«»«»«»«»«» 57:022 Principles of Design II «»«»«»«»«»
Homework Assignments, Spring 1996
Prof. Dennis L Bricker, Dept. of Industrial Engineering
University of Iowa

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #1 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Be sure to state what probability distribution you assume in each problem! You may find the necessary probabilities in tables, compute them manually with a calculator, or use the APL workspace "ProbLib" which can be found on the ICAEN fileserver (in the folder for IE). (Using the APL workspace requires that you have a disk containing APL.68000 Level II, an interpreter for the APL language.)

1. (a) Find the probability of throwing a "7 or 11" at least twice in six throws of a standard pair of dice.
   (b) What is the expected number of 7's & 11's obtained in 6 throws of a pair of dice?
   (c) What is the expected number of throws of a pair of dice required in order to obtain a 7 or 11?

2. A certain production process has a fraction defective of 8%. Four good pieces are required. Pieces are produced until the 4 good pieces are obtained.
   a. What are the expected value and standard deviation of the number of pieces produced?
   b. Compute the probabilities of producing no more than 6 pieces in order to obtain 4 good pieces.
   c. Suppose that instead of producing until 4 good pieces are obtained, a batch of size **n** is produced, and then the pieces are inspected. How large should **n** be in order to be 95% certain of obtaining at least 4 good pieces?

3. A telephone exchange contains 10 lines. A line can be busy or available for calls and all lines act independently. If each line is busy 72% of the noon period (so that the probability that a specified line will be busy at any given time during the noon period is 72%), what is the probability of there being at least three free lines at any given time during this period? What is the expected number of free lines at any time during this period?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #2 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Be sure to state what probability distribution you assume in each problem. You may find the necessary probabilities in tables, compute them manually with a calculator, or use the APL workspace "PROBLIB" .

1. Suppose that X is a *discrete* random variable with probability mass function given by:

   p(1) = 0.1 p(2) = 0.3 p(3) = 0.2 p(4) = 0.3 p(5) = 0.1

   (a.) Plot the probability mass function p(x).
   (b.) Compute and plot the CDF (cumulative distribution function) F(x).
   (c.) Compute $P\{1.4 \le X \le 4.2\}$.

2. Suppose that X is a *continuous* random variable with probability density function given by:

$$f(x) = \begin{cases} 4x/97.5 & 15.5 \le x \le 17.0 \\ 0 & \text{otherwise} \end{cases}$$

   (a.) Plot the density function f(x).
   (b.) Plot the CDF (cumulative distribution function) F(x).
   (c.) Compute $P\{X \le 16.0\}$.
   (d.) Compute $P\{X = 16.0\}$.

3. Along highway I-80 in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=2.5%, i.e, an average of one in forty drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

a. Each car may be considered as a "trial" in a ____________ process, with "success" defined as the car's stopping to pick up the hitchhiker.

b. Given that a hitchhiker has counted 25 cars passing him without stopping, what is the probability that he will be picked up by the 40^{th} car *or before?*

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 10 per minute. Define "success" for the hitchhiker to occur at time t provided that *both* an arrival occurs at t *and* that car stops to pick him up. Let T_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time $T_1=0$.

c. What is the arrival rate of "successes"? ______________

d. What is the name of the probability distribution of T_1? _______________

e. What is the value of $E(T_1)$?________ What's the value of $Var(T_1)$?_________

f. What is the probability that he must wait less than 5 minutes for a ride ($P\{T_1 < 5\}$?

g. What is the probability that he must wait more than 5 minutes for a ride($P\{T_1 > 5\}$?

h. What is the probability that he must wait *exactly* 5 minutes for a ride? ($P\{T_1 = 5\}$?
Note that your answers in f, g, and h must have a sum equal to 1!

i. Suppose that after 4 minutes (during which 42 cars have passed by) he is still there waiting for a ride. Compute the *conditional* expected value of T_1 (expected *total* waiting time, given that he has already waited 4 minutes).

4. A bearing in a Grass Chopper mower's PTO mechanism fails randomly, with an expected lifetime of 250 hours. Assume that the lifetime of the bearing has an exponential distribution.

(a.) What is the probability that the bearing lasts longer than 250 hours?

(b.) If the mower has already operated without failure, for 150 hours, what is the probability that the bearing will last at total of at least 250 hours?

(c.) If the bearing experiences its first failure at 238 hours and is replaced (with an identical bearing), what is the probability distribution, mean value, and variance of the time until the second failure?

(d.) What is the probability that the bearing will fail (& be replaced) three or more times in 750 hours of mowing?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #3 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

(1.) Run the SLAM simulation model with single teller window and with queue capacity equal to 4 by using the SLAM input file:

```
GEN,yourname,BANKTELLER,month/day/year,1,,,,,,72;
LIM,1,2,50;
INIT,0,480;
MONTR,TRACE;
NETWORK;
    CREATE,EXPON(5.0),,,20;
    QUE(1),0,4;
    ACT(1)/1,EXPON(2.0);
    TERM;
    END;
FIN;
```

Notes:
• *The "72" in the GEN statement specifies that the width of the output report is to be 72 columns.*
• *The "20" in the CREATE statement specifies that only 20 customers are to be simulated.*
• *The MONTR (monitor) statement specifies that a trace is being requested.*

Using the TRACE output, prepare a log recording, for each of the 20 customers:

a. the arrival time
b. length of queue (including current arrival) at time of arrival
c. status (busy or idle) of the teller immediately after the arrival
d. time that customer begins being served by the teller
e. departure time of each customer
f. length of the queue when the customer departs.

