56:171 Operations Research
Sample Quizzes -- Fall 1994

Quiz # 1

For each statement, indicate "+"=**true** or "o"=**false**.

_____ a. The number of basic variables in an LP is equal to the number of rows, *including* the objective function row.
_____ b. The Gauss-Jordan method for solving a system of equations requires an equal number of equations and variables.
_____ c. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.
_____ d. In the simplex method, every variable of the LP is either basic or nonbasic.
_____ e. If the columns of a 3x3 matrix are linearly independent, then the matrix is singular.
_____ f. In a basic LP solution, the nonbasic variables equal zero.
_____ g. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.
_____ h. It may happen that an LP problem has (exactly) two optimal solutions.
_____ i. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0.
_____ j. A "pivot" in a nonbasic column of a tableau will make it a basic column.
_____ k. A nonsingular square matrix has no inverse matrix.
_____ l. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.
_____ m. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.
_____ n. The value in the objective row of the simplex tableau is referred to as "reduced cost" or "relative profit", depending upon whether you are minimizing or maximizing, respectively.
_____ o. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.
_____ p. Every optimal solution of an LP is a basic solution.
_____ q. Basic solutions of an LP with constraints $Ax \leq b$, $x \geq 0$ correspond to "corner" points of the feasible region.
_____ r. In the simplex tableau, a linear inequality is written in the form of an equation by introducing a "slack" variable.
_____ s. In the simplex tableau, the objective row is written in the form of an equation.
_____ t. LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10"

Quiz # 1 Solutions

True a. The number of basic variables in an LP is equal to the number of rows, *including* the objective function row.
False b. The Gauss-Jordan method for solving a system of equations requires an equal number of equations and variables.
True c. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.
True d. In the simplex method, every variable of the LP is either basic or nonbasic.
False e. If the columns of a 3x3 matrix are linearly independent, then the matrix is singular.
True f. In a basic LP solution, the nonbasic variables equal zero.
False g. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations. *(This is done automatically by LINDO!)*

False h. It may happen that an LP problem has (exactly) two optimal solutions. *(There may be exactly two basic optimal solutions, but an infinite number of nonbasic optimal solutions on the line segment between them!)*

False i. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0. *(LINDO always assumes that the decision variables are nonnegative.)*

True j. A "pivot" in a nonbasic column of a tableau will make it a basic column.

False k. A nonsingular square matrix has no inverse matrix.

True l. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

True m. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.

True n. The value in the objective row of the simplex tableau is referred to as "reduced cost" or "relative profit", depending upon whether you are minimizing or maximizing, respectively.

True o. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.

False p. Every optimal solution of an LP is a basic solution. *(LINDO always produces a basic optimal solution, but there may be nonbasic optimal solutions as well!)*

True q. Basic solutions of an LP with constraints $Ax \leq b$, $x \geq 0$ correspond to "corner" points of the feasible region.

True r. In the simplex tableau, a linear inequality is written in the form of an equation by introducing a "slack" variable.

True s. In the simplex tableau, the objective row is written in the form of an equation.

True t. LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10"

SSSSSSSS Quiz # 2 SSSSSSSS

Statements (a) through (j) refer to the LP below. For each statement, indicate "+"=**true** or "o"=**false**.

$$\begin{aligned} \text{Maximize } & X_1 + 2X_2 + 3X_3 \\ \text{s.t. } & X_1 - X_2 + 2X_3 \leq 40 \\ & 2X_1 + 3X_2 + X_3 = 20 \\ & -X_1 + X_2 + 4X_3 \geq 10 \\ & X_j \geq 0,\ j=1,2,3 \end{aligned}$$

_____ a. Unlike the ordinary simplex method, the "Revised Simplex Method" never requires the use of artificial variables.

_____ b. If the LP above were a minimization rather than maximization problem, the first phase of the two-phase method would be exactly the same.

_____ c. In the first phase of the two-phase simplex method for the above LP, three artificial variables are required.

_____ d. At the beginning of the first phase of the two-phase simplex method, the phase-one objective function will have the value 0.

_____ e. At the beginning of the "Big-M" method used to solve the LP above, if M=100, then the objective function will have the value -100.

_____ f. If the "Big-M" method (with M=100) were used to solve the above LP, and an artificial variable is nonzero in the optimal solution, then this indicates that the problem has no feasible solution.

_____ g. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

_____ h. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.

_____ i. If an LP model has constraints of the form $Ax \leq b$, $x \geq 0$, and b is nonnegative, then there is no need for artificial variables.

_____ j. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.

Consider the simplex tableaus below, where the objective is to maximized or minimized, as specified. In each tableau, circle every element which could be selected for the next pivot.

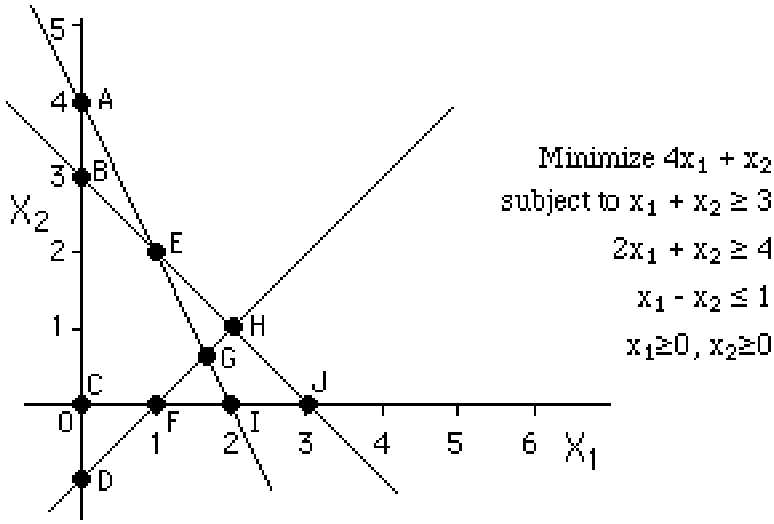
	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	RHS
MAX	1	3	0	-1	0	2	-2	0	0	1	-16
	0	0	0	2	0	-1	3	1	0	-2	8
	0	2	1	3	0	0	1	0	0	3	0
	0	-1	0	0	1	1	-1	0	0	1	6
	0	1	0	-4	0	2	0	0	1	0	12

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	RHS
MIN	1	3	0	-1	0	2	-2	0	0	1	-16
	0	0	0	2	0	-1	3	1	0	-2	8
	0	2	1	3	0	0	-1	0	0	3	0
	0	-1	0	0	1	1	1	0	0	1	6
	0	1	0	-4	0	2	0	0	1	0	5

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	RHS
MAX	1	-2	0	3	0	-2	4	0	0	2	-24
	0	0	0	2	0	-1	3	1	0	-2	4
	0	2	1	3	0	0	1	0	0	3	8
	0	-1	0	0	1	1	-1	0	0	1	2
	0	1	0	-4	0	2	0	0	1	0	1

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	RHS
MIN	1	-2	0	3	0	-2	4	0	0	2	-24
	0	0	0	2	0	-1	3	1	0	-2	4
	0	2	1	3	0	0	1	0	0	3	8
	0	-1	0	0	1	1	-1	0	0	1	2
	0	1	0	-4	0	2	0	0	1	0	1

§§§§§§§§ Quiz # 3 §§§§§§§§



a. Which of the ten points A-J are feasible solutions in the LP problem above?

(Circle all that apply): A B C D E F G H I J

b. Which of the ten points A-J are basic solutions?

