«»«»«»«»«» 57:022 Principles of Design II «»«»«»«»«»
Homework Assignments, Spring 1992
Prof. Dennis L Bricker, Dept. of Industrial Engineering

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #1 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

*Be sure to state what probability distribution you assume in each problem. You may find the necessary probabilities in tables, compute them manually with a calculator, or use the APL workspace "PROBLIB" which can be found on the ICAEN fileserver (in the folder for IE). (Using the APL workspace requires that you have a disk containing APL*PLUS, an interpreter for the APL language. If one is not available in the lab, and you wish to use APL, see the instructor or TA for the course.)*

1. The foreman of a casting section in a certain factory finds that on the average, 1 in every 5 castings made is defective.
   a. If the section makes 8 castings a day, what is the probability that 2 of these will be defective?
   b. What is the probability that 5 or more defective castings are made in one day?

2. A certain production process has a fraction defective of 8%. Three good pieces are required. Pieces are produced until the 3 good pieces are obtained.
   a. What are the expected value and standard deviation of the number of pieces produced?
   b. Compute the probabilities of producing no more than 5 pieces.
   c. Suppose that instead of producing until 3 good pieces are obtained, a batch of size **n** is produced, and then the pieces are inspected. How large should **n** be in order to be 95% certain of obtaining at least 3 good pieces?

3. A light bulb in an apartment entrance fails randomly, with an expected lifetime of 15 days, and is replaced immediately by the custodian. Assume that this lifetime has an *exponential* distribution.
   a. What is the probability that a bulb lasts longer than its expected lifetime?
   b. If the current bulb was inserted 10 days ago, what is the probability that its lifetime (since it was inserted) will exceed the expected lifetime of 15 days?
   c. If you were to test 5 of these bulbs, what is the probability that more than half will exceed the expected lifetime?
   d. If the custodian has 2 spare bulbs, what is the probability that these will be sufficient for the next 40 days?

4. A city's population is 60% in favor of a school bond issue. What is the probability that, if a random sample of ten citizens are polled, the majority of those polled will oppose the issue?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #2 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

*Be sure to state what probability distribution you assume in each problem. You may find the necessary probabilities in tables, compute them manually with a calculator, or use the APL workspace "PROBLIB" which can be found on the ICAEN fileserver (in the folder for IE). (Using the APL workspace requires that you have a disk containing APL*PLUS, an interpreter for the APL language. If one is not available in the lab, and you wish to use APL, see the instructor or TA for the course.)*

1. Trucks arrive at an unloading dock of a warehouse according to a Poisson process, at the rate of 4/hour.
   a. What is the probability distribution of the arrival time of the first truck?
      ...the arrival time of the 10^{th} truck?
      ...the time between the arrivals of the first and second truck?
   b. According to the Central Limit Theorem, the arrival time of the 10^{th} truck has approximately what distribution? (What is its mean and standard deviation?)

c. What is the probability distribution function of the maximum times between arrivals of the first 10 trucks?
d. Five minutes are required to unload a truck. What is the probability that one of the first ten trucks will arrive within less than five minutes of the preceeding truck? (In the case of the first truck, within five minutes of the beginning of the day.)

We wish to simulate this Poisson process by generating the arrival times of the trucks.

e. Generate 3 uniformly-distributed 4-digit numbers, using the Midsquare Method, with the last 4 digits of your ID# as the "seed".
f. Use the 3 numbers which you obtained in (e) to compute the arrival times of the first 3 trucks by the inverse transformation technique.
g. Why cannot the Rejection Technique be used in (f)?

2. An electronic device consists of many components, each subject to failure. The failure time of the device is, therefore, the minimum of the failure times of the individual components (all nonnegative random variables). We will therefore assume that the failure time of the device has approximately a Weibull distribution. Suppose that data has been collected on the failure times of a large number of these devices, and it is found that the average failure time is 200 days, with a standard deviation of 50 days.
   a. What is the coefficient of variation of the failure time?
   b. Using the plot or the table in the Hypercard stack on Extreme Value Distributions, estimate the parameter k of the Weibull distribution of failure time.
   c. Compute the parameter u of the Weibull distribution of failure time.
   d. What is the probability that the next device selected from the shelf will fail before its 90-day warranty expires?
   ... that it will still be operating one year from now?
   e. If your company buys 10 of these devices, what is the probability that no more than two of them fail within 90 days?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #3 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. The failure time of a light bulb is the minimum of the failure times of the individual components (all nonnegative random variables). We will therefore assume that the failure time of the light bulb has approximately a Weibull distribution. Suppose that 500 of a new type of bulb are operated simultaneously, and every 100 hours the total number of failed bulbs is recorded. These are:

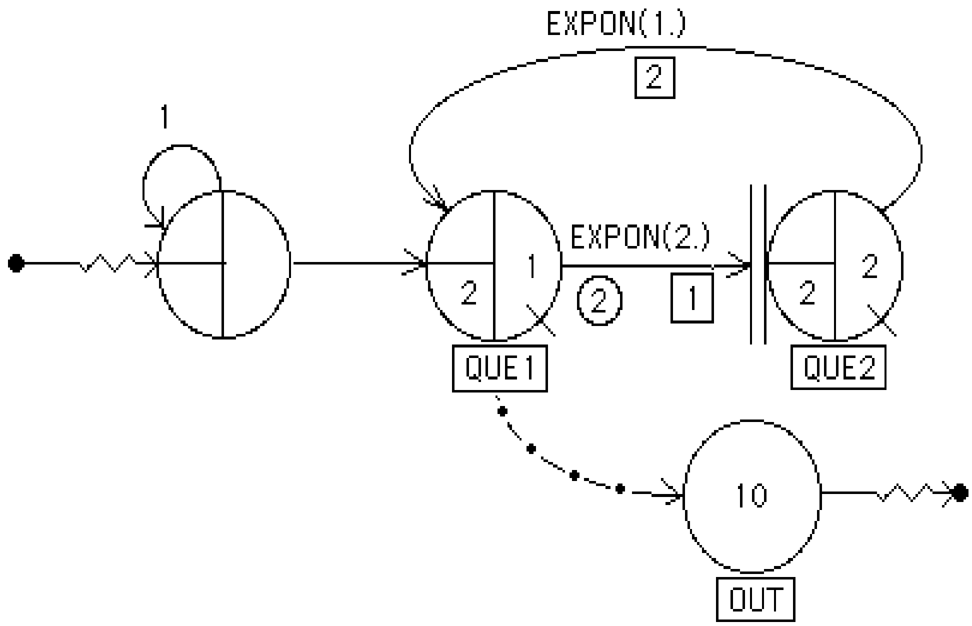
time (hrs.)	# failures
100	39
200	97
300	144
400	177
500	212
600	245

   We wish to estimate the Weibull distribution parameters u & k, without obtaining the mean and standard deviations of the failure times of all 500 bulbs, which would require an excessive amount of time.
   a. Enter the observed values of t and R (failure time & reliability, i.e., the fraction surviving) into the Cricket Graph program.
   b. Using "transform" on the menu:
      - compute ln t and place it into column 3 of the data matrix.
      - compute R^{-1} and place it into column 4 of the data matrix.
      - compute ln R^{-1} and place it into column 5 of the data matrix.
      - compute ln (ln R^{-1}) and place it into column 6 of the data matrix.
   c. Plot the scatter graph of the data, with ln t (column 3) on the horizontal axis, and ln (ln R^{-1}) (column 6) on the vertical axis. Do the points appear to lie on a straight line?
   d. Fit a line to the points, using the Cricket Graph program. What is the line?

e. Based upon the fitted line, what are the parameters u & k of the Weibull distribution?
f. According to these estimates of u & k, at what time should 5% of the bulbs have failed? ... 50% of the bulbs?
g. According to the value of k, is the failure rate increasing or decreasing with time?
h. Suppose that 6 of these bulbs are installed in ceiling lamps in an office. What is the probability that, after 150 hours, at least 3 bulbs are still functioning?

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #4 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

1. Write the SLAM statements for the network model below, and briefly describe the system:



2. Draw the network model which corresponds to the SLAM statements below, and briefly describe the system:

```
NETWORK;
        CREATE,EXPON(2.),,,50;
        ASSIGN,ATRIB(1)=5.0;
            ACTIVITY,,,WAIT;
        CREATE,EXPON(4.),,,50;
        ASSIGN,ATRIB(1)=10.0;
WAIT    QUEUE(1),,4,BALK(HUFF);
        ACTIVITY(2)/1,ATRIB(1)+RNORM(0.,1.);
        TERM;
HUFF    TERM,5;
        END;
```

When will the simulation be terminated?

3. At a drive-in bank, with only one teller, there is space for 5 waiting cars. Customers arrive according to a Poisson process at the average rate of 1 every 2 minutes. The time spent by a customer at the teller window is uniformly distributed between 1 minute and 3 minutes. If a customer arrives when the waiting line is full, the customer drives around the block (which requires an amount of time having normal distribution with average 2 minutes and standard deviation 30 seconds) and tries to join the waiting line

again. Initially, no customers are waiting, and the teller is idle. The simulation is to be terminated when 100 cars have been served. Draw the SLAM network model for this system.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #5 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

NOTE: If you choose to work in pairs, only one homework solution need be submitted for the pair of students.