Which customer waits longest in the queue? Which customer spends the most time in the system? (Note that these will not necessarily be the same customer!)

(2.) Now, run the simulation again five times (simulating five days), by
- changing the "1" in the NNRNS (# of runs) field of the GEN statement to "5".
- removing the MONTR statement so that TRACE output isn't generated.
- removing the "20" from the CREATE statement, so that more than 20 customers are simulated each day.

(a.) Verify that the results of the *first* day are identical to the results shown in the Hypercard stack. Explain how this can be. (Aren't the interarrival and service times *random* ?)
(b.) How many cars were unable to enter the queue each day, because the queue was filled to capacity?
(c.) What fraction of the time was the teller busy each day?

(3.) *Suppose that, after seeing the results of the simulation above, the board of directors has decided to consider the possibility of redesigning the layout so that the driveway for cars wishing to visit the teller window would be extended for a waiting capacity of 5 cars (not including the car at the window), if, in doing so, a single teller window would be sufficient.*

Again run the SLAM simulation model (specifying 5 runs on the GEN card) with the single teller window but with the queue capacity increased to 5.
(a.) How many cars were not able to enter the queue each day?
(b.) What was the maximum number of cars waiting in the driveway each day?
(c.) What fraction of the time was the teller busy each day?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #4 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. **Generating Arrival Times in Poisson Process.** Suppose, in preparation for performing a manual simulation of the arrivals in a Poisson process (e.g., parts randomly arriving at a machine to be processed), you wish to generate some inter-arrival times, where the arrival rate is 4/hour, i.e., one every fifteen minutes. First, you need some uniformly-distributed random numbers. To obtain these, select a row from the following random number table, based upon the last digit of your ID#: if 1, use row #1; if 2, use row #2; ... if 0, use row #10.

Uniformly-Distributed "Random" Numbers

1	3735	5010	6806	0727	5863	7548	2955	0176	9132	8283
2	6613	9328	5159	5559	4755	3992	2960	7498	1480	4217
3	8768	7158	4764	2101	3525	9297	4171	1247	8098	3589
4	4326	1074	2295	7662	5421	8747	1089	7609	9694	3363
5	5081	0461	7977	8573	3930	8291	9051	9449	3346	1644
6	6595	5202	5096	2783	3414	8637	6043	6052	9561	7252
7	4055	1579	2020	6620	2424	8275	1389	0217	3510	6474
8	5134	1844	2788	5399	0629	4932	3028	4722	7346	2755
9	9652	2277	8133	1447	5645	8730	2435	4076	2632	0775
10	4655	6443	5269	0004	3687	3189	8049	9378	6781	3383

a. What is the name of the probability distribution of the time T_1 of the first arrival?

b. What will be the name of the probability distribution of the time t_i *between* arrivals of parts i-1 and i (where i>1)?

c. Use the inverse-transformation method to obtain random inter-arrival times $t_1, t_2, \ldots t_{10}$ (where $T_1 = t_1$).

d. What are the arrival times ($T_1, T_2, \ldots T_{10}$) of the first ten parts in your simulation?

i	X_i	t_i	T_i
1	______	______	______
2	______	______	______
3	______	______	______
4	______	______	______
5	______	______	______
6	______	______	______
7	______	______	______
8	______	______	______
9	______	______	______
10	______	______	______

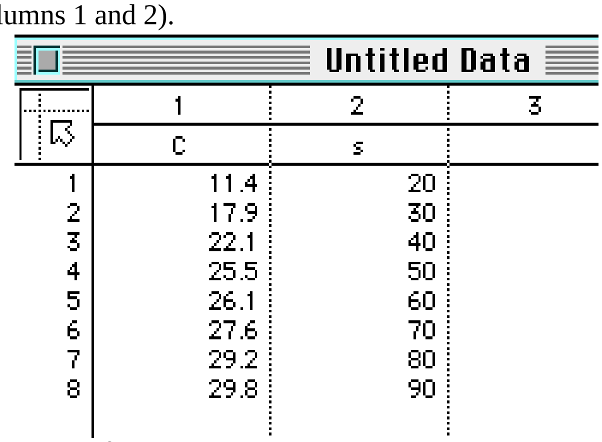
e. The expected number of arrivals in the first hour should, of course, be 4. In your simulation of the arrivals, however, what is the number of arrivals during the first hour?

2. **Regression Analysis.** Tests on the fuel consumption of a vehicle traveling at different speeds yeilded the following results:

Speed s (mph)	20	30	40	50	60	70	80	90
Consumption C (mile/gal.)	11.4	17.9	22.1	25.5	26.1	27.6	29.2	29.8

It is believed that the relation between the two variables is of the form $C = a + b/s$.

a. Run the "Cricket Graph" software package (on the Macintosh), and enter the above observed values of s and C (columns 1 and 2).