(Circle all that apply): A B C D E F G H I J

c. In order to formulate the LP using only equality constraints, 3 additional nonnegative variables (x_3, x_4, and x_5) were defined. Insert the correct sign (+ or -) in each constraint below:

$x_1 + x_2$ ___ x_3 $= 3$

$2x_1 + x_2$ ___ x_4 $= 4$

$x_1 - x_2$ ___ $x_5 = 1$

d. Indicate (by X) which variables are basic (in addition to -z in the objective row)

.... at point **A**: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4, _____x_5

.... at point **E**: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4, _____x_5

e. Suppose that the revised simplex method is being used to solve the above problem, and the current basic variables are x_1, x_2, and x_5, B={1, 2, 5}. Find the values of the quantities **a** through **h** in the computations below. *(Hint: Valid answers are all integers in the range -4 to +4.)*

a. ____ **b.** ____ **c.** ____ **d.** ____

e. ____ **f.** ____ **g.** ____ **h.** ____

The basis inverse matrix is computed, and used to compute the current basic solution:

$$(A^B)^{-1} = \begin{bmatrix}1&1&0\\2&1&0\\1&-1&1\end{bmatrix}^{-1} = \begin{bmatrix}-1&1&0\\2&-1&0\\3&-2&1\end{bmatrix}, \quad x_B = \begin{bmatrix}x_1\\x_2\\x_5\end{bmatrix} = (A^B)^{-1}\begin{bmatrix}3\\\mathbf{a}\\1\end{bmatrix} = \begin{bmatrix}1\\2\\\mathbf{b}\end{bmatrix}$$

The simplex multiplier vector is computed, and used to compute the reduced cost of x_3:

$$\pi_B = C_B(A^B)^{-1} = [\mathbf{c},\ 1,\ 0]\,(A^B)^{-1} = [-2,\ 3,\ \mathbf{d}], \quad \overline{C}_3 = C_3 - \pi_B A^3 = 0 - \pi_B\begin{bmatrix}-1\\0\\0\end{bmatrix} = \mathbf{e}$$

Because the reduced cost of x3 is negative, we decide to enter it into the basis, and therefore compute the "substitution rates":

$$\alpha = (A^B)^{-1}A^3 = (A^B)^{-1}\begin{bmatrix}-1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\\mathbf{f}\\-3\end{bmatrix}$$

The minimum ratio test indicates that we pivot in the first constraint row, and so the new basis will be B = {**g**, 2, 5}, with basis inverse matrix and basic solution:

$$(A^B)^{-1} = \begin{bmatrix}1&1&0\\0&1&0\\0&-1&1\end{bmatrix}^{-1} = \begin{bmatrix}-1&1&0\\0&1&0\\0&1&1\end{bmatrix}, \quad x_B = \begin{bmatrix}\mathbf{h}\\4\\5\end{bmatrix}$$

f. The pivot in the revised simplex method of part (e) is effectively a move from basic solution ___ to basic solution ____ in the diagram above. *(Select from labels* A *through* J *& write in the blanks.)*

§§§§§§§§ Quiz # 3 Solutions §§§§§§§§

a. Which of the ten points are feasible solutions in the LP problem above? *Points* ***A, E, & H***

b. Which of the ten points are basic solutions? ***All of the ten points indicated are basic solutions (but only the three above are feasible!)***

c. In order to formulate the LP using only <u>equality</u> constraints, 3 additional nonnegative variables (x_3, x_4, and x_5) were defined. The variables x_3 & x_4 are surplus variables, and have a coefficient of -1, while x_5 is a slack variable and has a coefficient of +1.

$$\begin{aligned} x_1 + x_2 - x_3 \quad\quad\quad &= 3 \\ 2x_1 + x_2 \quad\quad - x_4 \quad &= 4 \\ x_1 - x_2 \quad\quad\quad + x_5 &= 1 \end{aligned}$$

d. Indicate (by X) which variables are basic (in addition to $-z$ in the objective row)

.... at point **A**: _X_(-z) ___x_1, _X_x_2, __X_x_3, _____x_4, __X_x_5

.... at point **E**: _X_(-z) _X_x_1, _X_x_2, _____x_3, _____x_4, __X_x_5

e. Suppose that the revised simplex method is being used to solve the above problem, and the current basic variables are x_1, x_2, and x_5, B={1, 2, 5}. Find the values of the quantities **a** through **h** in the computations below. *(Hint: Valid answers are all integers in the range -4 to +4.)*

a. __**+4**_ **b.** __**+2**_ **c.** __**4**__ **d.** __ **0**_

e. __**-2**_ **f.** __**-2**_ **g.** __**+3**_ **h.** __**+1**_

The basis inverse matrix is computed, and used to compute the current basic solution:

$$\left(A^B\right)^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & 0 \\ 3 & -2 & 1 \end{bmatrix}, \; x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \left(A^B\right)^{-1} \begin{bmatrix} 3 \\ \mathbf{a} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \mathbf{b} \end{bmatrix}$$

The simplex multiplier vector is computed, and used to compute the reduced cost of x_3:

$$\pi_B = C_B\left(A^B\right)^{-1} = [\mathbf{c}, 1, 0]\left(A^B\right)^{-1} = [-2, 3, \mathbf{d}], \; \overline{C}_3 = C_3 - \pi_B A^3 = 0 - \pi_B \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{e}$$

Because the reduced cost of x3 is negative, we decide to enter it into the basis, and therefore compute the "substitution rates":

$$\alpha = \left(A^B\right)^{-1} A^3 = \left(A^B\right)^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \mathbf{f} \\ -3 \end{bmatrix}$$

The minimum ratio test indicates that we pivot in the first constraint row, and so the new basis will be B = {**g**, 2, 5}, with basis inverse matrix and basic solution:

$$\left(A^B\right)^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \; x_B = \begin{bmatrix} \mathbf{h} \\ 4 \\ 5 \end{bmatrix}$$

f. The pivot in the revised simplex method of part (e) is effectively a move from basic solution _**E**_ to basic solution __**A**_ in the diagram above.

§§§§§§§§ Quiz # 4 §§§§§§§§

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:
X1 = number of STANDARD golf bags manufactured per quarter
X2 = number of DELUXE golf bags manufactured per quarter
Four operations are required, with the time per golf bag as follows:

	STANDARD	DELUXE	Available

Cut-&-Dye	0.7 hr	1 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

LINDO provides the following output:

```
MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END

 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE
        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110           .000000
        X2       251.997800           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000          4.375086
        3)       111.602000           .000000
        4)          .000000          6.937440
        5)        18.000232           .000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1       10.000000         3.500135         3.700000
       X2        9.000000         5.285715         2.333400
```

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	630.000000	52.364582	134.400000
3	600.000000	INFINITY	111.602000
4	708.000000	192.000010	128.002800
5	135.000000	INFINITY	18.000232

THE TABLEAU

ROW	(BASIS)	X1	X2	SLK 2	SLK3	SLK 4	SLK 5	
1	ART	.00	.00	4.375	.00	6.937	.00	7668.012
2	X2	.00	1.00	1.875	.00	-1.312	.00	251.998
3	SLK 3	.00	.00	-1.000	1.00	.200	.00	111.602
4	X1	1.00	.00	-1.250	.00	1.875	.00	540.003
5	SLK 5	.00	.00	-.344	.00	.141	1.00	18.000

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If the profit on DELUXE bags were to decrease from $9 each to $7 each, the number of DELUXE bags to be produced would

|__| increase |__| decrease |__| remain the same |__| not sufficient info.

b. The LP problem above has

|__| exactly one optimal solution |__| exactly two optimal solutions

|__| an infinite number of optimal solutions

c. If an additional 10 hours were available in the sewing department, PAR would be able to obtain an additional $________ in profits.

d. If an additional 10 hours were available in the finishing department, PAR would be able to obtain an additional $________ in profits.

e. If the variable "SLK 4" were increased, this would be equivalent to

___ increasing the hours used in the cut-&-dye department
___ decreasing the hours used in the cut-&-dye department
___ increasing the hours used in the sewing department
___ decreasing the hours used in the sewing department
___ none of the above

f. If the variable "SLK 4" were increased by 10, X1 would |__| increase |__| decrease by ________ STANDARD golf bags/quarter.

g. If the variable "SLK 4" were increased by 10, X2 would |__| increase |__| decrease by ________ DELUXE golf bags/quarter.

h. If a pivot were to be performed to enter the variable SLK4 into the basis, then according to the "minimum ratio test", the value of SLK4 in the resulting basic solution would be approximately

__ $^{252}/_{1.312}$ __ $^{0.2}/_{111.6}$ __ $^{540}/_{1.875}$ __ $^{0.141}/_{18}$

__ $^{1.312}/_{252}$ __ $^{111.6}/_{0.2}$ __ $^{1.875}/_{540}$ __ $^{18}/_{0.141}$

__ not sufficient information

i. If the variable SLK 4 were to enter the basis, then the variable __________ will leave the basis.

$$$$$$$$ Quiz # 4 Solutions $$$$$$$$

a. If the profit on DELUXE bags were to decrease from $9 each to $7 each, the number of DELUXE bags to be produced would

|__| increase |__| decrease |_**X**_| remain the same |__| not sufficient info.

