56:171 Operations Research
Sample Quizzes -- Fall 1993

Quiz # 1

Indicate whether each statement is **true** or **false**.

_____ a. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot column, the resulting tableau is infeasible.

_____ b. The number of basic variables in an LP is equal to the number of rows, including the objective function row.

_____ c. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

_____ d. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

_____ e. A "pivot" in a nonbasic column of a tableau will make it a basic column.

_____ f. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

_____ g. If the columns of a 3x3 matrix are linearly independent, then the matrix is singular.

_____ h. It may happen that an LP problem has (exactly) two optimal solutions.

_____ i. If an LP model has 3 nonnegative variables, then when entering it into LINDO one must include three constraints of the type $Xj \geq 0$.

_____ j. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you cannot pivot in row i.

Quiz # 2

Indicate whether each statement is **true** or **false**. (a) through (e) refer to the following LP problem:

$$\text{Minimize } X_1 + 2X_2 + 3X_3$$
$$\text{s.t. } X_1 - X_2 + 2X_3 \leq 20$$
$$X_1 + 3X_2 + X_3 = 30$$
$$2X_1 + X_2 - X_3 \geq 40$$
$$X_j \geq 0,\ j=1,2,3$$

_____ a. In the first phase of the two-phase simplex method, only two artificial variables are required.

_____ b. At the beginning of both the "Big-M" method and (the first phase of) the 2-phase method, used to solve the LP above, the objective function will have only 2 terms.

_____ c. At the beginning of the first phase of the two-phase simplex method, the phase-one objective function will have the value 70.

_____ d. At the beginning of the "Big-M" method used to solve the LP above, if M=100, then the objective function will have the value 100.

_____ e. If the LP above were a maximization rather than minimization problem, the first phase of the two-phase method would be exactly the same.

_____ f. The initial basic solution in the two-phase method is infeasible in the original problem, but in the "Big-M" method it is feasible.

_____ g. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.

_____ h. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

_____ i. If an LP model is of the form $Ax \leq b$, $x \geq 0$, and b is nonnegative, then there is no need for artificial variables.

_____ j. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero.

SSSSSSSS Quiz # 2 Solutions SSSSSSSS

True a. In the first phase of the two-phase simplex method, only two artificial variables are required. *In the "less-than-or-equal" constraint, the slack variable,which will be introduced when converting the inequality to an equation, can be used as a basic variable.*

False b. At the beginning of both the "Big-M" method and (the first phase of) the 2-phase method, used to solve the LP above, the objective function will have only 2 terms. *In the 2-Phase method, the objective function in Phase One will be the sum of the two artificial variables, but in "Big-M" method, the objective function would be the original 3 terms plus M times the sum of the artificial variables.*

True c. At the beginning of the first phase of the two-phase simplex method, the phase-one objective function will have the value 70. *The artificial variables in the second and third constraints will have the values 30 and 40, respectively, so their sum (the Phase One objective) will be 70.*

False d. At the beginning of the "Big-M" method used to solve the LP above, if M=100, then the objective function will have the value 100. *The value of the objective function will be 100 times the sum of the artificial variables, which (see (c) above) will be 70, so that the beginning objective value will be 7000.*

True e. If the LP above were a maximization rather than minimization problem, the first phase of the two-phase method would be exactly the same. *The first phase will be a minimization problem, whatever the objective in phase two.*

False f. The initial basic solution in the two-phase method is infeasible in the original problem, but in the "Big-M" method it is feasible. *The initial basic solution in the two methods will be exactly the same, and will be infeasible in the original problem.*

False g. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i. *If the element in row i of the pivot column is zero or negative, it will not be selected as the pivot row; if it is positive, it must be selected as the pivot row (assuming no other row satisfies the same condition).*

True h. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

True i. If an LP model is of the form $Ax \leq b$, $x \geq 0$, and b is nonnegative, then there is no need for artificial variables. *The slack variables may be used as the initial basic variables.*

False j. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero. *If the original problem is not feasible, then the phase one objective function value will be nonzero, with an artificial variable having a positive value.*

SSSSSSSS Quiz # 3 SSSSSSSS

The following questions refer to the LP model for PAR, Inc. and its LINDO output. Select the answers from the list at the bottom of the quiz and **write only the alphabetic letter** in the blank.

@@@ PAR, inc. @@@

Processing times (hrs/golf bag):

	Cut & Dye	Sew	Finish	Inspect & Pack
Standard	0.7	0.5	1	0.1
Deluxe	1	0.8666	0.6666	0.25
Available hrs	630	600	708	135

Variables:

X1 = production of STANDARD golf bags (bags/quarter)
X2 = production of DELUXE golf bags (bags/quarter)

```
 MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END

        OBJECTIVE FUNCTION VALUE

        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110          .000000
        X2       251.997800          .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000         4.375086
        3)       111.602000          .000000
        4)          .000000         6.937440
        5)        18.000232          .000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1       10.000000         3.500135         3.700000
       X2        9.000000         5.285715         2.333400

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2      630.000000        52.364582       134.400000
        3      600.000000         INFINITY       111.602000
        4      708.000000       192.000010       128.002800
        5      135.000000         INFINITY        18.000232

THE TABLEAU
    ROW (BASIS)    X1      X2    SLK 2  SLK 3  SLK 4   SLK5
      1 ART        .000   .000   4.375   .000   6.937  .000 7668.012
      2  X2        .000  1.000   1.875   .000  -1.312  .000  251.998
      3 SLK 3      .000   .000  -1.000  1.000    .200  .000  111.602
      4 X1        1.000   .000  -1.250   .000   1.875  .000  540.003
      5 SLK 5      .000   .000   -.344   .000    .141 1.000   18.000
```

- There are __I__ unused hours in the inspect&pack department.
- The reduced cost of the variable X2 is __II__ .
- If the profit per STANDARD bag were to increase from $10 to $15, the quantity of these bags which should then be manufactured is _III_ .
- If the profit per DELUXE bag were to increase from $9 to $12, the quantity of these bags which should then be manufactured is _IV_ .
- If an additional hour were available in the Cutting&Dyeing Dept., the increase in profit would be _V_, the number of STANDARD bags would _VI_ (increase/decrease/stay the same/insufficient info.), and the number of DELUXE bags would _VII_ (increase/decrease/stay the same/insufficient info.)
- If an additional hour were available in the Sewing Dept., the increase in profit would be _VIII_, the number of STANDARD bags would _IX_ (increase/decrease/stay the same/insufficient info.), and the number of DELUXE bags would _X_ (increase/decrease/stay the same/insufficient info.)

