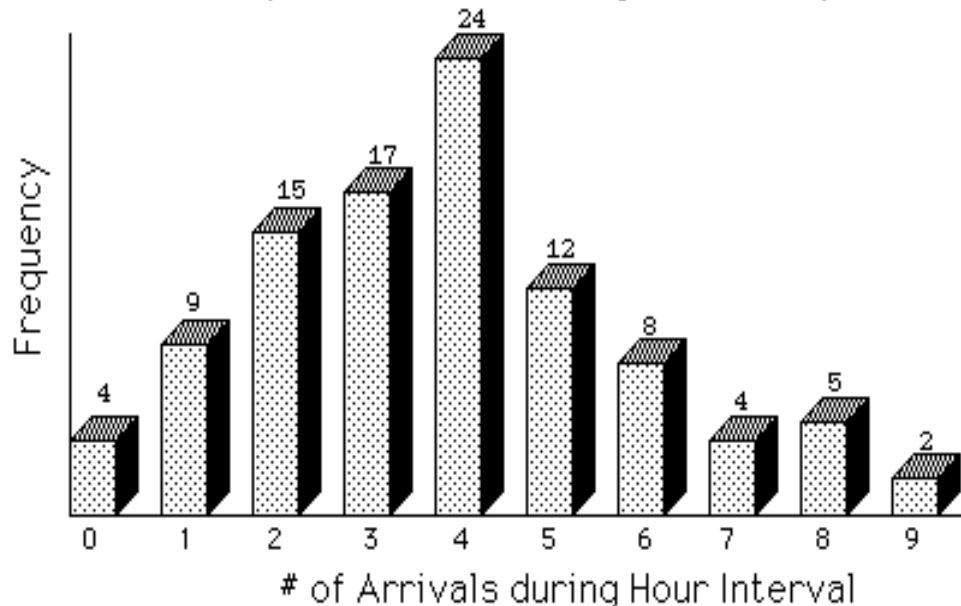


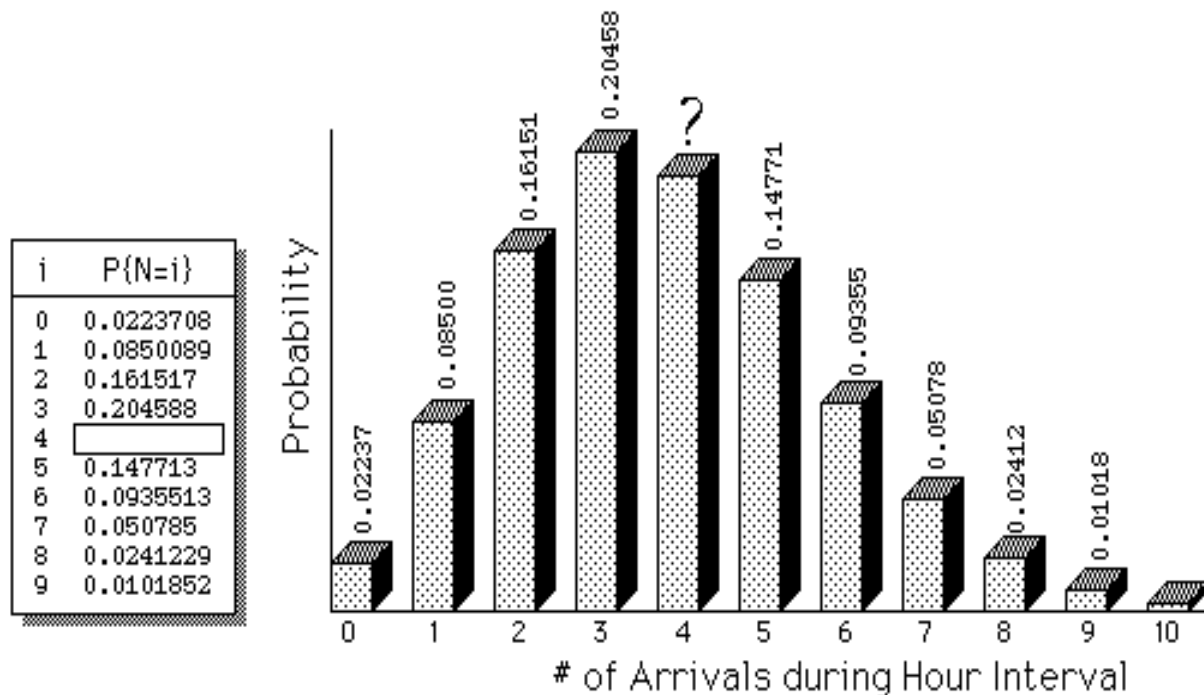
Select a column based upon the last digit of your ID#: if 1, use column #1; if 2, use column #2; ... if 0, use column #10.