Untitled Data

	1	2	3
	C	s	
1	11.4	20	
2	17.9	30	
3	22.1	40	
4	25.5	50	
5	26.1	60	
6	27.6	70	
7	29.2	80	
8	29.8	90	

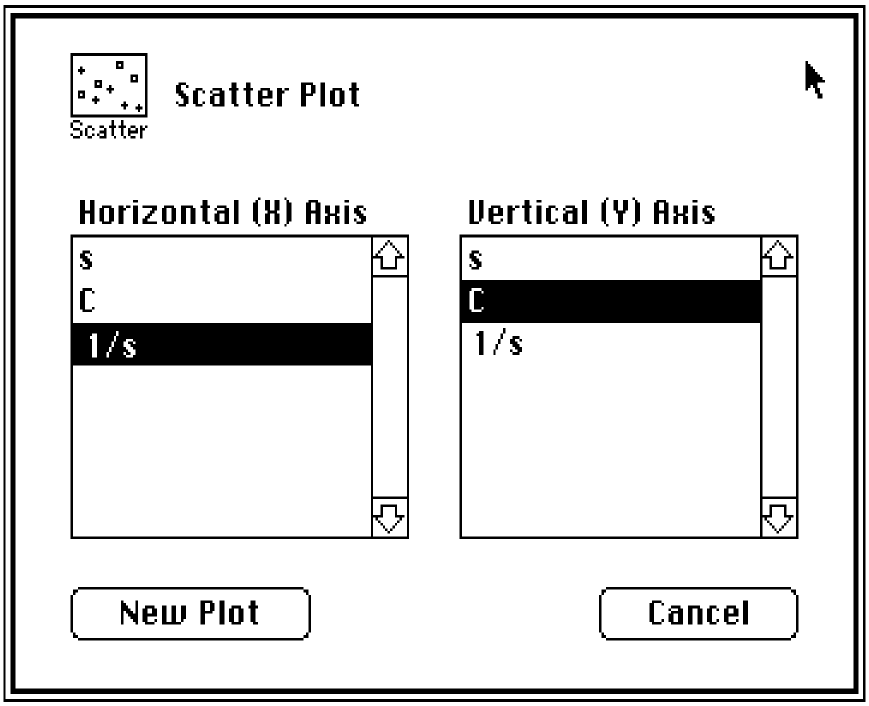
b. Plot the "scatter plot" of C versus s by choosing "scatter" on the Graph menu, and specifying s on the horizontal axis and C on the vertical axis. Does the plot appear to be linear? _______

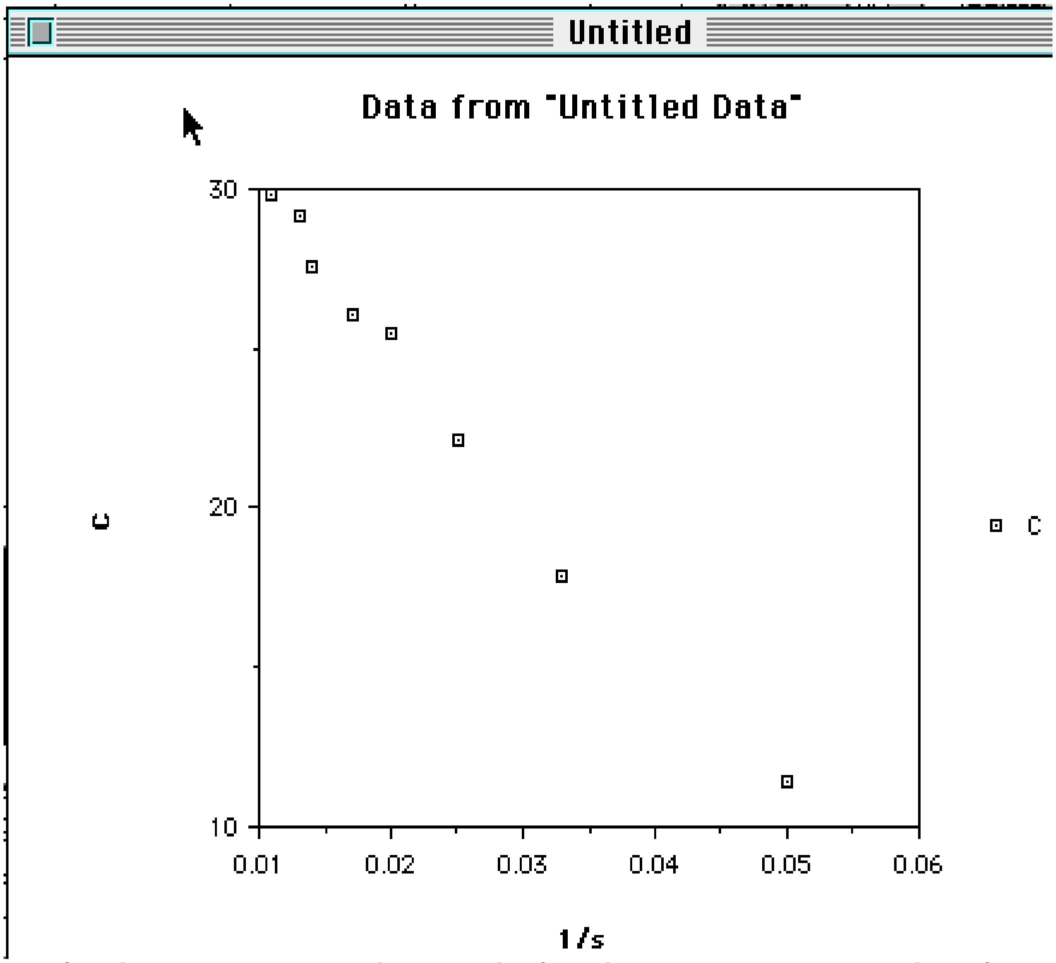
c. Choose "transform" from the "data" menu to create a new variable 1/s which is the *reciprocal* of s. (Put this new variable into Column 3.)

