56:171
Operations Research
Fall 1999

Quizzes

Dept of Mechanical & Industrial Engineering
University of Iowa

Name ______________________________

56:171 Operations Research Quiz #1 – September 8, 1999

For each statement, indicate "+"=**true** or "o"=**false**.

___ a. . A "pivot" in a nonbasic column of a tableau will make it a basic column.
___ b. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.
___ c. In the simplex method, every variable of the LP is either basic or nonbasic.
___ d. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0.
___ e. A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.
___ f. The feasible region is the set of all points that satisfy *at least* one constraint.
___ g. The number of basic variables in an LP is equal to the number of rows, *including* the objective function row.
___ h. A "pivot" in row i of the column for variable X_j will increase the number of basic variables.
___ i. Basic solutions of an LP with constraints $Ax \leq b$, $x \geq 0$ correspond to "corner" points of the feasible region.
___ j. In the simplex tableau, the objective row is written in the form of an equation.

Multiple-Choice:

____k. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau
(a) will be nonbasic
(b) will be infeasible
(c) will have a worse objective value
(d) *None of the above*

____l. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
(a) will be nonbasic
(b) will be infeasible
(c) will have a worse objective value
(d) *None of the above*

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Cattle feed can be mixed from oats, corn, alfalfa, and peanut hulls. The following table shows the current cost per ton (in dollars) of each of these ingredients, together with the percentage of recommended daily allowances for protein, fat, and fiber that a serving of it fulfills.

	Oats	Corn	Alfalfa	Peanut hulls
% protein	60	80	55	40
% fat	50	70	40	100
% fiber	90	30	60	80
Cost $/ton	200	150	100	75

We want to find a minimum cost way to produce feed that satisfies at least 60% of the daily allowance for protein and fiber while not exceeding 60% of the fat allowance.

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Define the variables OATS, CORN, etc. to be the quantity (in tons) mixed to obtain a ton of cattle feed. The model & LINDO output is below:

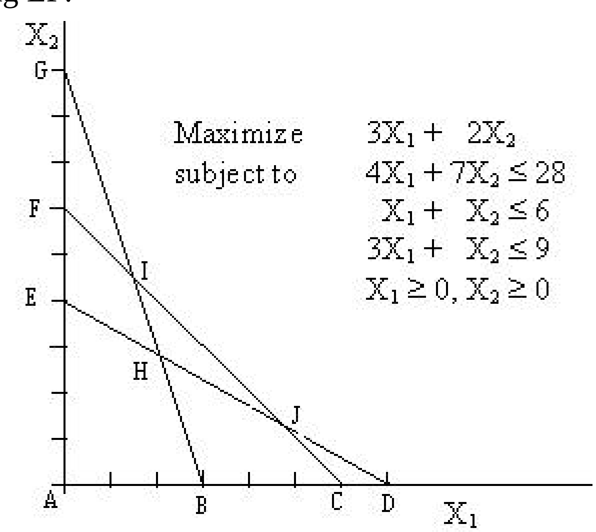
```
MIN     200 OATS + 150 CORN + 100 ALFALFA + 75 HULLS
  SUBJECT TO
         2)   60 OATS + 80 CORN + 55 ALFALFA + 40 HULLS >=   60
         3)   50 OATS + 70 CORN + 40 ALFALFA + 100 HULLS <=   60
         4)   OATS + CORN + ALFALFA + HULLS =    1
  END

OBJECTIVE FUNCTION VALUE
        1)      110.0000

  VARIABLE        VALUE          REDUCED COST
      OATS         0.000000         90.000000
      CORN         0.200000          0.000000
   ALFALFA         0.800000          0.000000
     HULLS         0.000000          5.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -2.000000
        3)        14.000000          0.000000
        4)         0.000000         10.000000
```

The optimal solution is to mix ______ pounds of corn and _______ pounds of alfalfa to obtain a ton (i.e. 2000 pounds) of feed. The cost of a ton of feed is $_________.

There are ____ basic variables in the optimal solution, in addition to –Z (in the cost equation).

_ _

Consider the following LP:



The feasible region is bounded by points ______________________.
At point **H**, the slack variable for constraint # _____ is positive.
Let X_3, X_4, and X_5 represent the slack in constraints 1, 2, and 3, respectively.
The coefficients of X_3, X_4, and X_5 in the *initial* tableau are ______.
Initially, the basic variables are _____________________ (plus –Z representing the profit function).
The optimal solution is at point _____,
where the basic variables are _____________________ (plus –Z).

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Quiz #2 – September 15, 1999

Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. *Also answer the question accompanying each classification, if any.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*

(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*

(**C**) Unique nondegenerate optimum.

(**D**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*

(**E**) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal

Warning: *Some of these classifications might be used for more than one tableau, while others might not be used at all!*

(i) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	-1	3	0	0	2	-2	-45	
0	0	0	-4	0	0	1	3	0	9	________
0	-4	1	2	-5	0	0	-2	1	0	
0	-6	0	3	-2	1	0	-4	3	5	

(ii) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	0	1	0	0	0	12	-45	
0	0	0	-4	0	0	1	3	0	9	______
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	5	

(iii) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	4	0	0	-2	2	-45	
0	0	0	-4	0	0	1	-3	0	3	________
0	4	1	2	-5	0	0	2	1	-8	
0	-6	0	3	-2	1	0	-4	3	15	

(iv) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	________
0	-6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

Name ________________________________

(v) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	1	0	0	-2	0	-45	
0	4	1	2	-5	0	0	2	1	5	_________
0	-6	0	3	2	1	0	-4	3	0	
0	0	0	-4	0	0	1	3	0	9	

(vi) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	2	0	-1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	_________
0	6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

(vii)-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	1	0	0	3	5	-45	
0	0	0	-4	0	0	1	3	0	3	_________
0	4	1	2	-5	0	0	2	1	7	
0	-6	0	3	-2	1	0	-4	3	15	

(viii)-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	-3	0	0	2	0	-45	
0	0	0	-1	0	0	1	3	0	9	_________
0	4	1	-4	-5	0	0	2	1	3	
0	-6	0	3	-2	1	0	-4	3	5	