b. The LP problem above has

|_**X**_| exactly one optimal solution |__| exactly two optimal solutions
|__| an infinite number of optimal solutions

c. If an additional 10 hours were available in the sewing department, PAR would be able to obtain an additional $___**zero**_____ in profits.

d. If an additional 10 hours were available in the finishing department, PAR would be able to obtain an additional $___**69.37**______ in profits.

e. If the variable "SLK 4" were increased, this would be equivalent to
- ___ increasing the hours used in the cut-&-dye department
- ___ decreasing the hours used in the cut-&-dye department
- **X**_ increasing the hours used in the sewing department
- ___ decreasing the hours used in the sewing department
- ___ none of the above

f. If the variable "SLK 4" were increased by 10, X1 would |__| increase |_**X**_| decrease by _**18.75**___ STANDARD golf bags/quarter.

g. If the variable "SLK 4" were increased by 10, X2 would |_**X**_| increase |__| decrease by __**13.12**__ DELUXE golf bags/quarter.

h. If a pivot were to be performed to enter the variable SLK4 into the basis, then according to the "minimum ratio test", the value of SLK4 in the resulting basic solution would be approximately

__ $^{252}/_{1.312}$ __ $^{0.2}/_{111.6}$ __ $^{540}/_{1.875}$ __ $^{0.141}/_{18}$

__ $^{1.312}/_{252}$ __ $^{111.6}/_{0.2}$ __ $^{1.875}/_{540}$ _**X**_ $^{18}/_{0.141}$

__ not sufficient information

i. If the variable SLK 4 were to enter the basis, then the variable _**SLK5**_ will leave the basis.

$$$$$$$ Quiz # 5 $$$$$$$

Gepbab Production Company uses labor and raw material to produce three products. The resource requirements and sales price for the three products are as shown in the tale below:

	Product 1	Product 2	Product 3
Labor	3 hours	4 hours	6 hours
Raw material	2 units	2 units	5 units
Sales price	$6	$8	$13

At present, 60 units of raw material are available. Up to 90 hours of labor can be purchased at $1 per hour. To maximize Gepbab profits, solve the following LP:

```
        MAX 6 X1 + 8 X2 + 13 X3 - L
ST      3 X1 + 4 X2 + 6 X3 - L <= 0
        2 X1 + 2 X2 + 5 X3 <= 60
        L <= 90
        END
```

Here, x_i = units of product i produced, and L = number of labor hours purchased.

```
        OBJECTIVE FUNCTION VALUE
        1)    97.5000000

  VARIABLE        VALUE          REDUCED COST
        X1         .000000           .250000
        X2       11.250000           .000000
        X3        7.500000           .000000
         L       90.000000           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         .000000          1.750000
```

3)	.000000	.500000
4)	.000000	.750000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	6.000000	.250000	INFINITY
X2	8.000000	.666667	.666667
X3	13.000000	3.000000	1.000000
L	-1.000000	INFINITY	.750000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	30.000000	18.000000
3	60.000000	15.000000	15.000000
4	90.000000	30.000000	18.000000

THE TABLEAU

ROW	(BASIS)	X1	X2	X3	L	SLK 2	SLK 3	SLK 4	
1	ART	.250	.000	.000	.000	1.750	.500	.750	97.500
2	X2	.375	1.000	.000	.000	.625	-.750	.625	11.250
3	X3	.250	.000	1.000	.000	-.250	.500	-.250	7.500
4	L	.000	.000	.000	1.000	.000	.000	1.000	90.000

1. If the quantity of product 1 which is produced were to increase by 20 units, then according to the "substitution rates" in the optimal tableau above, what is:

the change in quantity of product 2 produced? (+) (-)______
the change in quantity of product 3 produced? (+) (-)______
the change in labor hours purchased? (+) (-)______

(be sure to circle correct sign!)

2. Consider the raw material availability constraint, after it is transformed into equation form:

2 X1 + 2 X2 + 5 X3 (+) (-) SLK3 = 60

Circle the correct sign for SLK3 in the equation above.

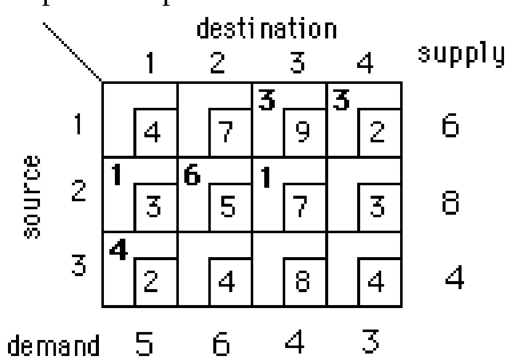
3. If the RHS remains unchanged, but the amount of raw material *used* were to decrease to 59, what would be the value of the slack variable? (+) (-) ________ *(be sure to specify the correct sign!)*

4. According to the substitution rates in the tableau above, if the slack variable (unused raw material) were to change from its current value to the value which you specified in (3), what would be

the change in quantity of product 1 produced? (+) (-)______
the change in quantity of product 2 produced? (+) (-)______
the change in quantity of product 3 produced? (+) (-)______
the change in labor hours purchased? (+) (-)______
the change in profit ? (+) (-)______

(be sure to circle correct sign!)

Consider the transportation problem with the initial solution specified below:



5. Is this a basic solution? (*circle:* Yes / No)
6. Did this initial solution result from the "NorthWest Corner Method"? (*circle:* Yes / No)
7. If X_{11} were to be increased, what would be the change in the cost function per unit increase in X_{11}? (+) (-)________ *(Be sure to indicate + or - !)*
8. If X_{11} were to enter the basis, what would be its value? _______
9. If X_{11} were to enter the basis, what variable(s) would leave the basis?

Circle: X_{13} X_{14} X_{21} X_{22} X_{23} X_{31}

10. If X_{11} were to enter the basis, would the new basic solution be degenerate?

(*circle:* Yes / No)

$$$$$$$$ Quiz # 5 Solutions $$$$$$$$

1. If the quantity of product 1 which is produced were to increase by 20 units, then according to the "substitution rates" in the optimal tableau above, what is:

the change in quantity of product 2 produced? (+) (-)_**-7.5**_
the change in quantity of product 3 produced? (+) (-)_**-5**__
the change in labor hours purchased? (+) (-)__**0**__

(be sure to circle correct sign!)

2. Consider the raw material availability constraint, after it is transformed into equation form:

$$2\,X1 + 2\,X2 + 5\,X3 \ (+)\ SLK3 = 60$$

Circle the correct sign for SLK3 in the equation above.

3. If the RHS remains unchanged, but the amount of raw material *used* were to decrease to 59, what would be the value of the slack variable? (+) (-) **+1** *(be sure to specify the correct sign!)*

4. According to the substitution rates in the tableau above, if the slack variable (unused raw material) were to change from its current value to the value which you specified in (3), what would be

the change in quantity of product 1 produced?	(+) (-) **0**
the change in quantity of product 2 produced?	(+) (-) **+0.75**
the change in quantity of product 3 produced?	(+) (-) **-0.5**
the change in labor hours purchased?	(+) (-) **0**
the change in profit ?	(+) (-) **-0.5**

(be sure to circle correct sign!)

5. Is this a basic solution? (*circle:* **Yes**)

6. Did this initial solution result from the "NorthWest Corner Method"? (*circle:* **No**)

7. If X_{11} were to be increased, what would be the change in the cost function per unit increase in X_{11}? (+) (-) **-1** *(Be sure to indicate + or - !)*

8. If X_{11} were to enter the basis, what would be its value? **1**

9. If X_{11} were to enter the basis, what variable(s) would leave the basis?

Circle: X_{13} X_{14} (**X_{21}**) X_{22} X_{23} X_{31}

10. If X_{11} were to enter the basis, would the new basic solution be degenerate?

(*circle:* **No**)

SSSSSSS Quiz # 6 SSSSSSS

Below, TP = transportation problem and AP = assignment problem.
Indicate whether true (+) *or false (*o*) :*

a.)____ Considered as a special case of the TP, the AP *always* has a degenerate basic optimal solution.

b.)____ After row reduction in the Hungarian method, each row contains at least one zero.

c.)____ An AP which has 16 variables will have 8 linear constraints.

d.)____ If an assignment (X^*) is optimal for the AP with cost matrix C, it is also optimal for the cost matrix obtained by adding 1 to each cost in column #1.

e.)____ The simplex method, if applied to AP, will always yield only integer (i.e., non-fractional) optimal solutions.

f.)____ In the Hungarian method, at each reduction step, i.e., row reduction, column reduction, or "mixed" reduction (in which the smallest cost without a line is used), the total number of zeroes in the cost matrix will increase.