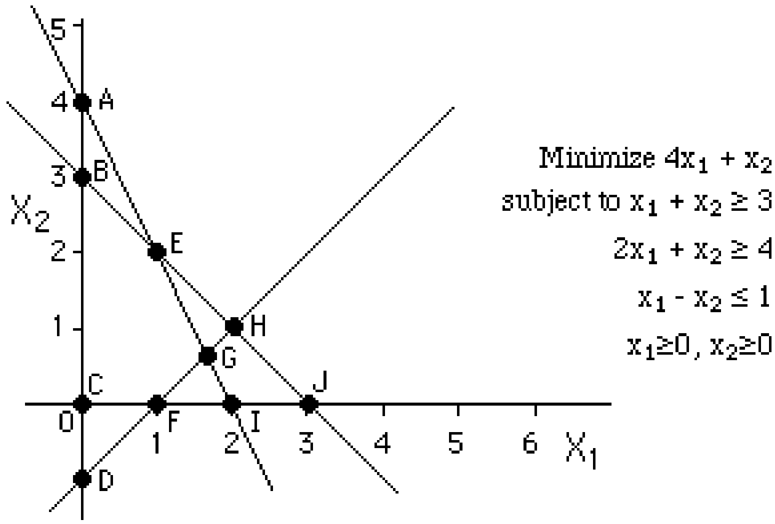
I. _____	II. ____	III. ____	IV. _____
V. _____	VI. ____	VII. ____	VIII. ____
IX. ____	X. ____		

a. Zero	b. Decrease	c. Increase
d. Not sufficient information	e. Remain the same	f. 18
g. 52.36	h. 111.6	i. 128
j. 134.4	k. 192	l. 370
m. $2.33	n. $3.50	o. $3.70
p. $4.38	q. $5.29	r. $6.94

Quiz # 3 Solutions

- There are **18** unused hours in the inspect&pack department.
- The reduced cost of the variable X2 is **zero** .
- If the profit per STANDARD bag were to increase from $10 to $15, the quantity of these bags which should then be manufactured is **Not sufficient information** .
- If the profit per DELUXE bag were to increase from $9 to $12, the quantity of these bags which should then be manufactured is **Remain the same** .
- If an additional hour were available in the Cutting&Dyeing Dept., the increase in profit would be **$4.38** , the number of STANDARD bags would **Decrease** , and the number of DELUXE bags would **Increase**.
- If an additional hour were available in the Sewing Dept., the increase in profit would be **zero**, the number of STANDARD bags would **Remain the same**, and the number of DELUXE bags would **Remain the same**.

Quiz # 4



a. Which of the points are feasible solutions in the LP problem above?
(Circle all that apply): A B C D E F G H I J

b. Which of the points are basic solutions?
(Circle all that apply): A B C D E F G H I J

c. In order to formulate the LP using only equality & nonnegativity constraints, 3 additional variables (x_3, x_4, and x_5) were defined. Insert the correct sign (+ or -) in each constraint below:

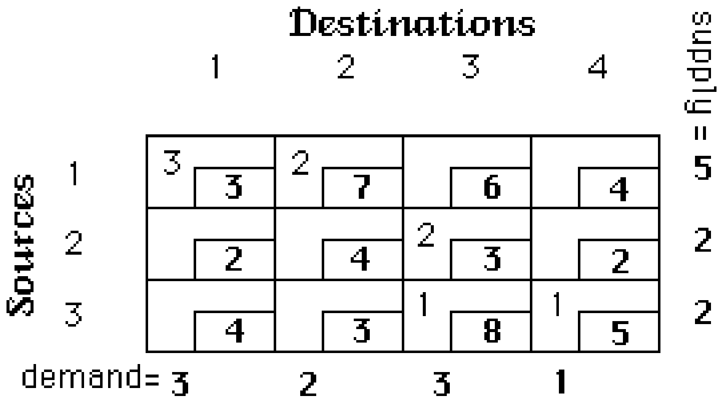
$x_1 + x_2$ ___ $x_3 = 3$
$2x_1 + x_2$ ___ $x_4 = 4$
$x_1 - x_2$ ___ $x_5 = 1$

d. Indicate (by X) which variables are basic (in addition to -z)
.... at point A: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4, _____x_5
.... at point E: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4, _____x_5

e. Which of the following does the dual LP for the original LP (with 2 variables) above? Indicate (by X) all that apply:
Objective type: ___ Minimize _____ Maximize
Objective function: ___ $3y_1 + 4y_2 + y_3$ ___ $- 3y_1 - 4y_2 + y_3$ ___$3y_1 + 4y_2 - y_3$
Subject to:
___ $y_1 + 2y_2 + y_3 \leq 4$ ___ $y_1 + 2y_2 + y_3 \geq 4$ _____ $y_1 + 2y_2 + y_3 = 4$
___ $y_1 + y_2 - y_3 \leq 1$ ___ $y_1 + y_2 - y_3 \geq 1$ _____ $y_1 + y_2 - y_3 = 1$
___ $y_1 \geq 0$ ___ $y_2 \geq 0$ _____ $y_3 \geq 0$
___ $y_1 \leq 0$ ___ $y_2 \leq 0$ _____ $y_3 \leq 0$
___ $y_1 = 0$ ___ $y_2 = 0$ _____ $y_3 = 0$

f. The optimal solution of this LP is at point H, where z=9. Indicate which of the following statements are true of the optimal solution of the dual LP, according to the duality theory (and complementary slackness theorem):
the objective value is _____ -9 _____ 9
y_1 is _____ basic ______ nonbasic
y_2 is _____ basic ______ nonbasic
y_3 is _____ basic ______ nonbasic