(ix) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	-3	0	1	1	0	0	2	3	-45	
0	0	0	-4	0	0	1	0	0	9	_______
0	-6	0	3	-2	1	0	2	3	5	
0	4	1	2	-5	0	0	1	1	8	

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Quiz #3 – September 22, 1999

Consider the LP problem:

$$\text{Max } w = 4Y_1 + 2Y_2 - Y_3$$
$$\text{s.t.} \quad Y_1 + 2Y_2 \leq 6$$
$$Y_1 - Y_2 + 2Y_3 = 8$$
$$Y_1 \geq 0,\ Y_2 \leq 0 \ (\ Y_3 \text{ is unrestricted in sign})$$

(Note: this differs somewhat from that in the HW exercise!) The dual of the above problem is

Min _____ X_1 + ______ X_2
s.t _____ X_1 + ______ X_2 _____
_____ X_1 + ______ X_2 _____
_____ X_1 + ______ X_2 _____
sign restrictions: X_1 __ 0, X_2 __ 0

For each statement, indicate "+"=**true** or "o"=**false**.

_____ 1. If you increase the right-hand-side of a "≤" constraint in a <u>min</u>imization LP, the optimal objective value will either increase or stay the same.
_____ 2. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
_____ 3. The "reduced cost" in an LP solution provides an estimate of the change in the objective value when a nonbasic variable increases.
_____ 4. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.
_____ 5. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.
_____ 6. When entering your LP model, the last constraint which you enter should be followed by "**END**".
_____ 7. If a minimization LP problem has a cost which is unbounded below, then its dual problem cannot be feasible.
_____ 8. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
_____ 9. The "minimum ratio test" is used to select the pivot column in the simplex method for linear programming..
____ 10. If a minimization LP problem is infeasible, then its dual problem has an objective (to be maximized) which must be unbounded above.

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Quiz #4 – September 29, 1999

Linear Programming sensitivity. SunCo processes oil into aviation fuel and heating oil. It costs $40 to purchase each barrel of oil, which is then distilled and yields 0.5 barrel of aviation fuel and 0.5 barrel of heating oil. Output from the distillation may be sold directly or processed in the catalytic cracker. If sold after distillation without further processing, aviation fuel sells for $60/barrel. If sold after distillation without further processing, heating oil sells for $40/barrel. It takes 1 hour to process 1000 barrels of aviation fuel in the catalytic cracker, and these can be sold for $130/barrel. It takes 45 minutes to process 1000 barrels of heating oil in the cracker, and these can be sold for $90/barrel. Each day, at most 20,000 barrels of oil can be purchased, and 8 hours of catalytic cracker time are available. Formulate an LP to maximize SunCo's profits.

Define the decision variables

OIL = # of barrels of oil purchased
HSOLD = # of barrels of heating oil sold
HCRACK = # of barrels of heating oil processed in catalytic cracker
ASOLD = # of barrels of aviation fuel sold
ACRACK = # of barrels of aviation fuel processed in catalytic cracker

The LP model to maximize profit is

```
Maximize 40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK - 40 OIL
subject to     OIL = 20000           (available supply)
     0.5 OIL = ASOLD + ACRACK           (aviation fuel & heating oil
     0.5 OIL = HSOLD + HCRACK            each constitute 50% of
                                          product of distilling)
     0.001 ACRACK + 0.00075 HCRACK =  8      (avail. time for cracker)
        OIL >= 0, ASOLD >=0, ACRACK >=0, HSOLD >=0, HCRACK >=0
```

The output of LINDO follows:

```
MAX     40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK - 40 OIL
SUBJECT TO
       2)   OIL <=   20000
       3) - ASOLD - ACRACK + 0.5 OIL =    0
       4) - HSOLD - HCRACK + 0.5 OIL =    0
       5)   0.00075 HCRACK + 0.001 ACRACK <=   8
END

LP OPTIMUM FOUND
       OBJECTIVE FUNCTION VALUE
       1)    760000.000

 VARIABLE        VALUE          REDUCED COST
    HSOLD     10000.000000           .000000
   HCRACK          .000000          2.500000
    ASOLD      2000.000300           .000000
   ACRACK      8000.000000           .000000
      OIL     20000.000000           .000000

ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000         10.000000
       3)          .000000        -60.000000
       4)          .000000        -40.000000
       5)          .000000      70000.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:
```

Name ______________________________

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
HSOLD	40.000000	INFINITY	2.500000
HCRACK	90.000000	2.500000	INFINITY
ASOLD	60.000000	3.333333	20.000000
ACRACK	130.000000	INFINITY	3.333333
OIL	-40.000000	INFINITY	10.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	20000.000000	INFINITY	4000.001000
3	.000000	2000.000300	INFINITY
4	.000000	10000.000000	INFINITY
5	8.000000	2.000000	8.000000

THE TABLEAU

ROW	(BASIS)	HSOLD	HCRACK	ASOLD	ACRACK	OIL	SLK 2
1	ART	.000	2.500	.000	.000	.000	10.000
2	OIL	.000	.000	.000	.000	1.000	1.000
3	ASOLD	.000	-.750	1.000	.000	.000	.500
4	HSOLD	1.000	1.000	.000	.000	.000	.500
5	ACRACK	.000	.750	.000	1.000	.000	.000

ROW	SLK 5	
1	0.70E+05	0.76E+06
2	.000	20000.000
3	-1000.000	2000.000
4	.000	10000.000
5	1000.000	8000.000

Using the LINDO output above, answer the following questions:

A. The optimal solution is to
purchase ___________ barrels of oil,
produce ___________ barrels of heating oil and ___________ barrels of aviation fuel.
sell ___________ barrels of heating oil without further processing, and process __________barrels in the catalytic cracker.
sell ___________ barrels of the aviation fuel without further processing, and process __________barrels in the catalytic cracker.

B. This plan should generate a profit of $_________.

C. If 21,000 barrels of oil is available for purchase, profit will be increased by $ _______________

D. If the selling price of (unprocessed) heating oil were to drop by 10%, will the optimal solution change?

E. A shutdown of the catalytic cracker for 15 minutes of repair will result in a $_________ loss in profit..

F. Shutting down the catalytic cracker for 15 minutes is equivalent to __________ (increasing/decreasing) the variable SLK5 by that amount.