Untitled Data

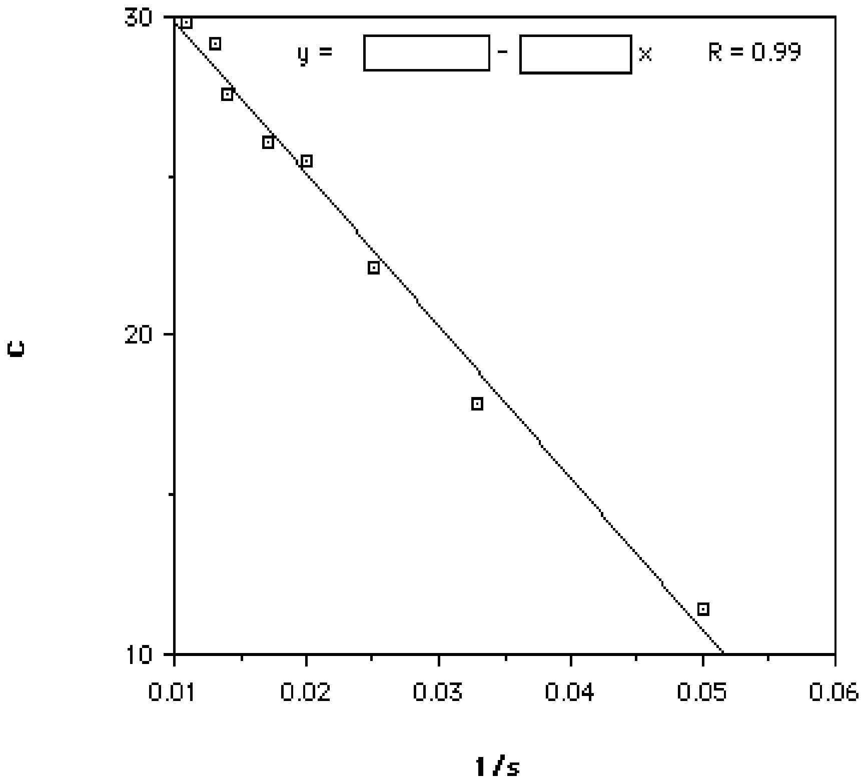
	1	2	3
	C	s	1/s
1	11.4	20	0.050
2	17.9	30	0.033
3	22.1	40	0.025
4	25.5	50	0.020
5	26.1	60	0.017
6	27.6	70	0.014
7	29.2	80	0.013
8	29.8	90	0.011

d. Plot the "scatter plot" of 1/s (horizontal axis) versus C (vertical axis). Does the plot appear to be linear? _______





e. After plotting C versus 1/s, select "Simple" from the "Curve Fit" menu, in order to fit a simple linear relationship between C and 1/s, i.e., to determine a and b such that $C \approx a + b(^1/_s)$. What is the value of a? __________ of b? _____________



(Note: the above data is complete fictitious!)

3. The numbers of arrivals during each of 100 one-minute intervals of what is believed to be a Poisson process were recorded.

Number of arrivals during each of the first 100 minutes

2	2	4	2	3	2	4	5	3	3	0	3	4	0	3	1	3	0	3	3	3	0	0	3	3
2	2	4	4	2	0	4	5	2	4	0	3	0	2	3	4	3	2	0	1	1	3	4	2	5
4	3	0	3	2	2	0	4	3	1	4	1	2	2	2	3	3	4	4	5	5	2	4	1	4
8	5	1	8	4	5	0	5	3	4	1	4	6	3	5	1	3	2	4	2	4	4	5	1	1

The observed numbers above ranged from zero to eight, with frequencies O_0 through O_8:

Number of arrivals i	0	1	2	3	4	5	6	7	8
observed frequency O_i	12	11	19	23	22	10	1	0	2

The average number of arrivals was 2.78/minute. We wish to test the "goodness-of-fit" of the assumption that the arrivals correspond to a Poisson process with arrival rate l = 2.78/minute.
The first step is to compute the probability p_i of each observed value, i=0 through 8:

i	p_i
0	0.0620385
1	0.1724670
2	0.2397292
3	
4	0.1543936
5	0.0858428
6	0.0397738
7	0.0157959
8	0.0054890

a. What is the value missing above? (That is, the probability that the number of arrivals is exactly 3.)