SSSSSSSS Quiz # 5 SSSSSSSS



Indicate whether true or false:

a.) ____ Either variable X_{22} or variable X_{13} should be basic in the set of shipments above.
b.) ____ The number of basic variables for this transportation problem is five.
c.) ____ The optimal dual variables for a transportation problem must be nonnegative.
d.) ____ If one unit were to be shipped from source #2 to destination #4, the result would be a reduction in the total cost .
e.) ____ Vogel's method will always yield an optimal solution, if it is nondegenerate.
f.) ____ If X_{24} were made a basic variable, then its value would be 2.
g.) ____ In the first step of Vogel's method for the above TP tableau, the penalty on column 1 will equal 1.
h.) ____ According to Complementary Slackness, if X* is optimal in the transportation problem and U* & V* in its dual problem, then if $X_{ij}^* > 0$, the slack in the dual constraint $U_i+V_j \leq C_{ij}$ will be positive.
i.) ____ For every basic solution in the TP tableau above, dual variable V_2 will be larger than V_1.
j.) ____ The transportation problem above is a special case of a linear programming problem.
k.) ____ The above transportation problem is "balanced".
l.) ____ The shipments indicated in the above TP tableau constitute a basic solution.
m.) ____ The Hungarian method might be used to solve the above transportation problem.
n.) ____ In the first step of Vogel's method for the above TP tableau, the penalty on row 1 will equal 7.
o.) ____ An assignment problem may be considered to be a special case of a transportation problem with all "transportation" costs equal to 1.
p.) ____ The above transportation problem will have 7 dual variables.
q.) ____ The shipments indicated in the above table are a feasible solution to this transportation problem.
r.) ____ The shipments indicated in the above table are a degenerate solution to this transportation problem.

SSSSSSSS Quiz # 6 SSSSSSSS

Below, TP = transportation problem and AP = assignment problem. Indicate whether true or false:

a.)____ Considered as a special case of the TP, the AP *always* has a degenerate basic solution.

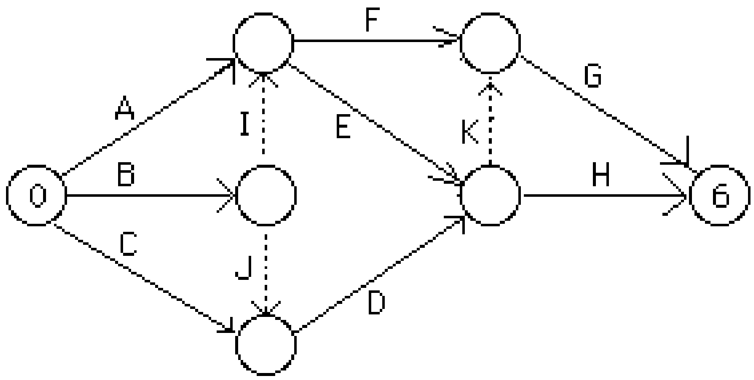
b.)____ After row reduction in the Hungarian method, each row contains at least one zero.
c.)____ An AP which has 25 variables will have 15 linear constraints.
d.)____ Both the Hungarian method and the transportation simplex method, applied to AP, will yield feasible solutions at each iteration.
e.)____ If the current solution of a TP is degenerate, the next iteration will not improve the objective function.
f.)____ If an assignment (X) is optimal for the AP with cost matrix C, it is also optimal for the cost matrix obtained by subtracting 1 from each cost in row #1.
g.)____ If n machines are to be assigned to n jobs, the AP will have n^2 variables and 2n linear equations.
h.)____ If no degenerate solution is encountered, the transportation simplex method gives an improvement in the objective function at every basis change.
i.)____ If an assignment (X) is optimal for the AP with cost matrix C, it is also optimal for the cost matrix obtained by adding 1 to each cost in column #1.
j.)____ After column reduction in the Hungarian method, each column will contain exactly one zero.
k.)____ The simplex method applied to AP might yield non-integer (fractional) solutions.
l.)____ In the AP, X_{ij}=1 means machine i is assigned to job j.
m.)____ If VAM (Vogel's Approximation Method) applied to AP (considered as a TP) yields the optimal assignment X^*, then the transportation simplex method will terminate at the first iteration.
n.)____ If 6 machines are each to be assigned one of 4 jobs, two "dummy" jobs must be defined before applying the Hungarian method.
o.)____ If a zero appears in row 1, column 1 of the cost matrix during row and column reduction in the Hungarian method, then a zero will occupy row 1, column 1 throughout the Hungarian method.