G. Shutting down the catalytic cracker for 15 minutes will result in the following revised optimal solution:
purchase ___________ barrels of oil,
produce ___________ barrels of heating oil and ___________ barrels of aviation fuel.
sell ___________ barrels of heating oil without further processing, and process __________barrels in the catalytic cracker.
sell ___________ barrels of the aviation fuel without further processing, and process __________barrels in the catalytic cracker.

Name ___________________________________

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Quiz #5 – October 6, 1999

Consider the following payoff table in which you must choose among three possible alternatives, after which two different states of nature may occur. (Nothing is known about the probability distribution of the state of nature.)

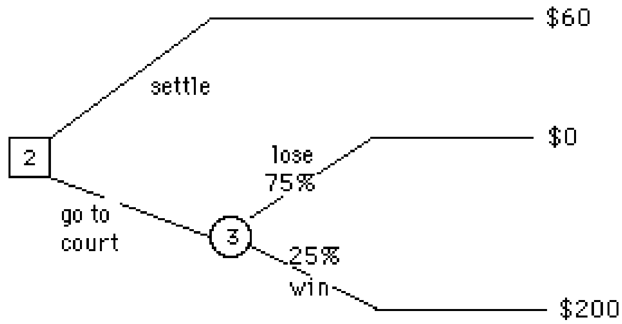
	State of Nature	
Decision	1	2
1	5	1
2	6	4
3	2	7

1. What is the optimal decision if the maximin criterion is used? _____
2. What is the optimal decision if the maximax criterion is used? _____
3. Create the regret table:

	State of Nature	
Decision	1	2
1	__	__
2	__	__
3	__	__

4. What is the optimal decision if the minimax regret is used? _____

General Custard Corporation is being sued by Sue Smith. Sue must decide whether to accept an offer of \$60,000 by the corporation to settle out of court, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive \$200,000, but if she loses, she will net \$0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



5. What is the decision which maximizes the expected value? ___ a. settle ___b. go to court

For \$20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 90% of the time, e.g., if the suit will win, the probability that the consultant predicts the win is 90%.

Bayes' Rule states that if S_i is one of the n states of nature and O_j is the outcome of an experiment,

$$P\left\{S_i \middle| O_j\right\} = \frac{P\left\{O_j \middle| S_i\right\}P\left\{S_i\right\}}{P\left\{O_j\right\}}, \text{ where } P\left\{O_j\right\} = \sum_{k=1}^{n} P\left\{O_j \middle| S_k\right\}P\left\{S_k\right\}$$

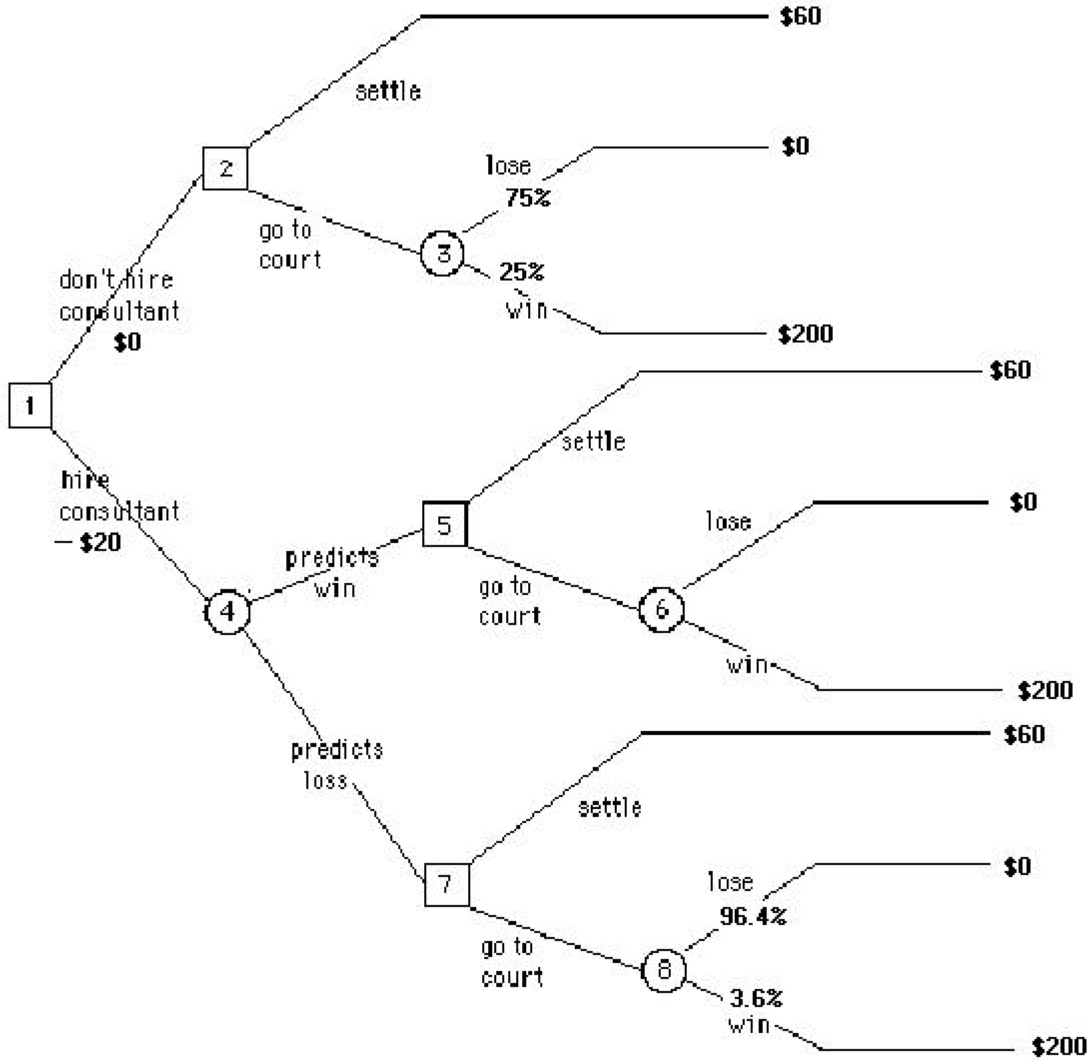
_____ 6. The probability that the consultant will predict a win, i.e. P{PW} is *(choose nearest value)*

a. ≤ 25%	b. 30%	c. 35%
d. 40%	e. 45%	f. ≥ 50%

_____ 7. According to Bayes' theorem, the probability that Sue *will* win, given that the consultant predicts a win, i.e. $P\left\{W \middle| PW\right\}$, is *(choose nearest value)*

a. ≤ 25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

The decision tree below includes Sue's decision as to whether or not to hire the consultant. Insert the probability that you computed in (7) above, together with its complement $P\{L \subseteq PW\}$.