- What is the name of the probability distribution of the time T_1 of the first arrival?
 - What will be the probability distribution of the time t_i between arrivals of parts $i-1$ and i (where $i > 1$)?
 - Use the inverse-transformation method to obtain (pseudo-)random inter-arrival times t_1, t_2, \dots, t_8 (where $T_1 = t_1$).
 - What are the arrival times (T_1, T_2, \dots, T_8) of the first eight parts in your simulation?
 - The expected number of arrivals in the first hour should, of course, be 4. In your simulation of the arrivals, however, what is the number of arrivals during the first hour?
- Manual Simulation of Drive-In Teller Window.** In the Hypercard stack, the arrival of the first three autos at the drive-in teller window was manually simulated. Repeat the simulation manually up to the departure of the 10th auto, except reverse the sequence of the 25 randomly-generated inter-arrival times. (That is, the first arrival should now be at $t=3$, the second 1 minute later, etc., so that $T_a = \{3, 1, 4, 1, 11, 7, 6, 3, 6, 5, 2, 1, 4, 8, 4, 5, 4, 13, 3, 1, 2, 2, 1, 6, 5\}$. Give the event log and the event schedule at that time. What was the maximum length of the waiting line during the simulation?
 - The numbers of arrivals during 100 hours of what is believed to be a Poisson process were recorded. The observed numbers ranged from zero to nine, with frequencies O_0 through O_9 :



The average number of arrivals was 3.8/hour. We wish to test the "goodness-of-fit" of the assumption that the arrivals correspond to a Poisson process with arrival rate 3.8/hour.

The first step is to compute the probability of each observed value, 0 through 9:



- a. What is the value missing above? (That is, the probability that the number of arrivals is exactly 4.)
- b. Now, we can compute the expected number of observations of each of the values 0 through 9, which we denote by E_0 through E_9 . What is the expected number of times in which we would observe five arrivals per hour? Did we observe more or fewer than the expected number?
- c. Complete the table below:

i	P(N=i)	E_i	O_i	$(E_i - O_i)^2$	$\frac{(E_i - O_i)^2}{E_i}$
0	0.0223708	2.23708	4	3.1079	1.38927
1	0.0850089	8.50089	9	0.249107	0.0293037
2	0.161517	16.1517	15	1.32641	0.0821218
3	0.204588	20.4588	17	11.9634	0.584756
4			24		
5	0.147713	14.7713	12	7.67991	0.519922
6	0.0935513	9.35513	8	1.83639	0.196298
7	0.050785	5.0785	4	1.16317	0.229037
8	0.0241229	2.41229	5	6.69625	2.77589
9	0.0101852	1.01852	2	0.9633	0.945782

- d. Ignoring the suggestion that cells should be aggregated so that they contain at least five observations, what is the observed value of

$$D = \sum_i \frac{(E_i - O_i)^2}{E_i} ?$$

- e. Keeping in mind that the assumed arrival rate $\lambda = 3.8/\text{hour}$ was estimated from the data, what is the number of "degrees of freedom"?
- f. Using a value of $\alpha = 5\%$, what is the value of χ^2_{α} such that D exceeds χ^2_{α} with probability 5% (if the assumption is correct that the arrivals form a Poisson process with arrival rate 3.8/hour)?

- g. Is the observed value greater than or less than $\frac{2}{5}\%$? Should we accept or reject the assumption that the arrival process is Poisson with rate 3.8/hour?

HW #4

- (1.) Consider again the proposed drive-up bank teller window in the Hypercard Stack "Intro. to Simulation". The SLAM model, together with output, appears below. According to this output...

- How many cars were unable to enter the queue each day, because the queue was filled to capacity?
- What fraction of the time was the teller busy each day?
- Estimate the mean (average) time in the system for the customers.

It has been suggested that a customer's time in the system will have exponential distribution, given that the service times have exponential distribution.