The statements below refer to the AP cost matrix:

$$\begin{bmatrix} 2 & 0 & 5 & 0 \\ 0 & 6 & 4 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 3 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \rightarrow \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$

g.)____ The cost matrix above might be the result after the row and column reduction steps of the Hungarian method applied to some AP cost matrix.

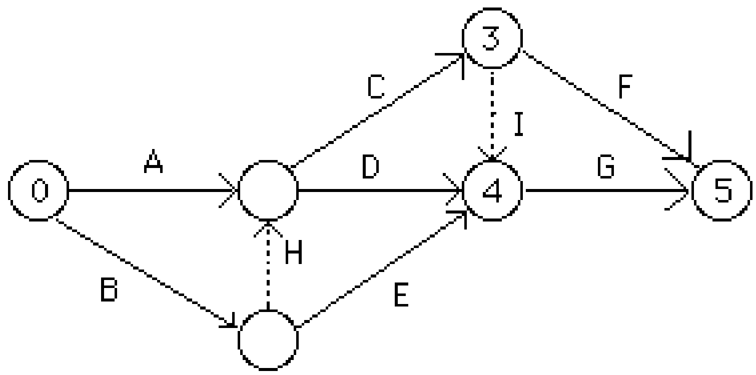
h.)____ After the <u>next</u> step of the Hungarian method, all of the elements currently occupied by zeroes in this matrix will again be occupied by zeroes.

i.)____ X_{23}=1 in the optimal solution of this AP.

j.)____ X_{14}=1 in the optimal solution of this AP.

Quiz # 7

Project Scheduling. Consider the project with the A-O-A (activity-on-arrow) network:

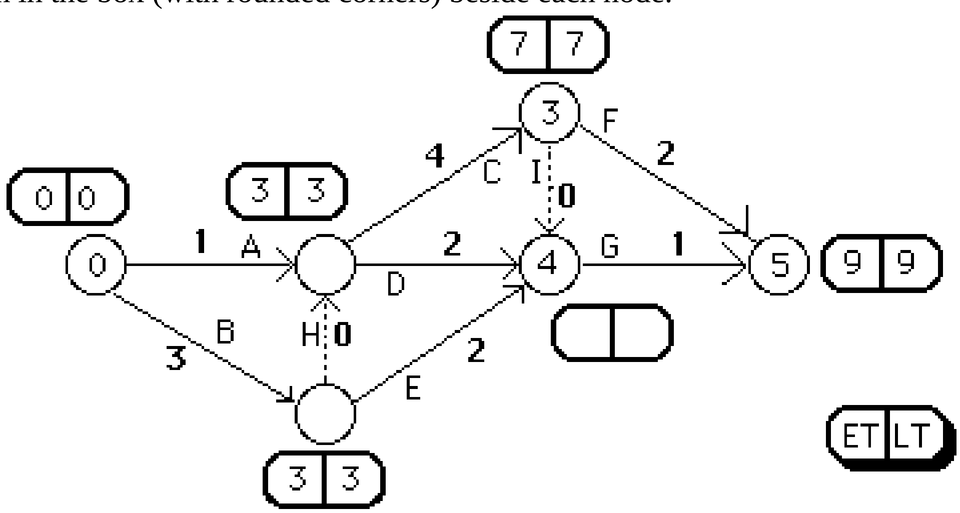


1. Complete the labeling of the nodes on the network above.

___ 2. The number of activities (i.e., tasks), <u>not</u> including "dummies", which are required to complete this project is

a. six
b. seven
c. eight
d. nine
e. none of the above

The activity durations are given below on the arrows. The Early Times (ET) and Late Times (LT) for each node are written in the box (with rounded corners) beside each node.



___ 3. The early time (ET) of node #4 in the network above is:

a. five
b. six
c. seven
d. eight
e. none of the above

___ 4. The late time (LT) of node #4 in the network above is:

a. five
b. six
c. seven
d. eight
e. none of the above

___ 5. The slack ("total float") for activity A is

a. zero
c. two

b. one d. three e. none of the above

6. Which activities are critical? (*circle:* A B C D E F G H I)

___ 7. The earliest completion time for the project is

a. six c. eight
b. seven d. nine e. none of the above

Suppose that the non-zero durations are *random*, with each value in the above network being the *expected* values and each *standard deviation* equal to 1.00. Then...

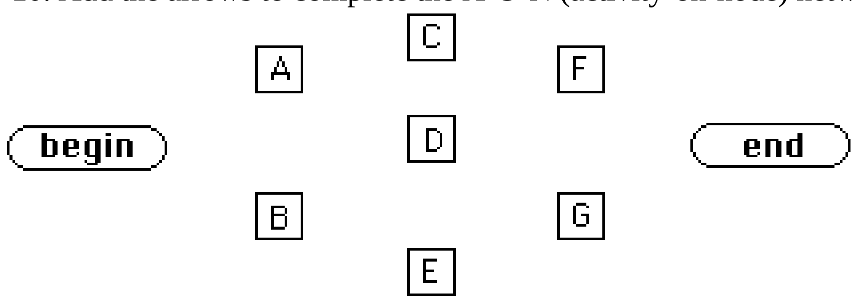
___ 8. The expected earliest completion time for the project is

a. six c. eight
b. seven d. nine e. none of the above

___ 9. The *standard deviation* σ of the earliest completion time for the project is

a. $\sqrt{3}$ c. 2 e. $\sqrt{7}$
b. 3 d. 4 f. 7 g. none of the above

10. Add the arrows to complete the A-O-N (activity-on-node) network below for this same project.



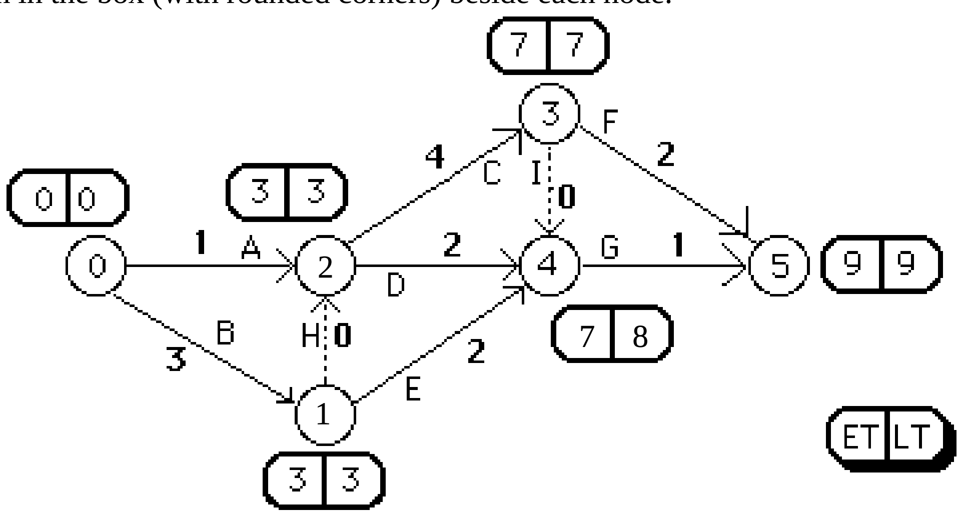
SSSSSSSS Quiz # 7 Solutions SSSSSSSS

1. Complete the labeling of the nodes on the network above.

_**b**_ 2. The number of activities (i.e., tasks), <u>not</u> including "dummies", which are required to complete this project is

a. six c. eight
b. seven d. nine e. none of the above

The activity durations are given below on the arrows. The Early Times (ET) and Late Times (LT) for each node are written in the box (with rounded corners) beside each node.