The statements below refer to the AP cost matrix:

2	0	0	5	1
3	6	4	0	1
0	0	2	3	0
7	3	1	0	5
3	4	1	0	3

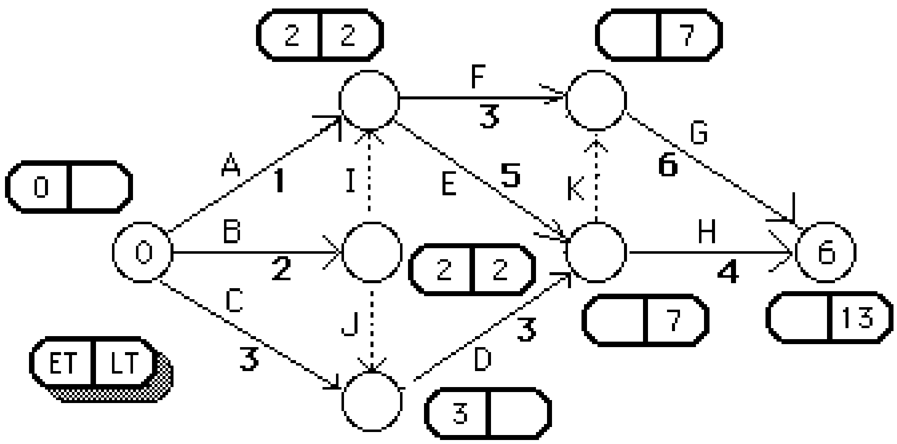
p.)____ This cost matrix could possibly result from the row and column reduction steps of the Hungarian method applied to some AP cost matrix.
q.)____ The Hungarian method, when this cost matrix is obtained, will terminate with an optimal assignment.
r.)____ After the next step of the Hungarian method, all of the elements occupied by zeroes in this matrix will again be occupied by zeroes.
s.)____ After the next step of the Hungarian method, three elements which are currently nonzero will be occupied by zeroes.
t.)____ X_{13}=1 in the optimal solution of this AP.

SSSSSSSS Quiz # 7 SSSSSSSS



a. Complete the labeling of the nodes on the A-O-A project network above.

b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.



d. Find the slack ("total float") for activity D. _________

e. Which activities are critical? (circle: A B C D E F G H I J K)

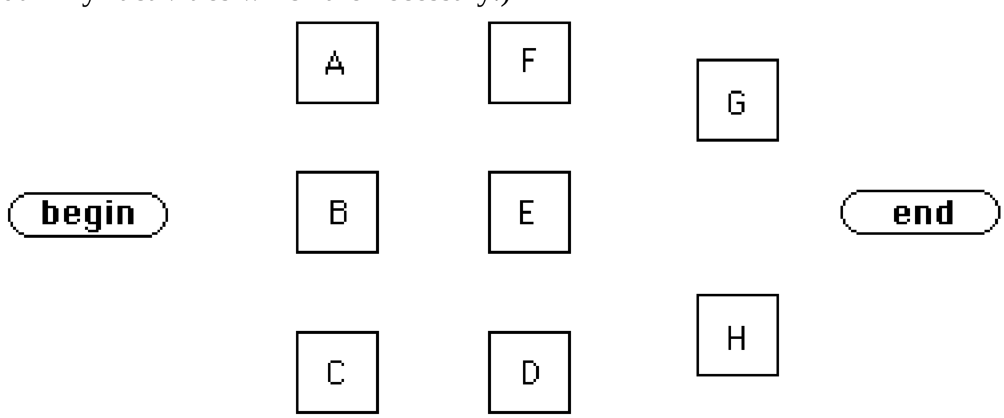
f. What is the earliest completion time for the project? ___________

g. Indicate by X which of the following constraint(s) would appear in the LP formulation of this problem:

____ $Y_F - Y_A \geq 3$ ____ $Y_E - Y_D \geq 3$ ____ $Y_E - Y_B \geq 2$

____ $Y_F - Y_A \geq 1$ ____ $Y_E - Y_A \geq 2$ ____ $Y_H - Y_C \geq 6$

h. Complete the A-O-N (activity-on-node) network below for this same project. (Add any "dummy" activities which are necessary.)



Quiz #8

To model a production planning problem, define

X_j = amount of item j which is produced (a continuous variable), and

Y_j = 1 if item j is produced, otherwise 0 (a binary integer variable),

for j=1,2,3,.... Select a constraint (or set of constraints) to model each situation below:

____ 1. *At most one* of items 1, 2, and 3 may be produced.
____ 2. If item 1 is produced, then *either* item 2 *or* item 3 must be produced.
____ 3. If item 1 is produced, then *both* items 2 and 3 must be produced.
____ 4. At least one of items 1, 2, and 3 may be produced.
____ 5. If item 1 is produced, then *at least* 100 units of the item must be produced.
____ 6. If both items 2 and 3 are produced, then item 1 must also be produced.
____ 7. At least two of items 1, 2, and 3 must be produced.
____ 8. If item 1 is produced, then item 2 must be produced.
____ 9. If it is decided to produce item 1, *at most* 100 units of the item may be produced.
____ 10. If neither item 2 nor item 3 are produced, then item 1 cannot be produced.

A. $Y_1 + Y_2 + Y_3 \geq 1$	B. $Y_1 + Y_2 + Y_3 \leq 2$	C. $X_1 + X_2 + X_3 \geq 2$
D. $Y_1 + Y_2 + Y_3 \geq 2$	E. $Y_1 \leq Y_2 + Y_3$	F. $Y_1 \leq Y_2 + Y_3$
G. $X_1 \leq Y_1$	H. $X_1 + X_2 + X_3 \geq 1$	I. $Y_1 + Y_2 + Y_3 \leq 1$
J. $X_1 \geq Y_1$	K. $X_1 + Y_1 = 1$	L. $X_1 \leq 100Y_1$
M. $X_1 \geq 100Y_1$	N. $X_1 \leq X_2$	O. $100X_1 \leq Y_1$
P. $100X_1 \geq Y_1$	Q. $X_2 \leq X_1$	R. $2Y_1 \leq Y_2+Y_3$
S. $2Y_1 = Y_2+Y_3$	T. $2Y_1 \geq Y_2+Y_3$	U. $X_1 = Y_2$
V. $X_1 \leq X_2 + X_3$	W. $Y_1 \leq Y_2 + Y_3$	X. $Y_1 = Y_2$
Y. $Y_1 \geq Y_2 + Y_3$	Z. *None of the above!*	

Quiz # 9

Careful study of a reservoir over the past 20 years has shown that, if the reservoir was full at the beginning of the summer, then the probability it would be full at the beginning of the following summer is **60%**, independent of its status in previous years. On the other hand, if the reservoir was <u>not</u> full at the beginning of one summer, the probability it would be full at the beginning of the following summer is only **20%**. Define a Markov chain model of this reservoir, with the states (1)"full" and (2)"not full".