Note that in this tree, the cost of the consultant is not included in the "payoffs" at the ends of the tree, but is included only when comparing the payoffs at nodes 2 & 4.

8. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	_____	5	150	2	60
7	60	4	______	1	______
6	150	3	50		

9. Should Sue hire the consultant? *Circle:* Yes No

____ 10. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤ 12.5 b. 15 c. 17.5 d. 20
e. 22.5 f. 25 g. 27.5 h. ≥ 30

Name ___________________________________

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Quiz #6 – October 22, 1999

Part A. The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS). The cost of signing each pitcher and the number of victories each pitcher will add to the Cubs are shown below.

Pitcher	Cost of signing ($million)	Right- or Left-handed?	Victories added to Cubs
RS	$6	Right	6
BS	$4	Right	5
DE	$3	Left	3
ST	$2	Left	3
TS	$2	Right	2

Define binary decision variables RS, BS, etc., e.g.,
RS = 1 if Rick Sutcliffe is signed, and 0 otherwise.

From the list below, select the linear inequality which imposes each of the following restrictions:
___ 1. If RS is signed, then TS cannot be signed.
___ 2. At most two right-handed pitchers can be signed.
___ 3. If DE is signed, then ST must be signed.
___ 4. At least one left-handed pitcher must be signed.
___ 5. The Cubs cannot sign both RS and BS.

a. $ST \geq DE$ b. $DE + ST \leq 1$ c. $RS + BS + TS \geq 2$ d. $RS + BS + TS \leq 2$

e. $RS + BS + TS \geq 1$ f. $RS + BS = 1$ g. $RS + BS = 0$ h. $ST \leq DE$

i. $RS + BS \leq 1$ j. $RS + BS \geq 1$ k. $ST + DE = 1$ l. $RS \leq TS$

m. $DE + ST \geq 1$ n. $RS + TS \leq 1$ o. $DE + ST \leq 1$ p. $RS + TS = 1$

q. *None of the above*

Name ____________________________________

Part B. A court decision has stated that the enrollment of each high school in Metropolis must be at least

are shown in the table below.

	White students		Total
1	80	30	110
2	70	5	75
3	90	10	100
4	50	40	90
5	60	30	90

The distance (in miles) that a student in each district must travel to each high school is:

District	HS #1	HS #2
1	1	2
2	0.5	1.7
3	0.8	0.8
4	1.3	0.4
5	1.5	0.6

School board policy requires that all the students in a given district must attend the same school.

Define the decision variables:

X_{ij} = 1 if all students in district *i* are assigned to school *j*
= 0 otherwise

For each of the following restrictions, select the corresponding linear constraint from the list below:

___ 6. Students in district 1 must be assigned to a school.
___ 7. The enrollment of school 1 must be at least 150.
___ 8. The enrollment of black students in school 1 must be at least 20% of its total enrollment.
___ 9. Districts 2 and 5 cannot be assigned to the same school.
___ 10. At least three districts must be assigned to school #1.

a. $110X_{11} + 75X_{21} + 100X_{31} + 90X_{41} + 90X_{51} \geq 150$

b. $110X_{11} + 75X_{21} + 100X_{31} + 90X_{41} + 90X_{51} \leq 150$

c. $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 3$

d. $X_{11} + X_{12} \leq 1$

e. $X_{11} + X_{12} = 1$

f. $X_{11} + X_{12} \geq 1$

g. $X_{21} + X_{51} = 1$ & $X_{22} + X_{52} = 1$

h. $X_{21} \leq X_{51}$ & $X_{22} \leq X_{52}$

i. $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \geq 3$

j. $30X_{11} + 5X_{21} + 10X_{31} + 40X_{41} + 30X_{51} \geq 20$

k. $X_{21} \times X_{51} = 1$

l. $X_{11} + X_{12} \geq 1$

m. $X_{11} + X_{12} = 1$

n. $X_{11} + X_{21} = 1$

o. $X_{21} + X_{51} \geq 1$ & $X_{22} + X_{52} \geq 1$

p. $30X_{11} + 5X_{21} + 10X_{31} + 40X_{41} + 30X_{51} \geq 0.2\,(110X_{11} + 75X_{21} + 100X_{31} + 90X_{41} + 90X_{51})$

q. $110X_{11} + 75X_{21} + 100X_{31} + 90X_{41} + 90X_{51} \geq 0.2\,(30X_{11} + 5X_{21} + 10X_{31} + 40X_{41} + 30X_{51})$

r. $30X_{11} + 5X_{21} + 10X_{31} + 40X_{41} + 30X_{51} \geq 30$

s. *None of the above*

Name ______________________________

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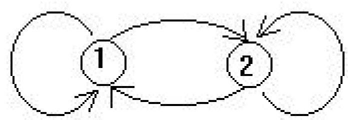
Quiz #7 – Fall 1999

Discrete-time Markov chains Let X_n denote the quality of the n^{th} item produced by a production system, with $X_n=1$ meaning "good" and $X_n=2$ meaning "defective". Suppose that $\{X_n: n=0,1,2,...\}$ is a Markov chain whose transition probability matrix P (and P^2 and P^3) are

$$P = \begin{bmatrix} .98 & .02 \\ .15 & .85 \end{bmatrix},\ P^2 = \begin{bmatrix} .963 & .037 \\ .275 & .725 \end{bmatrix},\ P^3 = \begin{bmatrix} .95 & .05 \\ .378 & .622 \end{bmatrix}$$

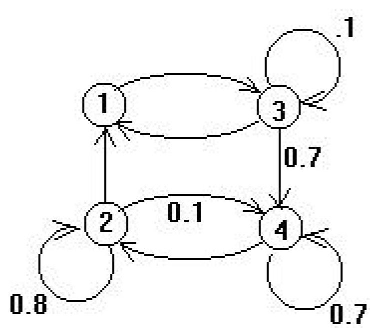
That is, if the previous item was "good", the probability of producing a defective item is 2%, but if the previous item was defective, there is an 85% probability that the next item will also be defective.