_**c**_ 3. The early time (ET) of node #4 in the network above is:

a. five c. seven
b. six d. eight e. none of the above

_**d**_ 4. The late time (LT) of node #4 in the network above is:
a. five c. seven
b. six d. eight e. none of the above

_**c**_ 5. The slack ("total float") for activity A is
a. zero c. two
b. one d. three e. none of the above

6. Which activities are critical? (*circle:* A B C D E F G H I)

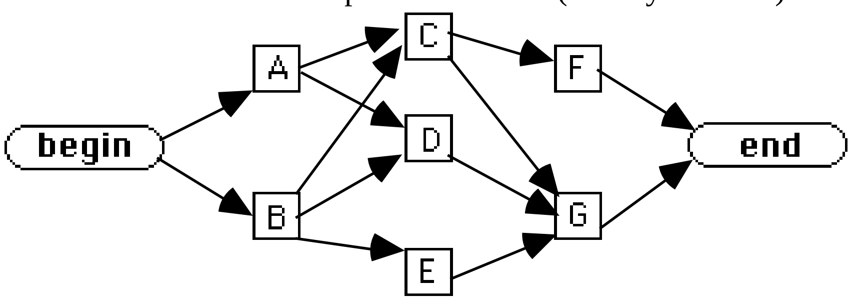
_**d**_ 7. The earliest completion time for the project is
a. six c. eight
b. seven d. nine e. none of the above

Suppose that the non-zero durations are *random*, with each value in the above network being the *expected* values and each *standard deviation* equal to 1.00. Then...

_**d**_ 8. The expected earliest completion time for the project is
a. six c. eight
b. seven d. nine e. none of the above

_**a**_ 9. The *standard deviation* σ of the earliest completion time for the project is
a. $\sqrt{3}$ c. 2 e. $\sqrt{7}$
b. 3 d. 4 f. 7 g. none of the above

10. Add the arrows to complete the A-O-N (activity-on-node) network below for this same project.



SSSSSSSS Quiz # 8 SSSSSSSS

To model a production planning problem, define
X_j = amount of item j which is produced (a continuous variable), and
Y_j = 1 if item j is produced, otherwise 0 (a binary integer variable),
for j=1,2,3,.... Select a constraint (or set of constraints) to model each situation below:

____ 1. At least one of items #1, #2, and #3 *nust* be produced.
a. $X_1 + X_2 + X_3 \leq 1$ c. $Y_1 + Y_2 + Y_3 \geq 1$ e. $Y_1 + Y_2 + Y_3 \leq 2$
b. $X_1 + X_2 + X_3 \geq 1$ d. $Y_1 + Y_2 + Y_3 \leq 1$ f. *NOTA*

____ 2. If it is decided to produce item #1, then *at most* 100 units of item #1 may be produced.
a. $X_1 \leq 100Y_1$ c. $100X_1 \leq Y_1$ e. $X_1 + Y_1 \leq 100$
b. $100X_1 \geq Y_1$ d. $X_1 \geq 100Y_1$ f. *NOTA*

____ 3. If neither item #2 nor item #3 are produced, then item #1 *cannot* be produced.
a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 4. If item #1 is produced, then *at least* 100 units of item #1 must be produced.
a. $X_1 \leq 100Y_1$ c. $100X_1 \leq Y_1$ e. $X_1 + Y_1 \leq 100$
b. $100X_1 \geq Y_1$ d. $X_1 \geq 100Y_1$ f. *NOTA*

____ 5. If both items #2 and #3 are produced, then item #1 must also be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 6. *At most one* of items #1, #2, and #3 may be produced.

a. $X_1 + X_2 + X_3 \leq 1$ c. $Y_1 + Y_2 + Y_3 \geq 1$ e. $Y_1 + Y_2 + Y_3 \leq 2$
b. $X_1 + X_2 + X_3 \geq 1$ d. $Y_1 + Y_2 + Y_3 \leq 1$ f. *NOTA*

____ 7. If item #1 is produced, then *either* item #2 *or* item #3 (or both) must be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 8. If item #1 is produced, then *both* items #2 and #3 must be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 9. At least two of items #1, #2, and #3 must be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 10. If item #1 is produced, then item #2 *must* be produced.

A. $100X_1 \geq Y_1$
B. $2Y_1 = Y_2+Y_3$
C. $Y_1 + Y_2 + Y_3 \geq 2$
D. $X_1 \leq X_2 + X_3$
E. $X_1 \leq Y_1$
F. $X_1 \geq Y_1$
G. $Y_1 + Y_2 + Y_3 \geq 1$
H. $X_1 \geq 100Y_1$
I. $Y_1 \geq Y_2 + Y_3$
J. $X_2 \leq X_1$
K. $2Y_1 \geq Y_2+Y_3$
L. $Y_1 \leq Y_2 + Y_3$
M. $Y_1 \leq Y_2 + Y_3$
N. $X_1 + X_2 + X_3 \geq 1$
O. $X_1 + Y_1 = 1$
P. $Y_1 + Y_2 + Y_3 \leq 2$
Q. $X_1 \leq X_2$
R. $100X_1 \leq Y_1$
S. $2Y_1 \leq Y_2+Y_3$
T. $X_1 = Y_2$
U. $Y_1 \leq Y_2 + Y_3$
V. $Y_1 = Y_2$
W. $Y_1 + Y_2 + Y_3 \leq 1$
X. $X_1 \leq 100Y_1$
Y. $X_1 + X_2 + X_3 \geq 2$
Z. *None of the above!*

SSSSSSSS Quiz # 8 Solutions SSSSSSSS

__d_ 1. If setup #1 is done, then *at least* 100 units of item #1 must be produced.
__f_ 2. If both setups #2 and #3 are done, then setup #1 must also be done.

The required constraint would be $Y_2 + Y_3 \leq Y_1+1$

__f_ 3. Setups for *at least two* different items must be done.

The required constraint would be $Y_1 + Y_2 + Y_3 \geq 2$

__e_ 4. If setup is done for item #1, then item #2 *must* be produced.
__d_ 5. Setups for *at most one* of items #1, #2, and #3 may be done.
__c_ 6. A setup for at least one of items #1, #2, and #3 *must* be done.
_a__ 7. If it is decided to produce item #1, then *at most* 100 units of item #1 may be produced.
_b__ 8. If setups are done for neither item #2 nor #3, then setup *cannot* be done for item #1.
_b__ 9. If setup is done for item #1, then a setup for *either* item #2 *or* #3 (or both) must be selected.
_e__ 10. If item #1 is produced, then setups must be done to produce *both* items #2 and #3.
Note that statements 8 & 9 are logically equivalent!

SSSSSSSS Quiz # 9 SSSSSSSS

Customers buy cars from three auto companies. Given the company from which a customer last bought a car, the probability that she will buy her next car from each company is as follows:

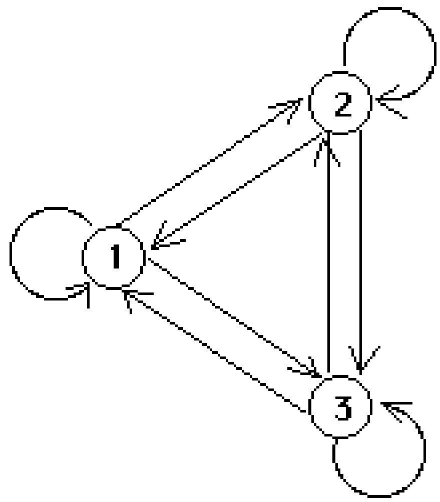
Last Bought From	Will Buy Next From Co. 1	Co. 2	Co.3
Company 1	.80	.10	.10
Company 2	.05	.85	.10
Company 3	.05	.15	.80

The second and third powers of the matrix (P) above, and the steadystate distribution, are:

$$P^2 = \begin{bmatrix} .65 & .18 & .17 \\ .0875 & .7425 & .17 \\ .0875 & .2525 & .66 \end{bmatrix},$$

$$P^3 = \begin{bmatrix} .5375 & .2435 & .219 \\ .1156 & .6653 & .219 \\ .1156 & .3223 & .562 \end{bmatrix}$$

$$\pi = [1/5\ ,\ 7/15\ ,\ 1/3]$$



1. On the diagram (above right) write the transition probabilities for the car purchased by "Jane Doe".

___ 2. If Jane currently owns a Company 2 car, what is the probability that her *next* car is a Company 1 car?
a. 5% c. 15% e. 85%
b. 10% d. 80% f. *NOTA*

___ 3. If Jane currently owns a Company 2 car, what is the probability that the car *following* her next car is a Company 1 car?
a. 5% c. 11.56% e. 65%
b. 8.75% d. 53.75% f. *NOTA*