____ 1. If the reservoir was full at the beginning of summer 1993, what is the probability that it will be full at the beginning of summer 1995 (rounded to the nearest 10%)?

a. 20%	b. 30%	c. 40%
d. 50%	e. 60%	f. none of the above

____ 2. The steadystate probability distribution π for this Markov chain must satisfy the following equation:

a. $\pi_1 + \pi_2 = 0$	b. $\pi_2 = .2\pi_1 + .8\pi_2$	c. $\pi_2 = .6\pi_1 + .4\pi_2$
d. $\pi_2 = .4\pi_1 + .8\pi_2$	e. $.6\pi_1 + .4\pi_2 = 1$	f. none of the above

____ 3. If the reservoir was full at the beginning of summer 1993, the expected number of years until it will next be "not full" is

a. 2	b. 2.5	c. 3
d. 3.5	e. 5	f. none of the above

____ 4. If the reservoir was full at the beginning of summer 1993, the probability that 1996 is the first year it is not full is (rounded to the nearest 10%)

a. 20%	b. 30%	c. 40%

d. 50% e. 60% f. none of the above

____ 5. If the reservoir was full at the beginning of summer 1993, the probability that in 1996 it is not full is (rounded to the nearest 10%)
a. 20% b. 30% c. 40%
d. 50% e. 60% f. none of the above

____ 6. During the next hundred years, in how many the reservoir be full at the beginning of the summer, as predicted by this model?
a. 20 b. 33 c. 40
d. 60 e. 67 f. none of the above

____ 7. If the reservoir is full at the beginning of **both** summer 1993 and summer 1994, the probability that it will be full at the beginning of summer 1995 is (rounded to the nearest 10%)
a. 40% b. 50% c. 60%
d. 70% e. 80% f. none of the above

____ 8. If the reservoir was full at the beginning of summer 1993, what is the expected number of years until it is expected to be full again?
a. 1.5 b. 2 c. 2.5
d. 3 e. 5 f. none of the above

____ 9. If the reservoir was full at the beginning of summer 1993, the probability that 1996 is the first year it is full again is (rounded to the nearest 10%)
a. 10% b. 20% c. 30%
d. 40% e. 50% f. none of the above

____ 10. If the reservoir was full at the beginning of summer 1993, the probability that it will next be not full in 1995 is (rounded to the nearest 10%)
a. 10% b. 20% c. 30%
d. 40% e. 50% f. none of the above

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \quad P^2 = \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix} \quad P^3 = \begin{bmatrix} 0.376 & 0.624 \\ 0.312 & 0.688 \end{bmatrix} \quad P^4 = \begin{bmatrix} 0.3504 & 0.6496 \\ 0.3248 & 0.6752 \end{bmatrix}$$

First Visit Probabilities

$$F = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \quad F^{(2)} = \begin{bmatrix} 0.08 & 0.24 \\ 0.16 & 0.08 \end{bmatrix} \quad F^{(3)} = \begin{bmatrix} 0.064 & 0.144 \\ 0.128 & 0.048 \end{bmatrix} \quad F^{(4)} = \begin{bmatrix} 0.0512 & 0.0864 \\ 0.1024 & 0.0288 \end{bmatrix}$$

i	π_i
1	0.33333333
2	0.66666667

$$M = \begin{bmatrix} 3 & 2.5 \\ 5 & 1.5 \end{bmatrix}$$

Quiz # 10 Solutions

At the beginning of each day, a company observes its inventory level. Then an order may be placed (and is instantaneously received). Finally, the day's demand is observed. We are given the following information:

- A $2 cost is assessed against each unit of inventory on hand at the end of a day.
- Placing an order costs 50¢ per unit plus a $5 ordering cost.
- During each day, demand is 1, 2, or 3 units with probability 1/4, 1/2, and 1/4, respectively.
- If the on-hand inventory is 1 unit or less, enough is ordered to bring the on-hand inventory level up to 4.
- Demand which cannot be satisfied immediately is backordered, with a cost of $3 per unit backlogged.

1. If the inventory had 3 units at the beginning of the day on Monday, what is the probability that there is a backorder at the beginning of the day Wednesday (rounded to the nearest 10%)? Answer: $p_{51}^{(3)}$=6.25% ≈ 5%
2. The steadystate probability distribution π for this Markov chain must satisfy the following equation:
   Answer: $\pi_5 = .25\pi_1 + .25\pi_2 + .25\pi_3$ (π times column 5 of P)
3. If the SOH = 0 Monday morning, the expected number of days until a backorder is (rounded to the nearest integer) Answer: m_{21}=12.889≈ 13
4. If the SOH=0 Monday AM, the probability that the next backorder is observed Thursday AM is (rounded to the nearest 1%): Answer: $f_{21}^{(3)}$=9.375%≈ 9% (not one of the options listed)
5. If the SOH=2 Monday AM, the probability that a backorder is observed Thursday AM is (rounded to the nearest 1%) Answer: $p_{21}^{(3)}$=12.5%≈ 12% (not one of the options listed)
6. During the next 100 days, the number of times backorders will occur, as predicted byu this model, is (rounded to the nearest integer) Answer: $100\pi_1$ =7.75≈ 8
7. If the SOH=0 Monday AM and SOH=2 Tuesday AM, the probability that SOH=0 Thursday AM is (rounded to the nearest 10%) Answer: $p_{42}^{(2)}$=0%
8. If the SOH=3 Monday AM, the number of orders during the remaineder of the week (mon-Fri) is (rounded to the nearest integer) Answer:
   0.22266+0.76953+1.3242=2.31639 (final row of $p^1+p^2+p^3+p^4$)
9. The average total cost per day (rounded to the nearest dollar) is... Answer: $6.6293≈ $7
10. If the SOH=3 Monday AM, the probability that the next time SOH=3 will be Friday is (rounded to the nearest 5%) Answer: $f_{21}^{(3)}$=12.1%≈ 10%