1. Sketch the diagram showing the states and transitions (with transition probabilities):



2. What's the probability that, if the 1st item is good, the next one (i.e., the 2nd) is defective? __________
3. What is the probability that, if the first item is defective, the second is defective? __________
4. What is the probability that, if the *first two* items are defective, the third is defective? __________
5. What is the probability that, if the first item is good, the third is defective? __________
6. What is the probability that, if the first item is defective, the third is also defective? __________

7. Write the transition probability matrix for the following Markov chain diagram:



(Note: some probabilities have not been specified in the diagram, but may be determined by the probabilities which are specified.)

$$P = \begin{bmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{bmatrix}$$

Name ______________________________

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Quiz #8 – Fall 1999

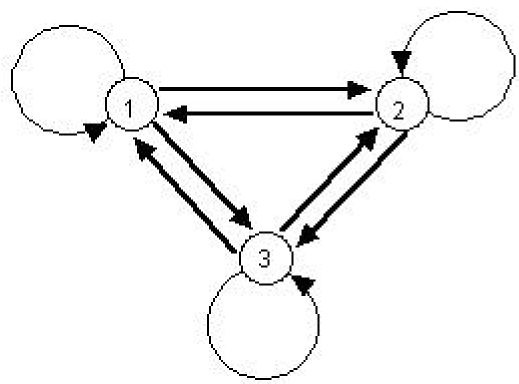
Discrete-time Markov chains On December 31 of each year I determine whether my car is in good, fair, or broken-down condition. If my car is broken-down, I replace it with a good used car.

- A good car will be good at the end of next year with probability 80%, fair with probability 15%, or broken-down with probability 5%.
- A fair car will be fair at the end of the next year with probability 50%, or broken-down with probability 50%.
- It costs \$10,000 to purchase a good used car; a fair car can be traded in for \$3000; and a broken-down car can be sold as junk for \$500.
- It costs \$1000 per year to operate a good car and \$1500 to operate a fair car.

Assume that the cost of operating a car during a year depends on the type of car on hand at the *beginning* of the year, and that any break-down occurs only at the end of a year.

My policy is to drive my car until it breaks down, at which time I replace it with a good used car. Define a Markov chain model representing the condition of the car which I own on Dec. 31, with three states:

1. Good condition
2. Fair condition
3. Broken-down



On the diagram to the right, indicate the transition probabilities.

Complete the transition probability matrix below

P=

_______Which one or more equations must be satisfied by the steadystate probabilities π_1, π_2, & π_3?

a. $\pi_1 + \pi_2 + \pi_3 = 1$

b. $\pi_1 + \pi_2 + \pi_3 = 0$

c. $0.8\pi_1 + 0.8\pi_3 = \pi_1$

d. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = \pi_1$

e. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = \pi_3$

f. $0.05\pi_1 + 0.5\pi_2 + 0.05\pi_3 = \pi_3$

g. $0.8\pi_1 + 0.8\pi_3 = 0$

h. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = 0$

Write the expression which represents my average cost per year:

_________ π_1 + _________ π_2 + _________ π_3

The matrix of *mean first passage times* is

```
f|      to
r|
o|   1       2      3
m|
1| 1.625  6.66667 6.5
2| 3.625  4.33333 2
3| 1.625  6.66667 6.5
```

I should expect to replace my car once every __________ years.

If my current car is in *fair* condition, I should expect to replace it in ___________ years.

Name ______________________________

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Quiz #9 – Fall 1999

A. ***Manufacturing System with Inspection & Rework***: Consider a system in which there are three machining operations, each followed by an inspection. Relevant data are:

OPERATION	TIME RQMT. (man-hrs)	OPERATING COST ($/hr.)	SCRAP RATE %	%SENT BACK FOR REWORK
Machine A	1.5	20.00	15	
Inspection A	0.25	8.00	4	8
Machine B	1.0	16.00	6	
Inspection B	0.25	8.00	5	4
Machine C	1.5	20.00	5	
Inspection C	.5	8.00	9	7
Pack & Ship	0.25	8.00		

The raw materials (blanks) cost $75.00 per part, and scrap value recovered is $10.00 per part. An order for 10 completed parts must be filled.
Consult the computer output below to answer the questions:

___ 1. What percent of the parts which are started are successfully completed? *Choose nearest value.*
a. 50% b. 55% c. 60% d. 65%
e. 70% f. 75% g. 80% h. 85%

___ 2. What is the **expected** number of blanks which are required to fill the order for 10 parts? *Choose nearest value*
a. 11 b. 12 c. 13 d. 14
e. 15 f. 16 g. 17 h. 18

___ 3. What is the probability that a part which passes inspection B will ultimately be scrapped? *Choose nearest value.*
a. 5% b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40%

___ 4. What is the expected number of times that a part is inspected? *Choose nearest value*
a. 1 b. 1.5 c. 2 d. 2.5
e. 3 f. 3.5 g. 4 h. 4.5

___ 5. If a part reaches Machine C, what is the probability that it will be successfully completed? *Choose nearest value*
a. 60% b. 65% c. 70% d. 75%
e. 80% f. 85% g. 90% h. 95%

Transition Probability Matrix

from	1	2	3	4	5	6	7	8
1	0	0.85	0	0	0	0	0	0.15
2	0.08	0	0.88	0	0	0	0	0.04
3	0	0	0	0.94	0	0	0	0.06
4	0	0	0.05	0	0.91	0	0	0.04
5	0	0	0	0	0	0.95	0	0.05
6	0	0	0	0	0.09	0	0.84	0.07
7	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	1