___ 4. If Jane currently owns a Company 2 car, what is the probability that *at least one* of the next two cars she buys will be a Company 1 car?
a. 5%+(85%)(5%)+(10%)(5%)=9.75% c. 80%+(10%)(5%)+(10%)(5%)= 81% e. 100%
b. 5%+8.75%+11.56% =25.31% d. 80%+5%+5% = 90% f. *NOTA*

___ 5. The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation
a. $P^t \pi = 0$ c. $P \pi = \pi$ e. $\pi P = 0$
b. $P \pi = 0$ d. $\pi (I-P) = 0$ f. *NOTA*

___ 6. The equations to be solved for the steadystate probabilities include:
a. $0.8\pi_1 + 0.05\pi_2 + 0.05\pi_3 = 0$ c. $0.8\pi_1 + 0.10\pi_2 + 0.10\pi_3 = 0$ e. $\pi_1 + \pi_2 + \pi_3 = 0$
b. $0.8\pi_1 + 0.05\pi_2 + 0.05\pi_3 = \pi_1$ d. $0.8\pi_1 + 0.10\pi_2 + 0.10\pi_3 = \pi_1$ f. *NOTA*

___ 7. Over a "long" period of time, which company would you expect to have the largest market share?
a. Company #1 c. Company #3 e. All 3 share equal
b. Company #2 d. Both Co. #1 & #3 equal f. *NOTA*

___ 8. The number of *transient* states in this Markov chain model is

a. 0 c. 2 e. 9
b. 1 d. 3 f. *NOTA*

___ 9. The number of *recurrent* states in this Markov chain model is
a. 0 c. 2 e. 9
b. 1 d. 3 f. *NOTA*

___ 10. The probabilities in a Markov chain transition matrix are
a. simple probabilities. c. conditional probabilities. e. *NOTA*
b. joint probabilities. d. more than one of the above are correct.

Quiz # 10

A baseball team consists of 2 stars, 13 starters, and 10 substitutes. For tax purposes, the team owner must place a value on the players. The value of each player is defined to be the total value of the salary he will earn until retirement. At the beginning of each season, the players are classified into one of four categories:

Category 1: Star (earns $1 million per year).
Category 2: Starter (earns $400,000 per year).
Category 3: Substitute (earns $100,000 per year).
Category 4: Retired (earns no more salary).

Given that a player is a star, starter, or substitute at the beginning of the current season, the probabiliites that he will be a star, starter, substitute, or retired at the beginning of the next season are as follows:

This Season	Next Season: Star	Starter	Substitute	Retired
Star	0.50	0.30	0.15	0.05
Starter	0.20	0.50	0.20	0.10
Substitute	0.05	0.15	0.50	0.30
Retired	0	0	0	1

$$P^2=\begin{bmatrix} 0.3175 & 0.3225 & 0.21 & 0.15 \\ 0.21 & 0.34 & 0.23 & 0.22 \\ 0.08 & 0.165 & 0.2875 & 0.4675 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F^{(2)}=\begin{bmatrix} 0.0675 & 0.1725 & 0.135 & 0.1 \\ 0.11 & 0.09 & 0.13 & 0.12 \\ 0.055 & 0.09 & 0.0375 & 0.1675 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.23375 & 0.288 & 0.217125 & 0.261125 \\ 0.1845 & 0.2675 & 0.2145 & 0.3335 \\ 0.087375 & 0.149625 & 0.18875 & 0.57425 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F^{(3)}=\begin{bmatrix} 0.04125 & 0.09975 & 0.1065 & 0.111125 \\ 0.066 & 0.0525 & 0.092 & 0.1135 \\ 0.044 & 0.053625 & 0.02625 & 0.10675 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

E =

from \ to	1	2	3
1	3.2	2.5090909	1.9636364
2	1.6	3.5272727	1.8909091
3	0.8	1.3090909	2.7636364

___ 1. The number of *transient* states in this Markov chain model is
a. 0 c. 2 e. 4
b. 1 d. 3 f. *NOTA*

___ 2. The number of *recurrent* states in this Markov chain model is
a. 0 c. 2 e. 4
b. 1 d. 3 f. *NOTA*

___ 3. If Joe Blough is a substitute in 1994, what is the probability that he is a star in 1997? (choose nearest answer)
a. 1% c. 5% e. 9%

b. 3% d. 7% f. *NOTA*

___ 4. If Joe Blough is a substitute in 1994, what is the probability that he is *first becomes* a star in 1997? (choose nearest answer)

a. 1% c. 5% e. 9%
b. 3% d. 7% f. *NOTA*

___ 5. If Joe Blough is a substitute at the beginning of 1994, what is the expected length of his playing career, in years? (choose nearest answer)

a. 1year c. 5 years e. 9 years
b. 3 years d. 7 years f. *NOTA*

mmm Part Two mmm

Assuming that the total number of players on the team must remain constant (at 25), a player must be replaced when he retires. Suppose that the team owner's policy is to replace a retiring player with a player in the "Starter" category. In the Markov chain model below, the state of the system is the classification of the player who is wearing a certain uniform number (which is inherited from player to player).

$$P=\begin{bmatrix}0.5 & 0.35 & 0.15\\ 0.2 & 0.6 & 0.2\\ 0.05 & 0.35 & 0.6\end{bmatrix}$$

i	π_i
1	0.21818182
2	0.46666667
3	0.31515152

$$M=\begin{bmatrix}4.5833 & 2.8571 & 5.7692\\ 6.6666 & 2.1428 & 5.3846\\ 8.3333 & 2.8571 & 3.1730\end{bmatrix}$$

___ 1. The equations to be solved for the steadystate probabilities include:

a. $0.15\pi_1 + 0.2\pi_2 + 0.6\pi_3 = \pi_3$ c. $0.05\pi_1 + 0.35\pi_2 + 0.6\pi_3 = \pi_3$ e. $\pi_1 + \pi_2 + \pi_3 = 0$
b. $0.15\pi_1 + 0.2\pi_2 + 0.6\pi_3 = 0$ d. $0.05\pi_1 + 0.35\pi_2 + 0.6\pi_3 = 0$ f. *NOTA*

___ 2. The states of this Markov chain model are

a. neither transient nor recurrent c. no transient & 3 recurrent e. *NOTA*
b. all states both transient & recurrent d. 3 transient & no recurrent

___ 3. Over a "long" period of time, the number of stars on the team will average (choose nearest integer)

a. 1 c. 5 e. 9
b. 3 d. 7 f. 11

___ 4. The number of years required for a new starter (or his successor) to first become a star is (choose nearest integer):

a. 1 c. 5 e. 9
b. 3 d. 7 f. 11

___ 5. In steady state, the annual salary for the team will average (choose nearest answer)

a. $3 million c. $7 million e. $11 million
b. $5 million d. $9 million f. $13 million

SSSSSSS Quiz # 10 Solutions SSSSSSS

_**d**_ 1. The number of *transient* states in this Markov chain model is
That is states 1,2, and 3.

_**b**_ 2. The number of *recurrent* states in this Markov chain model is
That is state 4.

_**e**_ 3. If Joe Blough is a substitute in 1994, what is the probability that he is a star in 1997? (choose nearest answer)

_**c**_ 4. If Joe Blough is a substitute in 1994, what is the probability that he is *first becomes* a star in 1997? (choose nearest answer)
Solution: 4.4%

_**c**_ 5. If Joe Blough is a substitute at the beginning of 1994, what is the expected length of his playing career, in years? (choose nearest answer)
Since 0.8+1.3+2.76=4.86

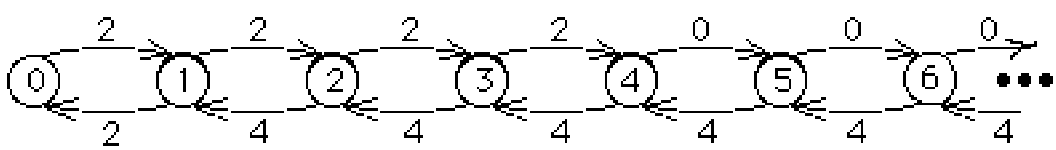
mmm Part Two mmm

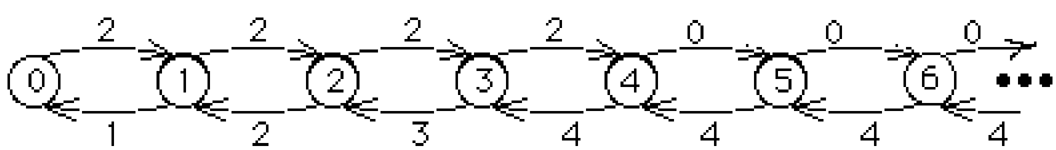
_a_ 1. The equations to be solved for the steadystate probabilities include:
_c_ 2. The states of this Markov chain model are
Since all steady state probabilites are all positive.
_c_ 3. Over a "long" period of time, the number of stars on the team will average (choose nearest integer)
Since $25\pi_1 = 5.45$.
_d_ 4. The number of years required for a new starter (or his successor) to first become a star is (choose nearest integer):
Solution = 6.6666
_e_ 5. In steady state, the annual salary for the team will average (choose nearest answer)
Since $25\pi_1(1) + 25\pi_2(0.4) + 25\pi_3(0.1) = 10.9$