$$P = \begin{bmatrix} 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0.25 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0.25 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.125 & 0.3125 & 0.3125 & 0.1875 & 0.0625 \\ 0.125 & 0.3125 & 0.3125 & 0.1875 & 0.0625 \\ 0.125 & 0.3125 & 0.3125 & 0.1875 & 0.0625 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0.0625 & 0.125 & 0.25 & 0.375 & 0.1875 \end{bmatrix}$$

$$F^2 = \begin{bmatrix} 0.125 & 0.3125 & 0.25 & 0.1875 & 0.0625 \\ 0.125 & 0.3125 & 0.25 & 0.1875 & 0.0625 \\ 0.125 & 0.3125 & 0.25 & 0.1875 & 0.0625 \\ 0 & 0 & 0.1875 & 0.5 & 0.25 \\ 0.0625 & 0.125 & 0.125 & 0.375 & 0.1875 \end{bmatrix}$$

$$M = \begin{bmatrix} 12.889 & 5.2727 & 3.5152 & 2.2222 & 6 \\ 12.889 & 5.2727 & 3.5152 & 2.2222 & 6 \\ 12.889 & 5.2727 & 3.5152 & 2.2222 & 6 \\ 10.667 & 3.6364 & 3.6364 & 3.2222 & 7 \\ 13.333 & 4.5455 & 2.7879 & 2.6667 & 7.25 \end{bmatrix}$$

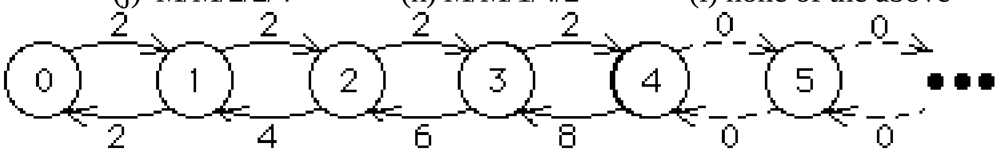
$$\sum_{n=1}^{4} P^n = \begin{bmatrix} 0.26953 & 0.66406 & 1.125 & 1.3359 & 0.6054 \\ 0.26953 & 0.66406 & 1.125 & 1.3359 & 0.6054 \\ 0.26953 & 0.66406 & 1.125 & 1.3359 & 0.6054 \\ 0.42188 & 0.92188 & 1.0781 & 1.0781 & 0.5 \\ 0.22266 & 0.76953 & 1.3242 & 1.2305 & 0.4531 \end{bmatrix}$$

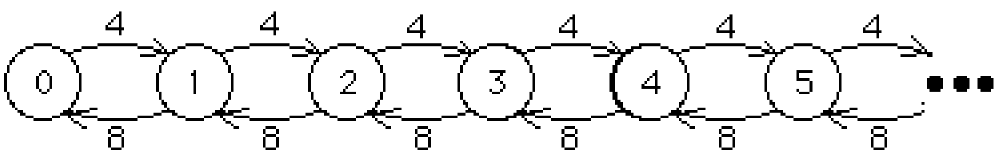
i	State	π_i	C_i	$\pi_i \times C_i$
1	SOH = ¯1	0.077586	10.5	0.81466
2	SOH = 0	0.18966	7	1.3276
3	SOH = 1	0.28448	8.5	2.4181
4	SOH = 2	0.31034	4	1.2414
5	SOH = 3	0.13793	6	0.82759
			total	6.6293

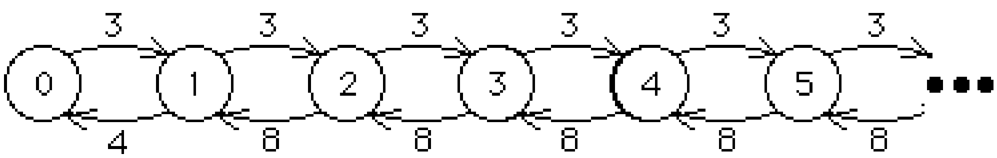
§§§§§§§§ Quiz # 11 §§§§§§§§

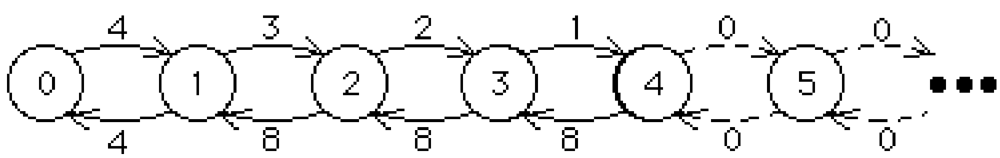
For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

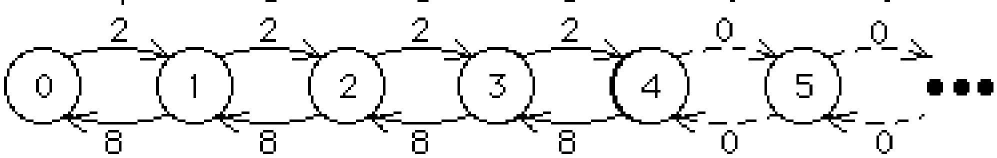
(a) M/M/1	(b) M/M/2	(c) M/M/1/4
(d) M/M/4	(e) M/M/2/4	(f) M/M/2/4/4
(g) M/M/1/2/4	(h) M/M/4/2	(i) M/M/4/4
(j) M/M/2/2/4	(k) M/M/1/4/2	(l) none of the above