b. Complete the table below:

i	p_i	E_i	O_i	$\lvert O_i - E_i \rvert$	$\frac{(O_i - E_i)^2}{E_i}$
0	0.062038507	6.2038507	12	5.7961493	5.4152409
1	0.17246705	17.246705	11	6.2467051	2.2625379
2	0.2397292	23.97292	19	4.97292	1.0315779
3			23		
4	0.1543936	15.43936	22	6.5606404	2.7878101
5	0.085842839	8.5842839	10	1.4157161	0.2334792
6	0.039773849	3.9773849	1	2.9773849	2.2288064
7	0.0157959	1.57959	0	1.57959	1.57959
8	0.0054890752	0.54890752	2	1.4510925	3.8361095

What is the expected number of times in which we would observe three arrivals per minute? Did we observe more or fewer than the expected number?

c. Because of the small number of observations of 6, 7, and 8 arrivals, we will group these observations together with 5 arrivals, so that we will have 6 cells (0, 1, ..., 4, and ≥5). Now, we can compute the expected number of observations in each of these cells, which we denote by E_0 through E_5. What is the expected number of times in which we would observe three arrivals per minute? Did we observe more or fewer than the expected number?

i	p_i	E_i	O_i	$\lvert O_i - E_i \rvert$	$\frac{(O_i - E_i)^2}{E_i}$
0	0.062038507	6.2038507	12	5.7961493	5.4152409
1	0.17246705	17.246705	11	6.2467051	2.2625379
2	0.2397292	23.97292	19	4.97292	1.0315779
3			23		
4	0.1543936	15.43936	22	6.5606404	2.7878101
5	0.14690166	14.690166	13	1.6901663	0.1944608

d. What is the observed value of

$$D = \sum_i \frac{(E_i - O_i)^2}{E_i} \ ?$$

e. Keeping in mind that the assumed arrival rate l=2.78/minute was estimated from the data, what is the number of "degrees of freedom"?

f. Using a value of a = 5%, what is the value of $X^2_{5\%}$ such that D exceeds $X^2_{5\%}$ with probability 5% (if the assumption is correct that the arrivals form a Poisson process with arrival rate 2.78/minute)?

g. Is the observed value greater than or less than $X^2_{5\%}$? Should we accept or reject the assumption that the arrival process is Poisson with rate 2.78/minute?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #5 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

The failure time of an electronic device is the minimum of the failure times of its individual elements (all nonnegative random variables). We will therefore assume that the failure time of the device has approximately a Weibull distribution. Suppose that 100 units of this device are operated simultaneously for 800 hours, at which time 60 have failed, and the failure times are recorded. These are:

Failure times (hrs.) for 60 units failing before 800 hours

9.17	50.76	64.81	76.37	150.55	162.52	173.21	180.25	206.48	211.79
213.47	220.26	230.50	243.49	248.93	271.52	300.68	300.92	306.69	320.16
324.14	394.69	399.12	404.24	405.06	411.12	412.07	414.73	434.94	435.10
447.62	450.84	475.89	490.71	492.67	501.36	501.86	510.83	530.87	533.67
555.69	556.86	559.31	604.51	607.05	669.16	674.71	678.42	678.47	681.88
691.15	695.02	698.77	709.68	714.47	722.95	732.41	743.64	760.81	768.37

The manufacturer wants to determine:

a.) the expected lifetime of the device
b.) the standard deviation of the lifetime
c.) the probability that the device will fail during its first 100 hours of operation.

To estimate the expected lifetime (and its standard deviation) by recording the failure times of all 100 units would require an excessive amount of time, since typically the last few surviving units fail at a low rate. Hence, we wish to estimate the Weibull parameters u & k from the data above, and use these to estimate µ and s.

a. Group the observations as follows:

Interval	# failures	% surviving
0-100	________	_______
100-200	________	_______
200-300	________	_______
300-400	________	_______
400-500	________	_______
500-600	________	_______
600-700	________	_______
700-800	________	_______

b. Enter the observed values of t and R (failure time & reliability, i.e., the fraction surviving) into the Cricket Graph program.

c. Using "transform" on the menu:

- compute ln t and place it into column 3 of the data matrix.
- compute R^{-1} and place it into column 4 of the data matrix.
- compute ln R^{-1} and place it into column 5 of the data matrix.
- compute ln (ln R^{-1}) and place it into column 6 of the data matrix.

d. Plot the scatter graph of the data, with ln t (column 3) on the horizontal axis, and ln (ln R^{-1}) (column 6) on the vertical axis. Do the points appear to lie on a straight line?

e. Fit a line to the points, using the Cricket Graph program. What is the equation of the line? What is its slope and y-intercept?

f. Based upon the fitted line, what are the parameters u & k of the Weibull distribution?

g. Using the parameters u & k, compute the expected number of failures (of the 100 units tested) in each of the intervals:

Interval	observed # failures	Expected # failures
0-100	_________	________
100-200	_________	________
200-300	_________	________
300-400	_________	________
400-500	_________	________
500-600	_________	________
700-800	_________	________