Project Scheduling: Hawkeye Construction Co. has prepared the following table listing the tasks required to complete construction of a house:

Task #	Task Description	Immediate Predecessor(s)	Most likely time (days)	Optimistic time (days)	Pessimistic time (days)
1	Walls & ceilings	2	5	3	7
2	Foundation	none	2	1	4
3	Roof timbers	1	2	1	2.5
4	Roof sheathing	3	3	2.5	3.5
5	Electrical wiring	1	4	2.5	6
6	Roof shingles	4	8	5	10
7	Exterior siding	8	5	2.5	8
8	Windows	1	2	1	3
9	Paint	6,7,10	2	1	5
10	Inside wallboard	8,5	3	2	4

a. Draw an A-O-N (activity on node) network representing this project.

b. Draw an A-O-A (activity on arrow) network representing this project. Are any "dummy" tasks required?

c. Number the nodes so that if there is an arrow from node i to node j, then i<j.

In parts (c)-(g), assume that the most likely completion times will be the actual completion times:
d. Compute the early times for each node.

e. What is the earliest completion time for the house?

f. Compute the latest times for each node in order to complete the house as early as possible.

g. For each task, compute the
- Early Start Time
- Early Finish Time
- Late Start Time
- Late Finish Time
- Total Float ("slack")

h. Which tasks are "critical"?

i. What, according to the assumptions of PERT (i.e., the Central Limit Theorem), is the probability distribution of the completion time of the house?

j. What are the mean and standard deviation of each activity on the critical path, assuming that each has the beta distribution?

k. What are the mean and standard deviation of the length of the critical path, under the assumptions of PERT? What is the name of the probability distribution of this project completion time, according to PERT?

l. Suppose that the company contracts to build the house in a length of time equal to your answer to (e) plus 5 days. Based on your answer to (k), what is the probability that the house can be completed within this time?

m. Draw a SLAM network corresponding to your A-O-A network in (b). Assume that activity durations have the triangular distribution (rather than beta), for simplicity.

n. Run the simulation of the construction project 500 times. Collect statistics on the project completion time, including a request for a histogram. For the histogram, choose the cell width to be one day, with the mean completion time you computed in (k) approximately in the middle of the histogram.
Be sure to indicate on the INITIALIZE statement that JJCLR is N, i.e., statistical arrays are not to be cleared after each of the 500 runs. Also indicate on the GEN statement that ISMRY/FSN is Y/500, i.e., that the summary report is to be printed only on the last (500th) run.

o. Answer the question in (l) again, based on your simulation results.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #6 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

NOTE: If you choose to work in pairs, only one homework solution need be submitted for the pair of students.

Draw a SLAM network model for a manufacturing department which processes items at 2 stations, or machines. (You need not run the simulation model, although doing so is a good way to test your model.)

- Items may visit the stations in any order, but must visit each station exactly once before it leaves the department.
- Each station processes only one item at a time, and has unlimited space for items awaiting service.
- When items arrive (at a constant rate of 2/minute), they are directed to the station with the smallest number of waiting items.
- At each station, items which have already visited the other station have priority for service.
- Processing time at station #1 is normally distributed with mean 0.4 minute and standard deviation 0.1 minute, while processing time at station #2 is normally distributed with mean 0.6 minute and standard deviation 0.2 minute.
- The arrival process stops after 8 hours, but the department continues working until all items currently in the department have been processed.

Statistics are desired on the number of items waiting at each station as well as the time spent by items in the department.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #7 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #8 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

A system has 6 components which are subject to failure, each having lifetimes with exponential distributions. The average lifetimes are:

Component	Average Lifetime
A	400 days
B	600 days
C	600 days
D	2000 days

E	2000 days
F	4000 days

The failure of either D, E, or F will result in system failure, but all three of A, B, and C must fail to cause system failure.