___ 1.


___ 2.


___ 3.


___ 4.


___ 5.


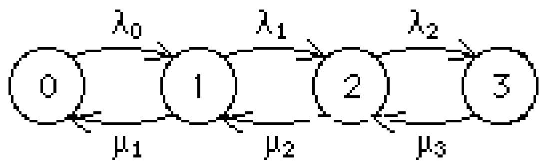
@@@ *PART TWO* @@@

A machine operator has the task of keeping three machines running. Each machine runs for an average of 1 hour before it becomes jammed or otherwise needs the operator's attention. He then spends an average of ten minutes restoring the machine to running condition. Define a continuous-time Markov chain, with the state of the system being*the number of machines which are **not** running.*

1. True or False (circle): This Markov chain is a birth/death process.
2. Specify the letter for each of the transition rates:

λ_0 ____ λ_1 ____ λ_2 ____

μ_1 ____ μ_2 ____ μ_3 ____



a. 1/hr	b. 2/hr	c. 3/hr
d. 4/hr	e. 6/hr	f. 8/hr
g. 12/hr	h. 18/hr	i. None of the above

____3. Which equation is used to compute the steady-state probability π_0?

(a.) $\pi_0 = \left(\frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3} \right)^{-1}$

(e.) $\pi_0 = \sum_{n=0}^{2} \frac{\lambda_n}{\mu_{n+1}}$

(b.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}$

(f.) $\pi_0 = \sum_{n=0}^{\infty} \left(\frac{\lambda_n}{\mu_{n+1}} \right)^n$

(c.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3}$

(g.) $\pi_0 = \sum_{n=0}^{2} \left(\frac{\lambda_n}{\mu_{n+1}} \right)^n$

(d.) $\pi_0 = \dfrac{1 + \frac{\lambda_0}{\lambda_1} + \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_3}}{1 + \frac{\mu_0}{\mu_1} + \frac{\mu_1}{\mu_2} + \frac{\mu_2}{\mu_3}}$

(h.) None of the above

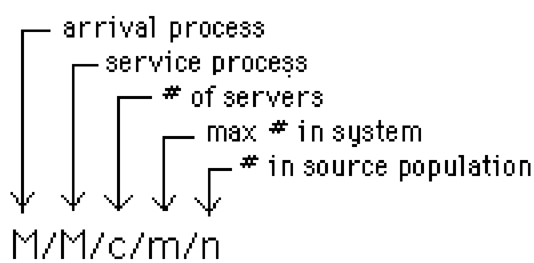
____4. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = 6\pi_0$	b. $\pi_1 = 2\pi_0$	c. $\pi_1 = \pi_0$
d. $\pi_1 = \frac{1}{6}\pi_0$	e. $\pi_1 = \frac{1}{2}\pi_0$	f. None of the above

____5. If the average number of machines not running is approximately 0.5 and the average time between machine jams is approximately 0.4 hr., what is the average turnaround time (including service time) to restore a machine to running condition? *(Choose nearest answer.)*

a. 0.1 hour	c. 0.2 hour	e. 0.3 hour
b. 0.4 hour	d. 0.5 hour	f. 0.6 hour

Note: Kendall's notation:



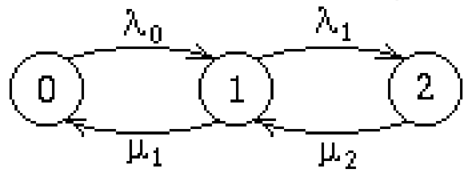
SSSSSSSS Quiz # 12 SSSSSSSS

For each question, select an answer (a) through (z) from the list at the end.

@@@@ *PART ONE* Z@@@@

"Bectol, Inc. is building a dam. A total of 10,000,000 cu ft of dirt is needed to construct the dam. A bulldozer is used to collect dirt for the dam. Then the dirt is moved via dumpers to the dam site. Only one bulldozer is available, and it rents for $100 per hour. Bectol can rent, at $40 per hour, as many dumpers as desired. Each dumper can hold 1000 cu ft of dirt. It takes an average of 12 minutes for the bulldozer to load a dumper with dirt, and it takes each dumper an average of five minutes to deliver the dirt to the dam and return to the bulldozer."

Suppose that two dumpers are rented. Assume that the system is a **birth/death process**, with the state of the system = **# of dumpers at the loading site**.



____ 1. The time required to load a dumper is assumed to have what distribution?

____ 2. The time required for a dumper to unload & return to the loading site is assumed to have what distribution?

____ 3. The "birth" rate λ_0 (arrivals/hour).

____ 4. The "birth" rate λ_1 (arrivals/hour).

____ 5. The "death" rate μ_1 (departures/hour).

____ 6. The "death" rate μ_2 (departures/hour).

____ 7. The steady-state probability π_0 is computed by what formula?

____ 8. The utilization of the bulldozer (expressed as %) if π_0 is 0.05774 ≈ 6%

____ 9. The output of the bulldozer (cubic ft per hour) at 100% utilization

____10. The achieved output of the bulldozer (cubic ft per hour)

____10. The number of hours required to complete the job (hours)

____11. The total rental cost of bulldozer and dumpers for this job ($)

PART TWO

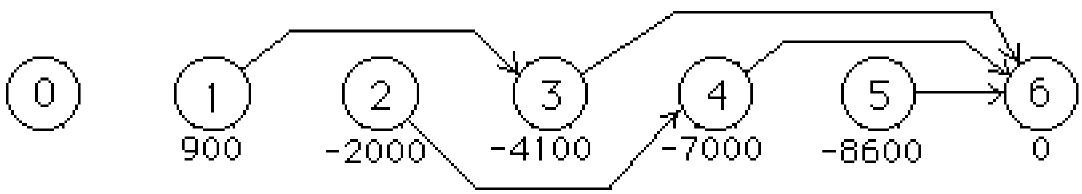
Suppose that a new car costs $12,000 and that the annual operating cost and resale value of the car are as shown in the table below. Suppose that at time=0 you have a new car (already paid for) and you require a car until time=6 years, after which no car is required.