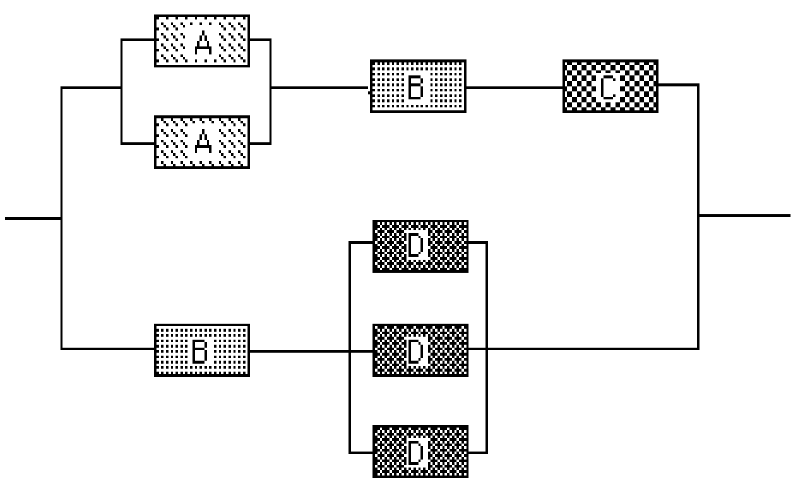
h. Perform a Chi-Square Goodness of Fit test to determine whether this distribution provides an acceptable fit to the observed data, with a = 5%.

i. According to these estimates of u & k, at what time should 5% of the units have failed? ... 50% of the units? ... 90% of the units?

j. According to the value of k, is the failure rate *increasing* or *decreasing* with time?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #6 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. A system contains 4 types of devices, with the system reliability represented schematically by



It has been estimated that the lifetime probability distributions of the devices are as follows:

A: Weibull, with mean 800 days and standard deviation 1200 days
B: Exponential, with mean 3000 days
C: Normal, with mean 1500 days and standard deviation 500 days
D: Exponential, with mean 1000 days

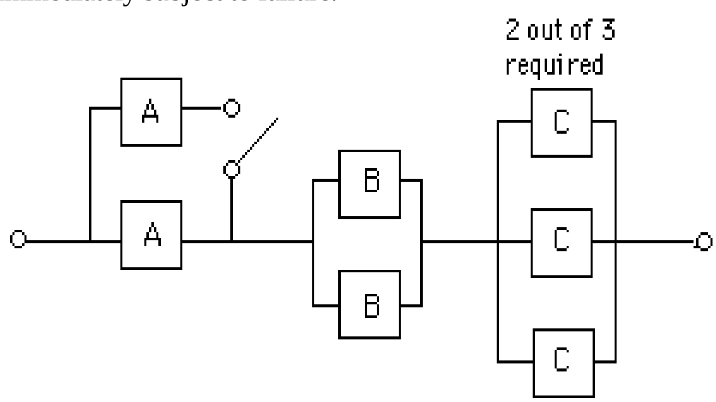
a.) Compute the reliabity of a unit of each device for a designed system lifetime of 1000 days:

Device	Reliability
A	________
B	________
C	________
D	________

b.) Using the reliabilities in (b), compute the system reliability:

Subsystem	Reliability
AA	________
AA+B+C	________
B+DDD	________
Total system:	________

2. A system has 3 types of components (A, B, & C) which are subject to failure. As shown below, the system requires at least one of components A and B, and at least two of component C. One of component A is in stand-by. When the first component A has failed, the second is to be switched on, and until then does not "age" or fail. Assume that the sensor/switch has 95% reliability. In the case of components B & C, on the other hand, all units of these components are in operation simultaneously, so that each unit is immediately subject to failure.



The lifetime distributions of the three component types are:

Component A: Erlang, being the sum of five random variables, each having exponential distribution with mean 50 days.
Component B: Exponential, with expected lifetime 200 days.
Component C: Exponential, with expected lifetime 250 days.

a. Draw a SLAM network which can simulate the lifetime of this system.

b. Enter the network into the computer, and simulate the system 1000 times, collecting statistics on the time of system failure. Request that a histogram be printed. Specify about 15-20 cells, with HLOW and HWID parameters which will give you a "nice" histogram with the mean approximately in the center and with small tails. (This may require a second simulation, with the histogram parameters selected after observing the results of the first simulation.)

Note: Be sure to specify on the GEN statement that you do NOT want intermediate results, and that the SUMMARY report is to be printed only after the 1000th run. Also specify on the INITIALIZE statement that the statistical arrays should not be cleared between runs. The following should work:

```
GEN,yourname,RELIABILITY,3/2/94,1000,,N,,N,Y/1000,72;
LIM,,,8;
INIT,,,NO;
NETWORK;
        (SLAM II network statements go here)
```

```
            END;
      FIN;
```

c. What is your estimate of the average lifetime of this system, based upon the simulation results?

d. Suppose that the system is required to survive for a 100-day mission. What is the estimated reliability of the system, i.e., the probability that the system survives 100 days?

e. Suppose your company will offer a warranty on this system, specifying the length of the warranty such that 95% of the systems will survive past the warranty period. What should be the length of the warranty?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #7 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. A bank currently has two tellers, each teller having his own queue. Customers arrive randomly at the rate of 1 per minute, and choose the shorter queue. The average service time by teller #1 is 60 seconds, with a standard deviation of 10 seconds, while for teller #2 the mean is 40 seconds with a standard deviation of 20 seconds. Assume that for each teller, the service time has a normal distribution, and that a customer cannot "jump" from one queue to the other, once he has selected a queue.