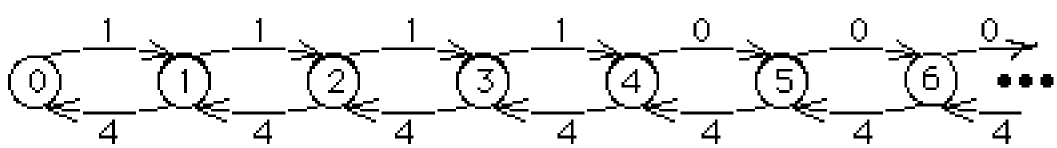
\$\$\$\$\$\$\$\$ Quiz # 11 \$\$\$\$\$\$\$\$

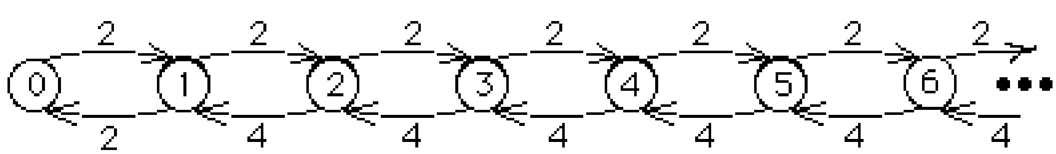
For each diagram of a Markov model of a queue in (a) through (f) below, indicate the correct Kendall's classification from among the following choices :

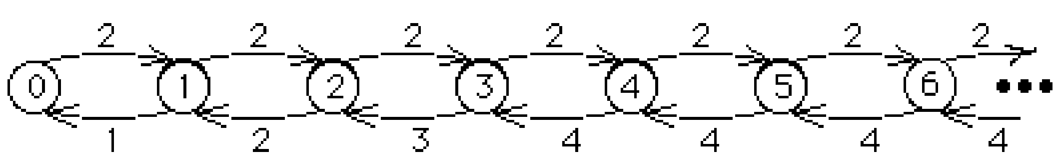
(1) M/M/1	(2) M/M/2	(3) M/M/1/4
(4) M/M/4	(5) M/M/2/4	(6) M/M/2/4/4
(7) M/M/1/2/4	(8) M/M/4/2	(9) M/M/4/4
	(10) none of the above	

____(a) 

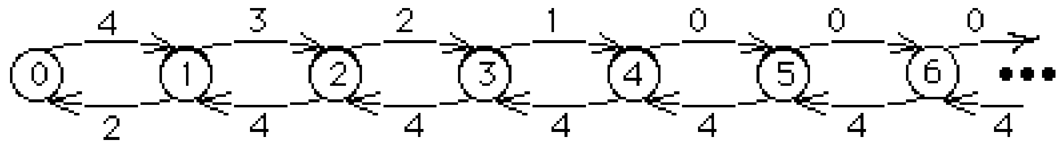
____(b) 

____(c) 

____(d) 

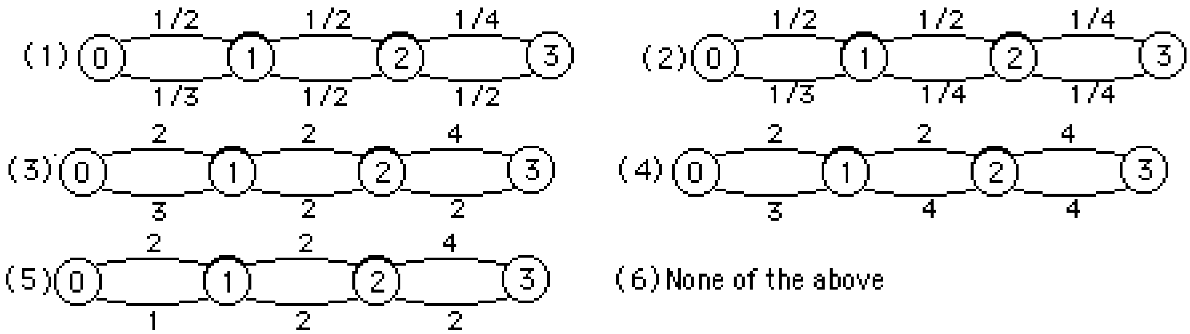
____(e) 

_____(f)



Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of 4 hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of 3 hours (also exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there is at least one idle mechanic, but one every 4 hours when both mechanics are busy. If 3 cars are in the shop, no cars arrive.

____ g. Choose the transition diagram below corresponding to this system.



The steady-state probabilities for this system are: π_0=4/19, π_1=6/19, π_2=6/19, π_3=3/19.

____ h. What fraction of the day will both mechanics be idle?

1. 2/19
2. 3/19
3. 4/19
4. 6/19
5. None of the above

____ i. What fraction of the day will both mechanics be working on the same car?

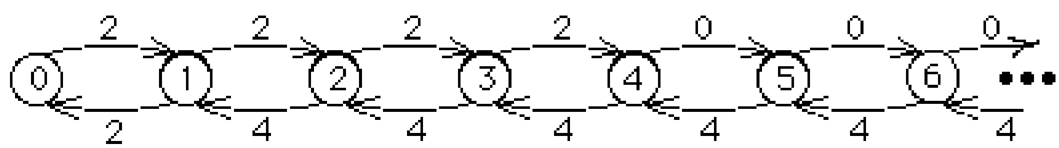
1. 2/19
2. 3/19
3. 4/19
4. 6/19
5. None of the above

____ j. What is the average number of cars in the shop?

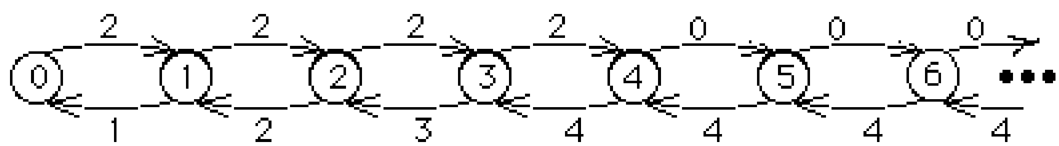
1. 10/19
2. 15/19
3. 27/19
4. 36/19
5. None of the above

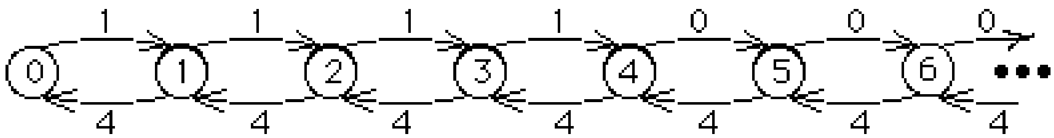
Quiz # 11 Solutions

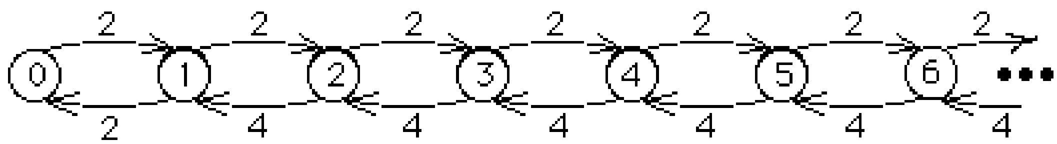
_5_ (a)