1. Draw a diagram showing the series/parallel configuration of the components, as in the Hypercard Stack "System Reliability".
2. Suppose that the system is required to survive for 500 days. What is the reliability of each component, i.e., the probability that it survives 500 days?
3. What is the reliability of the system, i.e., the probability that the system survives 500 days?
4. Draw a SLAM network which can simulate the lifetime of this system.
5. Run the SLAM simulation model, using 500 runs, collecting statistics on the time of system failure. Request that a histogram be printed. Select about 15 cells, with HLOW and HWID parameters which will give you a "nice" histogram with the mean approximately in the center and small tails.
6. Suppose you will offer a warranty on this system, such that 95% of the systems will survive past the length of the warranty. What should be the length of the warranty?
7. If the time to system failure has a Weibull distribution with the mean and standard deviation obtained from the simulation model, what are the two parameters (shape and location) of the model?
8. Based upon the Weibull model with these parameters, what is the *expected* number of observations in each cell of the histogram obtained from the simulation run?
9. Do a "Chi-square goodness-of-fit" test of the Weibull model with the parameters you specified in (7) and alpha=5%. What is your conclusion?

The APL workspace "PROBLIB" can be used for the calculation of the probabilities and the Weibull parameters. (Recall that an Exponential distribution is a special case of Erlang, with k=1.)

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #9 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Exercise 6.7 (Chapter 6 of SLAM text by Pritsker).

Hints & suggestions:

1. In your SLAM network, model the airplanes as entities, with the cargo as RESOURCE.
2. Draw separate network segments for large planes and small planes.
3. Pritsker probably intended that you use an ALTER node to increase the amount of resource available for allocation to the planes. You can avoid this by specifying the initial amount of resource available on the RESOURCE block to be some large number (> the maximum amount that will ever be waiting for a plane). Let's arbitrarily use 1000. (Total cargo carried by the five planes is only 520 units.)
4. You need to create a single entity which arrives at an AWAIT node and "gobbles" up this 1000 units of resource initially, to be FREEd later.
5. On the RESOURCE block, specify that the small planes are given priority over the large planes in allocating the resource (i.e., the cargo).
6. In another network segment, you need to have an entity circulating through a loop, so that each 0.5 minutes (does the book mean exactly, or average?) 1 unit of cargo (the resource) is FREEd.
7. When planes are allocated the resource (cargo) at their AWAIT nodes, they then leave and return to the AWAIT nodes after a random round-trip time.
8. To generate the round trip times, the probability distribution should be a "truncated normal", i.e., a normal distribution with mean 3 hours, standard deviation 1 hour, but truncated on the left at 2 hours and the right at 4 hours. To do this, assign a system variable XX(1) the value RNORM(3,1). Draw 2 branches leaving the ASSIGN node, one representing the round trip with the condition XX(1).LE.4.AND.XX(1).GE.2, and the other branch looping back to the same ASSIGN node if the first branch is not selected. In this way, SLAM will continue to generate random numbers until it obtains one within the desired range.
9. Use the system variable just assigned, i.e., XX(1), for the duration of the round-trip activity.

If you can discover another way to model this system, please let me know! I failed in every attempt which I made to use the cargo as entities, and ACCUMULATE nodes to create the loads, because of the complication of the 2 types of planes. The model which is described above is suggested by the example in the book which models the inventory control system, with the demand modeled as RESOURCE.

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«» HW #10 «»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

The following exercises should be done assuming that the queueing system operates in steady state.

1. Each airline passenger and his/her luggage must be checked to determine if he/she is carrying weapons onto the airplane. Suppose that at C.R. Airport an average of 10 passengers/minute arrive (with exponentially-distributed times between arrivals). To check passengers for weapons, the airport must have a checkpoint consisting of a metal detector and baggage X-ray machine. Whenever a checkpoint is in operation, two employees are required. A checkpoint can check an average of 12 passengers/minute (one at a time), with the time for each having an exponential distribution. Assume that the airport has only one checkpoint.
   a. What is the probability that a passenger will have to wait before being checked?
   b. On the average, how many passengers are waiting in line to enter the checkpoint?
   c. On the average, how long will a passenger spend at the checkpoint?

2. ***(Revised)*** An average of 100 customers arrive each hour at a bank, forming a Poisson process. The service time per customer has exponential distribution, with mean 1 minute. The manager wants to ensure that the *average* time which customers will have to wait in line is no more than 0.5 minute. If the bank follows the policy of having all customers join a single queue to wait for a teller, how many tellers should the bank hire?

3. An average of 60 cars per hour arrive (forming a Poisson process) arrive at the MacBurger's drive-in window. However, if a total of more than 4 cars are in line (including the car at the window), an arriving car will not enter the line. It takes an average of 3 minutes (exponentially distributed) to serve a car.
   a. What is the average number of cars waiting for the drive-in window (not including a car at the window)?
   b. On the average, how many cars will be served per hour?

c. If you have just joined the line, how many minutes will you expect to pass before you receive your food?

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