   A new arrangement is being considered by the management, in which a single queue forms, and the customer at the head of the queue chooses the next available teller. (If both tellers are available when the customer arrives, assume that the choice of teller is random.) Simulate both systems (using SLAM II) for an 8-hour day, and compare the average time in the systems for the customers. (What, if any, is the % reduction in average time in the system if the new plan is implemented?)

2. The arrival of copper coils at an annealing oven forms a Poisson process, with an average arrival rate of 2/minute. When enough coils have arrived to fill the oven (with a capacity of 12), the annealing process begins. This process is timed so that the coils remain in the oven for exactly 5 minutes. At that time they are removed to be packaged, 12 coils to a carton. The empty cartons arrive in bundles of 10, with time between arrival of the bundles having normal distribution with mean 60 minutes and standard deviation 10 minutes. One person works at this packaging station, with the time required to fill the carton, seal it, and carry it to the shipping dock having a normal distribution with mean 4 minutes and standard deviation 1 minute.

   Using SLAM II, simulate this system for a 24-hour day. What are:
   a. The number of cartons filled during this day?
   b. The average and maximum number of annealed coils waiting to be packaged?
   c. The maximum and minimum busy intervals for the packager?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #8 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Project Scheduling. A building contractor is preparing a project schedule for the construction of a house. The activity descriptions and estimated durations (in days) for the project are:

			Duration				
Activity	Description	Prede-cessor(s)	opti-mistic	most likely	pessi-mistic	Mean	Std Dev'n
A	Excavate foundations	none	1	2	3	____	____
B	Pour footings	A	-	1	-	1	0
C	Pour foundations, including placing & removing forms	B	2	3	5	____	____
D	Framing floors, walls, & roof	C	6	9	13	____	____

E	Construct brick chimney	C	2	3	4	____	____
F	Install drains & rough plumbing	D	2	3	4	____	____
G	Pour basement floor	F	-	1	-	1	0
H	Install rough wiring	D	1	2	3	____	____
I	Install water lines	D	2	3	5	____	____
J	Install heating ducts	D,E,G	4	5	7	____	____
K	Lath & plaster walls	H,I,J	7	11	14	____	____
L	Finish flooring	K	-	2	-	2	0
M	Install kitchen equipment	L	2	3	5	____	____
N	Install bath plumbing	L	-	1	-	1	0
O	Cabinetwork	M,N	4	6	8	____	____
P	Lay roofing	D	2	3	5	____	____
Q	Install downspouts & gutters	P	-	1	-	1	0
R	Paint walls & trim	O	4	6	7	____	____
S	Sand & varnish floors	R	1	2	3	____	____
T	Install electric fixtures	H,O	1	2	3	____	____
U	Grade lot	C,Q	1	2	4	____	____
V	Landscape	U	4	5	8	____	____

a. Assuming that random durations have the BETA probability distribution, compute the expected value of each and write in the table above. (You need not compute the standard deviations, except for those activities which are determined later to be 'critical'.)

b. Draw the AON (activity-on-node) network representing this project.

c. Schedule this project by entering the AON network into MacProject II (found on several of the Macintosh II computers in the computer lab on 3rd floor of the Engineering Building.) Specify that the start time for the project will be April 1, 1996. What is the earliest completion time for the project? (Note that 5-day work weeks are assumed by default.)

d. Draw the AOA (activity-on-arrow) network representing this project. Explain the necessity for any "dummy" activities which you have included.

e. Label the nodes of the AOA network, so that i<j if there is an activity with node i as its start and node j as its end node.

Determining the critical path. *In questions (f) through (j), use the "most likely" as the duration:*

f. Perform the forward pass through the AOA network to obtain for each node i, ET(i) = earliest possible time for event i.

g. What is the earliest completion time (in work days) for this project?

h. Perform the backward pass through the AOA network to obtain, for each node i, LT(i) = latest possible time for event i (assuming the project is to be completed in the time which you have specified in (e).)

i. For each activity, compute:

ES = earliest start time EF = earliest finish time
LS = latest start time LF = latest finish time
TF = total float (slack)

j. Which activities are "critical", i.e., have zero float ("slack")?

Now, assume that the durations are random, with BETA distribution.

k. What is the expected duration of the critical path found in (j)?

l. What is the standard duration of the critical path found in (j)?

m. According to the PERT technique, what is the probability that the project duration will not exceed the expected duration by more than 5 days?

Simulation of construction project. *(For convenience, use triangular distribution instead of BETA distribution for the random durations-- the BETA parameters required by SLAM are not the three parameters given above, and their meanings aren't well specified!)*

n. Draw a SLAM network to simulate this project, and simulate it 1000 times.

o. What is the average duration of the project according to SLAM? (Compare it to your answer in (k).)

p. What is the standard deviation of the project duration according to SLAM? (Compare it to your answer in (l).)