- d. Test the "goodness-of-fit" of the exponential probability distribution having the mean value in (c). (Use $\alpha = 10\%$, and group the cells as necessary so that there are at least 5 observations in each cell.)
- e. What fraction of the customers spend more than 5 minutes (total of both waiting and being served) at the bank?

```

1 GEN,BRICKER,BANKTELLERS,2/11/1993,,,,,,,,,72;
2 LIM,2,1,50;
3 INIT,0,480;
4 NETWORK;
5     CREATE,EXPON(5.0),,1;
6     QUE(1),0,4,BALK(OVFLO);
7     ACT(1)/1,EXPON(2.0);
8     COLCT,INTVL(1),CUSTOMER_TIME,20/.5/.5;
9     TERM;
10 OVFLO COLCT,FIRST;
11     TERM,1;
12     END;
13 FIN;

```

SLAM II SUMMARY REPORT
SIMULATION PROJECT BANKTELLERS BY BRICKER

```
DATE 2/ 1/1993                                RUN NUMBER 1 OF 1
```

```
CURRENT TIME      0.4081E+03
STATISTICAL ARRAYS CLEARED AT TIME  0.0000E+00
```

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	0.303E+01	0.286E+01	0.944E+00	0.345E-01	0.110E+02	88
	0.408E+03	0.000E+00	0.000E+00	0.408E+03	0.408E+03	1

```

**FILE  STATISTICS**

```

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	0.300	0.724	4	4	1.317

2		0.000	0.000	0	0	0.000
3	CALENDAR	1.439	0.496	3	2	2.669

SERVICE ACTIVITY STATISTICS

ACT	ACT	LABEL	OR	SER	AVERAGE	STD	CUR	AVERAGE	MAX	IDL	MAX	BSY	ENT
NUM	START	NODE		CAP	UTIL	DEV	UTIL	BLOCK	TME/SER	TME/SER			CNT

1	QUEUE	1	0.439	0.50	1	0.00	17.35	29.23	88
---	-------	---	-------	------	---	------	-------	-------	----

HISTOGRAM NUMBER 1

CUSTOMER TIME

OBS	RELA	UPPER	0	20	40	60	80	100
FREQ	FREQ	CELL LIM	+	+	+	+	+	+
10	0.114	0.500E+00	*****					+
9	0.102	0.100E+01	*****	C				+
16	0.182	0.150E+01	*****		C			+
13	0.148	0.200E+01	*****			C		+
4	0.045	0.250E+01	***			C		+
7	0.080	0.300E+01	****				C	+
5	0.057	0.350E+01	****				C	+
1	0.011	0.400E+01	+				C	+
3	0.034	0.450E+01	***				C	+
0	0.000	0.500E+01	+				C	+
2	0.023	0.550E+01	+				C	+
1	0.011	0.600E+01	+				C	+
3	0.034	0.650E+01	***				C	+
0	0.000	0.700E+01	+				C	+
4	0.045	0.750E+01	***				C	+
1	0.011	0.800E+01	+				C	+
2	0.023	0.850E+01	+				C	+
2	0.023	0.900E+01	+				C	+
2	0.023	0.950E+01	+				C	+
2	0.023	0.100E+02	+				C	+
0	0.000	0.105E+02	+				C	+
1	0.011	INF	+				C	+
---			+	+	+	+	+	+
88			0	20	40	60	80	100

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	0.303E+01	0.286E+01	0.944E+00	0.345E-01	0.110E+02	88

HW #5

1. The failure time of a light bulb is the minimum of the failure times of the individual elements (all nonnegative random variables). We will therefore assume that the failure time of the light bulb has approximately a Weibull distribution. Suppose that 500 of a new type of bulb are operated simultaneously, and every 100 hours the total number of failed bulbs is recorded. These are:

time (hrs.)	# failures
100	42
200	98
300	141

400	177
500	214
600	248

The manufacturer wants to determine:

- the expected lifetime of the bulb
- the standard deviation of the lifetime
- the probability that the bulb will fail during its first 100 hours of operation.

To estimate the expected lifetime (and its standard deviation) by recording the failure times of all 500 bulbs would require an excessive amount of time, since typically the last few surviving bulbs fail at a low rate. Hence, we wish to estimate the Weibull parameters u & k from the data above, and use these to estimate μ and s .

