the expected number of blanks which should be required to fill an order for 100 completed parts?

Solution: $\dfrac{100}{A_{1,OK}} = \dfrac{100}{0.6335} = 157.8532$

f. What percent of the parts arriving at machine #2 will be successfully completed?

Solution: $A_{3,OK} = 0.7909 = 79.09\ \%$

g. What is the expected total number of inspections which entering parts will undergo?

Solution: $E_{12} + E_{14} + E_{16} = 2.4069$

h. Explain the meaning of the number appearing in row 3, column 2 of the A matrix.

Solution: A_{32} = .2091 indicates that parts arriving at machine 2 have a 20.91 % probability of being failed.

i. Explain the meaning of the number appearing in row 3, column 3 of the E matrix.

Solution: $E_{33} = 1.029$ indicates the expected number of times that parts arriving at machine 2 returns to machine 2 before successfully finished or failed.

j. To fill the order for 100 completed parts, what is the expected man-hour requirement for each machine? for each inspection station?

Solution:

Operation	state	Man-hour / 100 completed part
Machine 1	1	1.047 × 0.5 × (1/0.6335) × 100 = 82.64
Inspection 1	2	1.047 × 0.1 × (1/0.7039) × 100 = 14.87
Machine 2	3	1.029 × 0.75 × (1/0.7909) × 100 = 97.58
Inspection 2	4	1.029 × 0.2 × (1/0.8325) × 100 = 24.72
Machine 3	5	1.02 × 0.25 × (1/0.9296) × 100 = 27.43
Inspection 3	6	1.02 × 0.25 × (1/0.9486) × 100 = 26.88
Pack-&-Ship	7	1 × 0.1 × (1/1) × 100 = 10

Total = 284.12

k. What are the expected direct costs (row materials + operating costs - scrap value of rejected parts) per completed part?

Solution:

Each completed part requires an expected 1/0.6335, i.e., 1.5785 entering parts.

Materials : \$ 50 × 1.5785 = \$ 78.925

Scrap value of rejected parts : \$ 10 × 1.5785 × 0.3665 = \$ 5.785

Estimated Man-hour requirements per completed part

Operation	state	Man-hour / completed part
Machine 1	1	1.047 × 0.5 × (1/0.6335) = 0.8264
Inspection 1	2	1.047 × 0.1 × (1/0.7039) = 0.1487
Machine 2	3	1.029 × 0.75 × (1/0.7909) = 0.9758
Inspection 2	4	1.029 × 0.2 × (1/0.8325) = 0.2472
Machine 3	5	1.02 × 0.25 × (1/0.9296) = 0.2743
Inspection 3	6	1.02 × 0.25 × (1/0.9486) = 0.2688
Pack-&-Ship	7	1 × 0.1 × (1/1) = 0.1

Total = 2.8412

Operation cost :

Operation	state	Hourly Rate × Man-Hours
Machine 1	1	20 × 0.8264 = 16.528
Inspection 1	2	15 × 0.1487 = 2.2305
Machine 2	3	20 × 0.9758 = 19.516
Inspection 2	4	15 × 0.2472 = 3.708
Machine 3	5	20 × 0.2743 = 5.486
Inspection 3	6	15 × 0.2688 = 4.032
Pack-&-Ship	7	10 × 0.1 = 1

Total = $ 52.5005

Total Direct Cost = row material + operating costs - Scrap value of rejected parts

= $ 78.925 + $ 52.5005 - $ 5.785

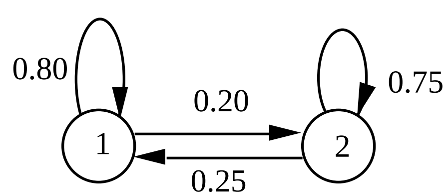
= 125.6405

3. A city's water supply comes from a reservoir. Careful study of this reservoir over the past 20 years has shown that, if the reservoir was full at the beginning of the summer, then the probability it would be full at the beginning of the following summer is 80%. On the other hand, if the reservoir was not full at the beginning of one summer, the probability it would be full at the beginning of the following summer is only 25%.

a. Defining the states to be "full" and "not full", draw a transition diagram for a Markov chain model of this reservoir.

Solution:

state	definition
1	full
2	not full



b. Write the transition probability matrix.

Solution:

$$P = \begin{bmatrix} 0.80 & 0.20 \\ 0.25 & 0.75 \end{bmatrix}$$

c. If the reservoir was full at the beginning of summer 1996, what is the probability that it will be full at the beginning of summer 1998?

Solution: $P_{11}^2 = 0.69 \quad \left(\text{from } P^2 = \begin{bmatrix} 0.6900 & 0.3100 \\ 0.3875 & 0.6125 \end{bmatrix} \right)$

d. Write equations which determine the steadystate probability distribution for this Markov chain, and solve them.

Solution:

$$\begin{cases} \pi_1 = 0.80\pi_1 + 0.25\pi_2 \\ \pi_1 + \pi_2 = 1 \end{cases} \quad \text{or} \quad \begin{cases} \pi_2 = 0.20\pi_1 + 0.75\pi_2 \\ \pi_1 + \pi_2 = 1 \end{cases}$$

$$\pi_1 = \frac{5}{9}, \quad \pi_2 = \frac{4}{9}$$

e. Over a long period of time, in what percent of the years would we expect the reservoir be full at the beginning of the summer, according to this model?

Solution: $\pi_1 = \frac{5}{9} = 55.56\ \%$