A = Absorption Probabilities

from	7	8

Name ______________________________

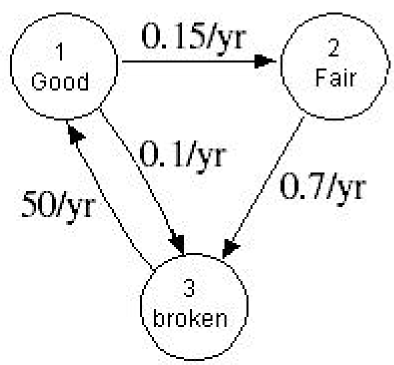
1	0.62861	0.37139
2	0.739541	0.260459
3	0.783241	0.216759
4	0.833235	0.166765
5	0.872608	0.127392
6	0.918535	0.0814653

E = Expected No. Visits to Transient States

from	1	2	3	4	5	6
1	1.07296	0.912017	0.842156	0.791627	0.787732	0.748345
2	0.0858369	1.07296	0.990772	0.931326	0.926743	0.880406
3	0	0	1.04932	0.986359	0.981505	0.93243
4	0	0	0.0524659	1.04932	1.04415	0.991947
5	0	0	0	0	1.09349	1.03882
6	0	0	0	0	0.0984144	1.09349

B. Continuous-time Markov Chains. Consider the vehicle replacement problem:

I own one car. At any time, my current car is in <u>good</u>, <u>fair</u>, or <u>broken-down</u> condition. My policy is to drive my car until it breaks down, at which time I replace it. I have modeled the process as a continuous-time Markov chain, with the transition diagram below. (Transition rates are shown.) It costs me \$9000 to purchase a good car; a broken-down car has no trade-in. It costs me \$1000/yr to operate a good car and \$1500/yr to operate a fair car.



1. What is the value of the matrix L of transition rates?

$$\Lambda = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

_____2. The probability distribution of the length of time between purchase of a car and when it has deteriorated to a "fair" car is

a. Uniform   b. Normal   c. Exponential
d. Markov   e. Gamma   f. *None of the above*

_____3. Suppose that I have just purchased a car. What is the probability that this (good) car will change its state within the next year?

a. $1 - e^{0.25}$ b. $1 - e^{-0.25}$
c. $e^{0.25}$ d. $e^{-0.25}$ e. *None of the above*

_____4. Suppose that I purchased my current car one year ago. Then the probability that one year from now my car will <u>not</u> have deteriorated into a "fair" car is

a. $1 - e^{0.25}$ b. $1 - e^{-0.25}$ c. $1 - e^{0.5}$ d. $1 - e^{-0.5}$
e. $e^{0.25}$ f. $e^{-0.25}$ g. $e^{0.5}$ h. $e^{-0.5}$ i.. *None of the above*

_____ 5. Which (one <u>or more</u>) of the following equations describe the steadystate probability distribution?

a. $\pi_1 + \pi_2 + \pi_3 = 0$ d. $\pi_1 + \pi_2 + \pi_3 = 1$ g. $0.15\pi_1 = 0.7\pi_2 + 0.1\pi_3$
b. $\pi_1 = 0.15\pi_2 + 0.1\pi_3$ e. $\pi_1 = 0.15\pi_1 + 0.7\pi_2 + 50\pi_3$ h. $0.25\pi_1 = 50\pi_3$
c. $0.15\pi_1 = 0.7\pi_2$ f. $0.25\pi_1 = 0.7\pi_2 + 50\pi_3$ i. $0.15\pi_1 = 50\pi_3$

_____ 6. Suppose that the steadystate probabilities are $\pi = (0.8, 0.195, 0.005)$. *(Not the actual values!)* Then the expected time T between replacements, measured in years, is *(choose nearest value):*

a. 1   b. 1.5   c. 2
d. 2.5   e. 3   f. 3.5
g. 4   h. 4.5   i. 5

Name ___________________________

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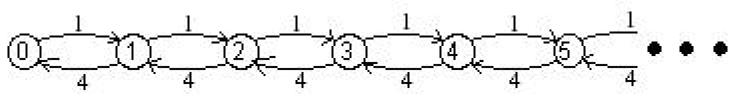
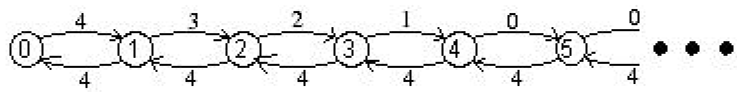
Quiz #10 – Fall 1999

For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

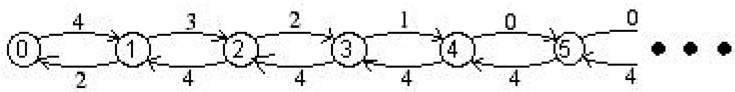
(a) M/M/1	(b) M/M/2	(c) M/M/1/4
(d) M/M/4	(e) M/M/2/4	(f) M/M/2/4/4
(g) M/M/1/4/4	(h) M/M/4/2	(i) M/M/4/4
(j) M/M/2/2/4	(k) M/M/1/4/2	(l) *none of the above*



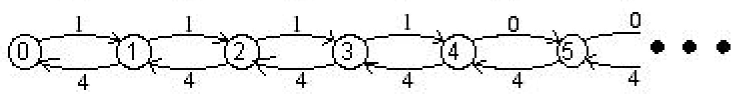
___ 1.



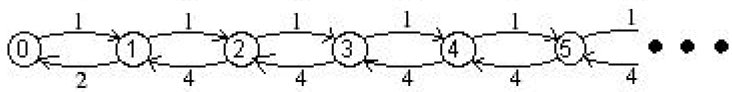
___ 2.