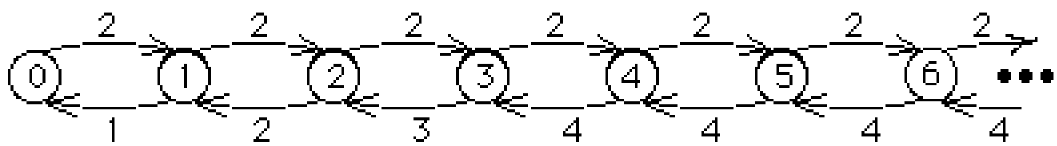


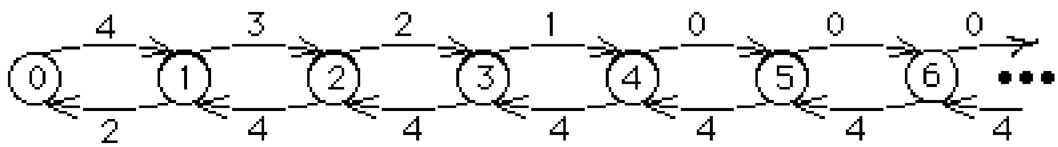
_9_ (b)



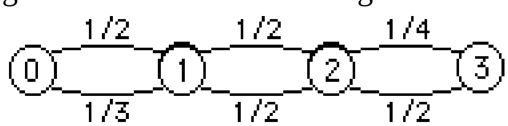
_3_ (c) 

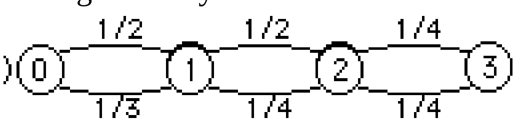
_2_ (d) 

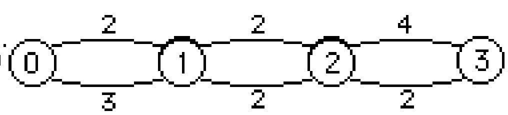
_4_ (e) 

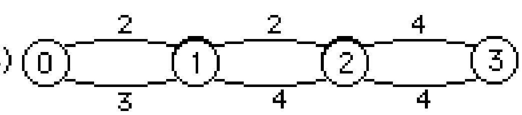
_6_ (f) 

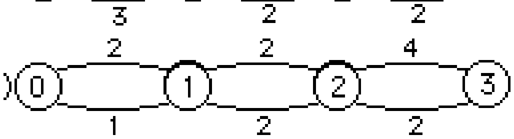
_**6**_ g. Choose the transition diagram below corresponding to this system.

(1) 

(2) 

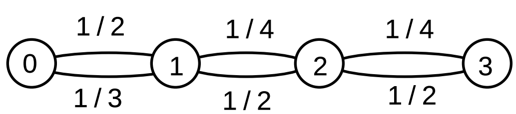
(3) 

(4) 

(5) 

(6) None of the above

Note: The correct answer should be



The steady-state probabilities for this system are: π_0=4/19, π_1=6/19, π_2=6/19, π_3=3/19.

_**3**_ h. What fraction of the day will both mechanics be idle?

1. 2/19 2. 3/19
3. 4/19 4. 6/19 5. None of the above

_**4**_ i. What fraction of the day will both mechanics be working on the same car?

1. 2/19 2. 3/19
3. 4/19 4. 6/19 5. None of the above

_**3**_ j. What is the average number of cars in the shop?

1. 10/19 2. 15/19
3. 27/19 4. 36/19 5. None of the above

$$$$$$$$ Quiz # 12 $$$$$$$$

Match Problem. Suppose that there are 27 matches originally on the table, and you are challenged by your dinner partner to play this game. Each player must pick up either 1, 2, 3, or 4 matches, with the player who picks up the last match pays for dinner.

Define F(i) to be the minimal cost to you (either 1 or 0) if it is your turn to pick up matches, and i matches remain on the table. Thus, F(1) = 1, F(2) = 0 (since you can pick up one match, forcing your opponent to pick up the last match), etc.

_____ 1. What is the value of F(3)?

_____ 2. What is the value of F(4)?

_____ 3. What is the value of F(6)?

_____ 4. What is the value of F(27)?

Auto Replacement Problem. Suppose that a new car costs $15,000 and that the annual operating cost and resale value of the car are as shown in the table below:

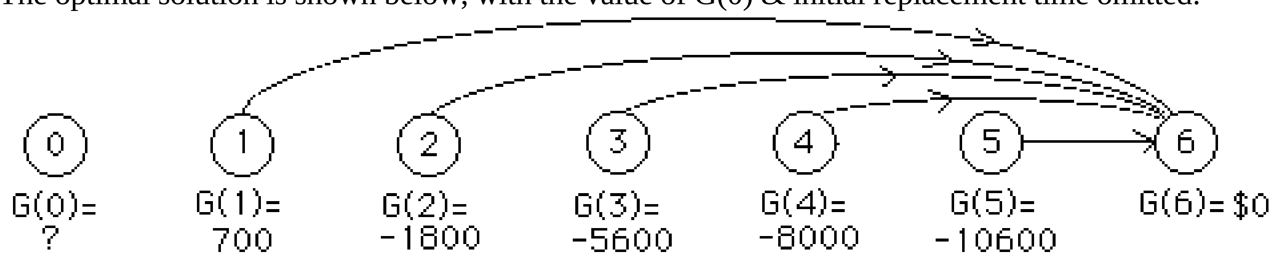
Age of Car (years)	Resale Value	Operating Cost
1	$11000	$400 (year 1)
2	$9000	$600 (year 2)
3	$7500	$900 (year 3)
4	$5000	$1200 (year 4)
5	$4000	$1600 (year 5)
6	$3000	$2200 (year 6)

(The operating cost specified above is for the year which is ending.) If I have a new car now (time 0, and this initial car is assumed to be "free", i.e. a "sunk" cost), I wish to determine a replacement policy that minimizes my net cost of owning and operating a car for the next six years.

Define G(t) = minimum cost of owning and operating car(s) through the end of the sixth year, **given that I have a new car at the end of year t.**

(As in the example solved in class, this includes the cost of the replacement car if I trade in my current car before the end of the sixth year, but does not include the cost of the car which is new at the beginning of this period.)

The optimal solution is shown below, with the value of G(0) & initial replacement time omitted:



___ 5. The value of G(5), i.e., the cost for the final year if I have a new car at the end of year 5, is

a. 0 | c. 2200 | e. 11000-2200 = 8800
b. 400 | d. 400-11000 = -10600 | f. *NOTA*

___ 6. If I have a new car at the end of year 4 and replace it after one year, my cost for the remainder of the six-year period is

a. 0 | c. 2200 | e. 11000-2200 = 8800
b. 400 | d. 400-11000 = -10600 | f. *NOTA*

___ 7. If I have a new car at the end of year 4 and keep it until the end of the sixth year, my cost for that period is

a. 600 | c. 1000-9000=-8000 | e. 9000-600 = 8400
b. 1000 | d. 600-3000 = -2400 | f. *NOTA*

___ 8. If I have a new car at the end of year 2, how old will it be when I should replace it?

a. 1 year old | c. 3 years old | e. 5 years old
b. 2 years old | d. 4 years old | f. 6 years old

___ 9. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the first year, my total cost for the six-year period is

a. 400 | c. 15000-11000=4000 | e. 9000-600 = 8400
b. 400+700=1100 | d. 400+15000-11000 = 4400 | f. *NOTA*

___ 10. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the second year, my total cost for the six-year period is

a. 600 | c. 400+600-9000-1800=-6200 | e. 400+600-1800=-800
b. 400+600=1000 | d. 400+600+15000-9000=7000 | f. *NOTA*

SSSSSSSS Quiz # 12 Solutions SSSSSSSS

__1_ 1. What is the value of F(3)?

__1_ 2. What is the value of F(4)?

__0_ 3. What is the value of F(6)?

__1__ 4. What is the value of F(27)?

_d_ 5. The value of G(5), i.e., the cost for the final year if I have a new car at the end of year 5, is
_f_ 6. If I have a new car at the end of year 4 and replace it after one year, my cost for the remainder of the six-year period is

The correct value is 400 + 15000 - 11000 + (-10600) = - 6200

_c_ 7. If I have a new car at the end of year 4 and keep it until the end of the sixth year, my cost for that period is
_d_ 8. If I have a new car at the end of year 2, how old will it be when I should replace it?
_f_ 9. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the first year, my total cost for the six-year period is

The correct value is (400 + 15000 - 11000) + 700 = 5200

_f_ 10. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the second year, my total cost for the six-year period is

The correct answer is 400 + 600 + 15000 - 9000 + (-1800) = +5200