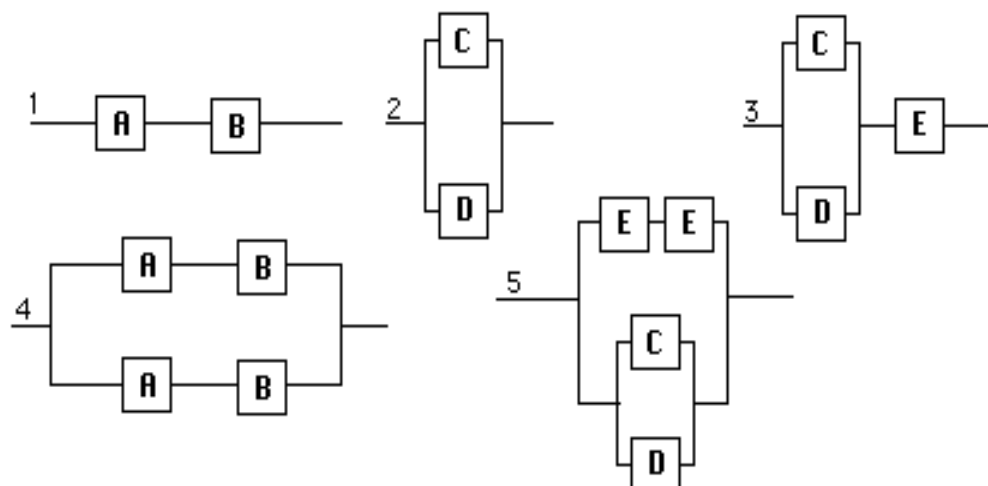
- Enter the observed values of t and R (failure time & reliability, i.e., the fraction surviving) into the Cricket Graph program.
- Using "transform" on the menu:
 - compute $\ln t$ and place it into column 3 of the data matrix.
 - compute R^{-1} and place it into column 4 of the data matrix.
 - compute $\ln R^{-1}$ and place it into column 5 of the data matrix.
 - compute $\ln(\ln R^{-1})$ and place it into column 6 of the data matrix.
- Plot the scatter graph of the data, with $\ln t$ (column 3) on the horizontal axis, and $\ln(\ln R^{-1})$ (column 6) on the vertical axis. Do the points appear to lie on a straight line?
- Fit a line to the points, using the Cricket Graph program. What is the line? What is its slope and y-intercept?
- Based upon the fitted line, what are the parameters u & k of the Weibull distribution?
- According to these estimates of u & k , at what time should 5% of the bulbs have failed? ... 50% of the bulbs? ... 90% of the bulbs?
- According to the value of k , is the failure rate *increasing* or *decreasing* with time?
- Suppose that 6 of these bulbs are installed in ceiling lamps in an office. What is the probability that, after 150 hours of continuous operation, at least 3 bulbs are still functioning? *Hint: Use the binomial distribution... what is the probability p that a single bulb has not failed after 150 hours? What is the probability of 3 "successes" in 6 "trials"?*

HW #6

1. Devices A through E are basic components of five different systems (#1 through #5). They are subject to failure, with the probability that each fails during its first year of use:

Device	Failure Probability
A	10%
B	5%
C	20%
D	25%
E	15%

Compute, for each system below, the reliability, i.e., the probability that it will survive for at least one year.



2. A system has 6 components which are subject to failure, each having lifetimes with exponential distributions. The average lifetimes are:

Component	Average Lifetime
A	2000 days
B	3000 days
C	800 days
D	800 days
E	500 days
F	500 days
G	500 days

The system will fail if any one of the following situations occurs:

- A fails
- B fails
- Both C and D fail
- Components E, F, and G all fail.

- Draw a diagram showing the series/parallel configuration of the components, as in the previous exercise.
- Suppose that the system is required to survive for 1000 days. What is the reliability of each component, i.e., the probability that it survives 1000 days?
- What is the reliability of the system, i.e., the probability that the system survives 1000 days?
- Draw a SLAM network which can simulate the lifetime of this system. (At a later time, you may be required to perform the simulation.)

HW #7

Project Scheduling. A building contractor is preparing a project schedule for the construction of a house. The activity descriptions and estimated durations for the project are:

Activity	Description	Predecessor(s)	Duration			Mean	Std Dev'n
			opti- mistic	most likely	pessi- mistic		
A	Excavate foundations	none	1	2	3	—	—
B	Pour footings	A	-	1	-	1	0
C	Pour foundations, including placing & removing forms	B	3	5	8	—	—
D	Framing floors, walls, & roof	C	8	10	14		

E	Construct brick chimney	C	2	3	4	—	—
F	Install drains & rough plumbing	D	2	3	4	—	—
G	Pour basement floor	F	-	1	-	1	0
H	Install rough wiring	D	1	2	3	—	—
I	Install water lines	D	2	3	5	—	—
J	Install heating ducts	D,E,G	4	5	7	—	—
K	Lath & plaster walls	H,I,J	7	10	12	—	—
L	Finish flooring	K	-	2	-	2	0
M	Install kitchen equipment	L	1	2	4	—	—
N	Install bath plumbing	L	-	1	-	1	0
O	Cabinetwork	M,N	4	6	8	—	—
P	Lay roofing	D	1	2	4	—	—
Q	Install downspouts & gutters	P	-	1	-	1	0
R	Paint walls & trim	O	3	4	5	—	—
S	Sand & varnish floors	R	1	2	3	—	—
T	Install electric fixtures	H,O	2	3	5	—	—
U	Grade lot	C,Q	1	2	4	—	—
V	Landscape	U	4	5	8	—	—
W	End	S,T,V	-	0	-	0	0

- Draw the AON (activity-on-node) network representing this project.
- Draw the AOA (activity-on-arrow) network representing this project. Explain the necessity for any "dummy" activities which you have included.
- Label the nodes of the AOA network, so that $i < j$ if there is an activity with node i as its start and node j as its end node.