___ 3.

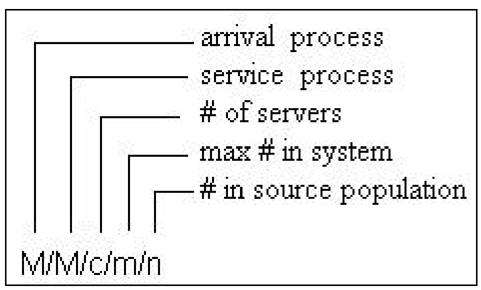


___ 4.



___ 5.

Note: Kendall's notation:



Name ________________________

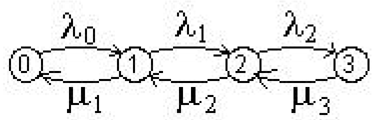
A machine operator has the task of keeping three machines running. Each machine runs for an average of 1 hour before it becomes jammed or otherwise needs the operator's attention. He then spends an average of twelve minutes restoring the machine to running condition. Define a continuous-time Markov chain, the state of the system being the number of machines not running.

6. True or False (circle): This Markov chain is a birth/death process.

7. Specify the letter for each of the transition rates:

λ_0 ____ λ_1 ____ λ_2 ____

μ_1 ____ μ_2 ____ μ_3 ____



a. 1/hr b. 2/hr c. 3/hr

d. 4/hr. e. 5/hr. f. 0.2/hr

g. 0.4/hr h. 0.5/hr. i. *None of the above*

____8. Which equation is used to compute the steady-state probability π_0?

(a.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3}$

(b.) $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2} + \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2}\frac{\lambda_2}{\mu_3}$

(c.) $\pi_0 = 1 \times \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu_3}$

(d.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu_3}$

(e.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2} + \frac{\lambda_0}{\mu_1}\frac{\lambda_1}{\mu_2}\frac{\lambda_2}{\mu_3}$

(f.) $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3}$

(g.) $\frac{1}{\pi_0} = 1 \times \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2} \times \frac{\lambda_2}{\mu_3}$

(h) None of the above

____9. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \pi_0$ b. $\pi_1 = 0.1\ \pi_0$ c. $\pi_1 = 0.6\ \pi_0$

d. $\pi_1 = 1/6 \times \pi_0$ e. $\pi_1 = 3\ \pi_0$ f. *None of the above*

____10. If the average number of machines not running were 0.5 and the average time between machine jams were 0.4 hr, what is the average turnaround time (waiting plus service time) to restore a machine to running condition? *(Choose nearest answer)*

a. 0.1 hour c. 0.2 hour e. 0.3 hour

b. 0.4 hour d. 0.5 hour f. 0.6 hour

Name ________________________________

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Quiz #11 – Fall 1999

Part I: Suppose that a new car costs \$10,000 and that the annual operating cost & trade-in value are as follows

Age of car (years)	Trade-in value	Operating cost in previous year
1	\$7000	\$300
2	\$6000	\$500
3	\$4000	\$800
4	\$3000	\$1200
5	\$2000	\$1600
6 or more	\$1000	\$2200

I wish to determine the replacement policy that, starting with a new car, minimizes my net cost of owning and operating a car for the next ten years (from t=0 until t=10)? (Do not include the cost of the initial car.)

As in the class notes, define:
G(t) = minimum total cost incurred from time t until the end of the planning period, if a new car has just been purchased. *(Note: this does not include the cost of purchasing this initial new car.)*
X*(t) = optimal replacement time for a car which has been purchased at the beginning of period t.

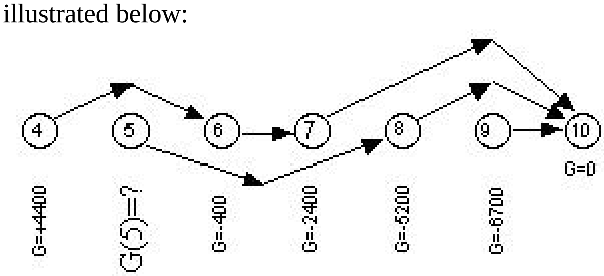
The optimal value function G(t) is defined recursively by

$$G(t) = \underset{t+1 \le x \le T}{\text{minimum}} \left\{ \sum_{i=1}^{x-t} C_i - S_{x-t} + P_x + G(x) \right\}$$

where

$G(10) = 0$
P_t = purchase price of a new car at time t
C_i = cost of operation & maintenance of a car in its i^{th} year.
S_j = trade-in value of a car of age j

The computation of G(4) through G(10), i.e., for the final 6 years, has already been done in the example presented in class, and is illustrated below:



1. What is the value of G(5)? \$______
2. If I purchase a new car at the beginning of year 4, how many additional cars should I purchase until the end of the planning period? ____
3. If I purchase a new car at the beginning of year 4, what is my average cost/year until the end of the planning period? \$______/year

Name ________________________________

Part II. ***Optimal Reliability by means of redundancy.*** A system consists of three components, each of which is necessary for the operation of the system. The weight and the reliability of each component, i.e., the probability that the component survives for the system's intended lifetime, is shown in the table below:

Component	Weight (kg)	Reliability (%)
1	1	70
2	2	80
3	1	75

The total weight of the system is to be no more than 7 kg. We will use dynamic programming to determine how many redundant units of each component should be included in order to maximize the reliability of the system.

The stages correspond to the three types of components. We will perform a backward recursion, in which we imagine that we are deciding first how many units of type 3 are to be included, then type 2, and finally type 1. The state ***s*** of the system at stage ***n*** is the number of kg remaining to be filled with components n, n-1, … 1, and the optimal value $\boldsymbol{V_n(s)}$ is the maximum reliability that can be attained for the subsystem consisting of components of type n, n-1, …1 if s kg are available. The computations are done first for stage 1, then stage 2, and finally stage 3.