(Some discrepancy in (o) and (p) can be attributed to the use of the triangular distribution instead of the BETA distribution in the SLAM model.)

q. According to the SLAM simulation, what is the probability that the project duration will not exceed the expected duration by more than five days? Compare with PERT's estimate which you found in (m).

r. Does your SLAM output provide you with information about which path in each simulation run was "critical", i.e. the longest path? If so, how often was the path determined in (j) the longest path? If there isn't enugh information to answer this, suggest a modification to your SLAM network which would provide this information. (You need not actually modify nor re-run your simulation model, however.)

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #9 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

A certain large shop doing light fabrication work uses a single central storage facility (dispatch station) for material in in-process storage. The typical procedure is that each employee personally delivers his finished work (by hand, tote box, or hand cart) and receives new work and materials at the facility. Although this procedure worked well in earlier years when the shop was smaller, it appears that it may now be advisable to divide the shop into two semi-independent parts, with a separate storage facility for each one. You have been assigned the job of comparing the use of two facilities and of one facility from a cost standpoint.

The factory has the shape of a rectangle 150 by 100 yards. Thus, by letting 1 yard be the unit of distance, the (x,y) coordinates of the corners are (0,0), (150,0), (150,100), and (0,100). With this coordinate system, the existing facility is located at (50,50) and the location available for the second facility is (100,50).

Each facility would be operated by a single clerk. The time required by a clerk to service a caller has an Erlang-2 distribution, with a mean of 2 minutes. Employees arrive at the present facility according to a Poisson input process at a mean rate of 24 per hour. The employees are rather uniformly distributed throughout the shop, and if the second facility were installed, each employee would normally use the nearer of the two facilities. Employees walk at an average speed of about 5K yards/hour. All aisles are parallel to the outer walls of the shop. The net cost of providing each facility is estimated to be about $20/hour, plus $15/hour for the clerk. The estimated total cost of an employee being idled by traveling or waiting at the facility is $25/hour.

Given the preceding information, build and simulate a SLAM model in order to determine which alternative minimizes the average total cost per hour.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #10 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. Consider a check-out counter at a grocery store. Customers arrive at the check-out randomly, with a rate of one every 2 minutes. The grocery store clerk requires an average of one minute and 30 seconds to serve each customer. However, as soon as the waiting line exceeds 2 customers, plus the customer being served, the manager helps by packing the groceries, which reduces the average service time to one minute. If there are 4 customers in the waiting line (plus the customer being served), a new check-out counter is opened, and no more customers enter the waiting line. Assume a Poisson arrival process and exponentially-distributed service times.

a. Draw the flow diagram for a birth-death model of this system.
b. Compute the steady-state distribution of the number of customers at the check-out.
c. What fraction of the time will the check-out clerk be idle?
d. What is the expected number of customers in the check-out area?
e. What is the expected length of time that a customer spends in the check-out area?
f. Simulate this system using SLAM. According to the simulation results, what are:
   • fraction of the time that the check-out clerk willbe idle?
   • the expected number of customers in the check-out area?
   • the expected length of time that a customer spends in the check-out area?
   Compare these results with the theoretical results in (c)-(e).

2. A job shop has four numerically controlled machines that are capable of operating on their own (i.e., without a human operator) once they are set up with the proper cutting tools and all adjustments are made. Each setup requires the skills of an experienced machinist, and the time need to complete a setup is exponentially distributed with a mean of 30 minutes. When the setup is complete, the machinist just pushes a button, and the machine requires no further attention until it has finished its job and is ready for another setup. The job tiimes are exponentially distributed with a mean of one hour. The question is, "how many machinists should there be to tend the machines?" At opposite extremes, there could be one machinist tending all four machines, or there could be one machinist for each machine. The optimal number to have obviously depends on a trade-off between the cost of machinists and the cost of idle machines. Of course, machinists are paid the same regardless of how much work they do, but each machine incurs idle-time costs only when it is idle.

Assume that the cost of a machinist (including fringe benefits, etc.) is $20 per hour, and that the cost of an idle machine (including lost revenues, etc.) is $60 per hour of idleness. (Assume also that the machinists are not assigned to specific machines, but are responsible for all machines.) For each alternative (i.e., 1, 2, 3, or 4 machinists) answer (a) through (f):

a. Sketch the transition diagram and set up the transition rate matrix.
b. Compute the steady-state distribution.
c. What is the percent of the time that each machinist is busy?
d. What is the average number of machines in operation?
e. What is the percent of the time that each machine is busy (i.e., the utilization)?
f. What is the total cost of the alternative?
g. What is the optimal number of machinists?
h. Using SLAM, simulate this system with the number of machinists found in (g). According to the simulation results, what are:
   • the average number of machines in operation?
   • the utilization of the machines?
i. Repeat (h), but with job times having a Normal distribution with mean 1 hour and standard deviation 15 minutes. Is the utilization higher or lower than in (h)? What was the standard deviation of the job times in (h)?

«»