In questions (d) through (h), use the "most likely" as the duration:

- Perform the forward pass through the AOA network to obtain for each node i , $ET(i)$ = earliest possible time for event i .
- What is the earliest completion time (# work days) for this project?
- Perform the backward pass through the AOA network to obtain, for each node i , $LT(i)$ = latest possible time for event i (assuming the project is to be completed in the time which you have specified in (e).)
- For each activity, compute:

ES = earliest start time

LS = latest start time

EF = earliest finish time

LF = latest finish time

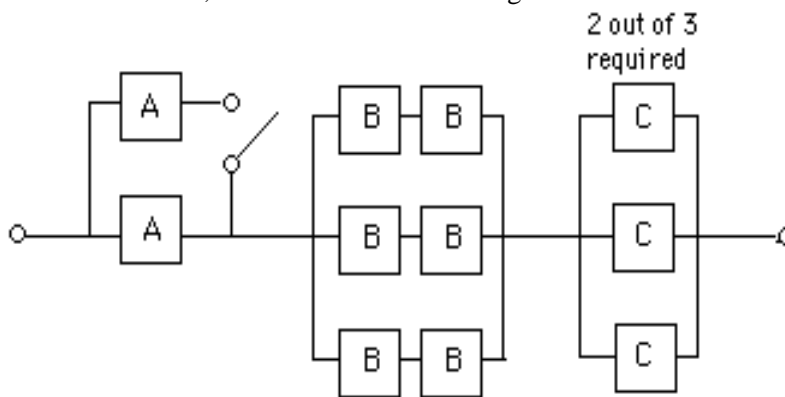
TF = total float (slack)

Activity	Description	ES	EF	LS	LF	TF
A	Excavate foundations	—	—	—	—	—
B	Pour footings	—	—	—	—	—
C	Pour foundations, including placing & removing forms	—	—	—	—	—
D	Framing floors, walls, & roof	—	—	—	—	—
E	Construct brick chimney	—	—	—	—	—
F	Install drains & rough plumbing	—	—	—	—	—
G	Pour basement floor	—	—	—	—	—
H	Install rough wiring	—	—	—	—	—
I	Install water lines	—	—	—	—	—

- b. What is the average duration of the project according to SLAM? (Compare it to your answer in (k) in HW#7.)
- c. What is the standard deviation of the project duration according to SLAM? (Compare it to your answer in (k) in HW#7.)
- d. According to the SLAM simulation, what is the probability that the project duration will not exceed the expected duration by more than five days?

@@

2. A system has 3 types of components (A, B, & C) which are subject to failure. As shown below, the system requires at least one of component A, one pair of components B, and at least two of the three components C. One of component A is in stand-by. When the first component A has failed, the second is to be switched on, and until then does not "age" or fail. Assume that the sensor/switch has 98% reliability.



The lifetime distributions of the three component types are:

Component A: Erlang, being the sum of five random variables, each having exponential distribution with mean 100 days.

Component B: Exponential, with expected lifetime 300 days.

Component C: Exponential, with expected lifetime 250 days.

- a. Draw a SLAM network which can simulate the lifetime of this system.
- b. Enter the network into the computer, and simulate the system 1000 times, collecting statistics on the time of system failure. Request that a histogram be printed. Specify about 15-20 cells, with HLOW and HWID parameters which will give you a "nice" histogram with the mean approximately in the center and with small tails.

Note: Be sure to specify on the GEN statement that you do NOT want intermediate results, and that the SUMMARY report is to be printed only after the 1000th run. Also specify on the INITIALIZE statement that the statistical arrays should not be cleared between runs.

- c. What is the average lifetime of this system?
- d. Suppose that the system is required to survive for a 100-day mission. What is the reliability of the system, i.e., the probability that the system survives 100 days?
- e. Suppose you will offer a warranty on this system, and want to be confident that 95% of the systems will survive past the length of the warranty. What should be the length of the warranty?

Note: Homework #5 of Spring '94 has a somewhat similar exercise, with the SLAM code given in the solutions document. If you are having difficulty coding the SLAM statements, refer to that document.

HW #9

[illegible]