Optimal System Reliability Using Redundancy

Recursion type: backward

---Stage 1---

s \ x:	1	2	3
1	0.7000	-∞	-∞
2	0.7000	0.9100	-∞
3	0.7000	0.9100	0.9730
4	0.7000	0.9100	0.9730
5	0.7000	0.9100	0.9730
6	0.7000	0.9100	0.9730
7	0.7000	0.9100	0.9730

State s	Optimal Values $V_1(s)$	Optimal Decisions	Resulting State
1	0.7000	1	0
2	0.9100	2	0
3	0.9730	3	0
4	0.9730	3	1
5	0.9730	3	2
6	0.9730	3	3
7	0.9730	3	4

---Stage 2---

s \ x:	1	2	3
3	0.5600	-∞	-∞
4	0.7280	-∞	-∞
5	0.7784	0.6720	-∞
6	0.7784	0.8736	-∞
7	0.7784	0.9341	0.6944

State s	Optimal Values $V_2(s)$	Optimal Decisions	Resulting State
3	0.5600	1	1
4	0.7280	1	2
5	0.7784	1	3
6	0.8736	2	2
7	0.9341	2	3

Name ________________________________

	---Stage 3---		
s \ x:	1	2	3
4		0.4200	-∞	-∞
5		0.5460	0.5250	-∞
6		0.5838	0.6825	0.5512
7		**??????**	0.7298	0.7166

State s	Optimal Values $V_3(s)$	Optimal Decisions	Resulting State
4	0.4200	1	3
5	0.5460	1	4
6	0.6825	2	4
7	0.7298	2	5

___ 4. What is the reliability of a subsystem consisting of 2 units of component #1?

a. 0.7^2 b. 0.3^2 c. $1–0.7^2$
d. 2×0.7 e. $1–0.3^2$ f. *None of the above*

5. What is the missing value in the table at stage 3? ______%

6. What is the maximum reliability that can be obtained using redundant units with a weight restriction of 7 kg.? _______

7. If only six kg were available, the maximum reliability that could be achieved is _____%

8. If only six kg. were available, the optimal design would include:

___ units of component #1
___ units of component #2
___ units of component #3

Name ___________________________

Part I: ***Production Planning*** We wish to plan production of an expensive, low-demand item for the next three months (January, February, & March).

- the cost of production is $10 for setup, plus $5 per unit produced, up to a maximum of 4 units.
- the storage cost for inventory is $2 per unit, based upon the level at the beginning of the month.
- a maximum of 3 units may be kept in inventory at the end of each month; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each month:

demand d	0	1	2
P{D=d}	0.2	0.5	0.3

- there is a penalty of $25 per unit for any demand which cannot be satisfied. Backorders are not allowed.
- the initial inventory (i.e., the inventory at the end of December) is 1.
- a salvage value of $4 per unit is received for any inventory remaining at the end of the last month (March)

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 3 = January, stage 2 = February, etc.** (i.e., n = # months remaining in planning period.)

a. What is the optimal production quantity for January? _______
b. What is the total expected cost for the three months? __________
c. If, during January, the demand is 1 unit, what should be produced in February? ______
d. Three values have been blanked out in the computer output, What are they?
   i. the optimal value $f_2(1)$ ___________
   ii. the optimal decision $x_2^*(1)$ ___________
   iii. the cost associated with the decision to produce 1 unit in February when the inventory is 0 at the end of January. ____________

```
============================================================
                     ---Stage 1---
   s \ x:   0        1        2        3        4

   0 |  27.5000  21.7000  16.4000  17.4000  19.2000
   1 |   8.7000  13.4000  14.4000  16.2000  20.0000
   2 |  -0.4000  11.4000  13.2000  17.0000  22.0000
   3 |  -1.6000  10.2000  14.0000  19.0000  24.0000

        Optimal  Optimal
 State  Values   Decision

   0 |   16.4000 |    2
   1 |    8.7000 |    0
   2 |   -0.4000 |    0
   3 |   -1.6000 |    0

============================================================
                    ---Stage 2---
   s \ x:   0        1        2        3        4

   0 |  43.9000  ???????  29.3500  27.4900  29.0000
   1 |  24.3600  26.3500  24.4900  26.0000  30.4000
   2 |  13.3500  21.4900  23.0000  27.4000  32.4000
   3 |   8.4900  20.0000  24.4000  29.4000  34.4000
```

Name ___________________________

```
        Optimal  Optimal
 State  Values   Decision

   0 |   27.4900 |    3
   1 |  ??????? |    ?
   2 |   13.3500 |    0
   3 |    8.4900 |    0

============================================================
                    ---Stage 3---
   s \ x:   0        1        2        3        4

   0 |  54.9900  49.3640  43.0970  40.6810  39.9480
   1 |  36.3640  40.0970  37.6810  36.9480  40.4900
   2 |  27.0970  34.6810  33.9480  37.4900  42.4900
   3 |  21.6810  30.9480  34.4900  39.4900  44.4900

        Optimal  Optimal
 State  Values   Decision

   0 |   39.9480 |    4
   1 |   36.3640 |    0
   2 |   27.0970 |    0
   3 |   21.6810 |    0

============================================================
```

Part II. ***Markov chains*** The Green Valley Christmas Tree Farm owns a plot of land with 5000 evergreen trees. Each year they allow individuals to select and cut Christmas trees. However, they protect small trees (usually less than 4 feet tall) so that they will grow and be available for sale in future years. Currently 2000 trees are classified as protected trees, while the remaining 3000 are available for cutting. However, even though a tree is available for cutting in a given year, it possibly might not be selected for cutting until future years. While most trees not cut in a given year live until the next year (protected or unprotected), approximately 15% are lost to disease. Each year, approximately 50% of the unprotected trees are cut, and 40% of the protected trees surviving from the previous year have matured sufficiently to be made available for cutting.

Define a Markov chain model of the system consisting of a **single** tree, with states (1) protected, (2) unprotected, (3) dead, (4) cut & sold. The transition probability matrix is

$$P = \begin{bmatrix} 0.51 & 0.34 & 0.15 & 0 \\ 0 & 0.425 & 0.075 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The following computations were performed:

$$\begin{bmatrix} 0.49 & -0.34 \\ 0 & 0.575 \end{bmatrix}^{-1} = \begin{bmatrix} 2.0408 & 1.2067 \\ 0 & 1.7391 \end{bmatrix}$$

$$\begin{bmatrix} 2.0408 & 1.2067 \\ 0 & 1.7391 \end{bmatrix} \begin{bmatrix} 0.15 & 0 \\ 0.075 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.3966 & 0.6034 \\ 0.1304 & 0.8696 \end{bmatrix}$$

1. What are the absorbing states of this model? ___________
2. What is the probability that a tree which is protected is eventually sold? _________
3. What is the probability that a protected tree eventually dies of disease? _________
4. How many of the farm's 5000 trees are expected to be sold eventually_____________
5. If a tree is initially protected, what is the expected number of years until it is either sold or dies? ____________