Age of Car (years)	Resale Value	Operating Cost
1	$9000	$400 (year 1)
2	$8000	$600 (year 2)
3	$6000	$900 (year 3)
4	$4000	$1200 (year 4)
5	$3000	$1600 (year 5)
6	$2000	$2200 (year 6)

Using dynamic programming to find the optimal replacement time for the first car, we define

G(t) = your minimum total cost (operating+purchase-resale value) of owning a car from time=t until the end of the sixth year.

The values of G(1) through G(6) and some of the optimal replacement times are as shown in the diagram below:



____ 1. G(0), measured in $

____ 2. Age at which your initial car should be replaced (years).

____ 3. Age at which your second car should be replaced (years).

____ 4. Total number of cars you will own during this six-year period, according to the optimal strategy.

ANSWERS

a. Normal
b. Uniform
c. Exponential
d. Poisson
e. Markov

f. $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2}$

g. $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2}$

h. $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2}$

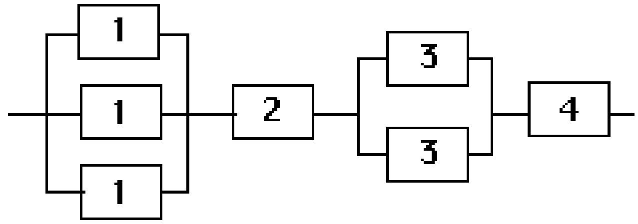
i. $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2}$

j. 1
k. 2
l. 3
m. 4
n. 5
o. 6
p. 12
q. 94
r. 940
s. 1000
t. 2000
u. 2128
v. 4700
w. 5000
x. 6000
y. 360000
z. 383000
aa. 24
bb. 3000

Quiz # 13

Optimization of System Reliability: A system consists of 4 devices, each subject to possible failure, such that the system fails if any one or more of the devices fail:

Device	Reliability (%)	Weight (kg.)
1	70	1
2	90	2
3	80	2
4	95	3



Suppose that redundant units of devices 1 and 3 are included as shown on the right above. (That is, system failure occurs if all 3 of device 1, or both of device 3, or device 2, or device 4 were to fail.)

____1. The reliability of **device 3** in this system is:

a. $1-.2^2 = 96\%$ b. $.8^2 = 64\%$ c. $1-e^{-2\times 0.2} = 32.97\%$
d. $.2^2 = 4\%$ e. $1-e^{-2\times 0.8} = 79.8\%$ f. $1-.8^2 = 36\%$
g. None of the above

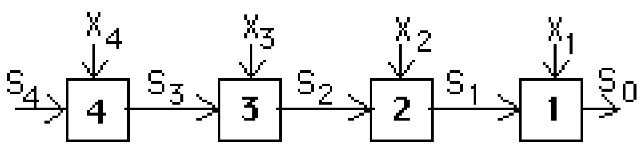
____2. The reliability of this entire **system** is:

a. $\left(1-0.7^3\right)(1-0.9)\left(1-0.8^2\right)(1-0.95) = 79.8\%$
b. $1-\left(0.7^3\right)(0.9)\left(.8^2\right)(0.95) = 81.2\%$
c. $1-\left(1-e^{-3\times 0.3}\right)\left(1-e^{-0.1}\right)\left(1-e^{-2\times 0.2}\right)\left(1-e^{-0.05}\right) = 99.1\%$
d. $1-\left(0.3^3\right)(0.1)\left(.2^2\right)(0.05) = 99.995\%$
e. $\left(1-e^{-3\times 0.7}\right)\left(1-e^{-0.9}\right)\left(1-e^{-2\times 0.8}\right)\left(1-e^{-0.95}\right) = 25.5\%$
f. $\left(1-0.3^3\right)(1-0.1)\left(1-0.2^2\right)(1-0.05) = 0.12\%$
g. None of the above

____3. The weight of this system is:

a. 10 kg. c. 11 kg. e. 12 kg.
b. 13 kg. d. 14 kg. f. none of the above

Suppose that we wish to find the system design having maximum reliability subject to a limit of 14 kg. weight.



____4. The dynamic programming model used in the homework assignment and in the Hypercard stack defines a function $f_n(S)$, where $f_n(S)$ is

a. the reliability of S redundant units of device #n.
b. the maximum reliability of the system if n redundant units are allowed.
c. the maximum reliability of devices 1 through n, if S kg. of weight is allocated to them.
d. the maximum reliability of devices n through 4, if S kg. of weight is allocated to them.

____5. The value of S_4 is (choose one or more!):

a. the safety factor for device 4 b. 30%
c. the state of the DP system at stage 4 d. 1 kg.
e. the reliability of device #4 f. 14 kg.

6. The following output is obtained during the solution of the DP model, where several values have been omitted. Enter the correct letter which indicates the missing value for each.

___ α ____ β ____ δ ____ ε ____ γ ____ φ

a. 0.9009 b. 0.7862 c. 0.8757 d. 0.7207
e. 0.9663 f. 0.7006 g. 0 h. 1
i. 2 j. 3 k. 4 l. 5

_____7. The optimal design, weighing 14 kg., has reliability: *(choose nearest value)*
a. 0.85 b. 0.90 c. 0.925 d. 0.95 e. 0.975

Stage 1

s \ x:	1	2	3
1	0.70	-999.99	-999.999
2	0.70	0.91	-999.999
3	0.70	0.91	0.973
4	0.70	0.91	0.973
5	0.70	0.91	0.973
6	0.70	0.91	0.973
7	0.70	0.91	0.973
8	0.70	0.91	0.973
9	0.70	0.91	0.973
10	0.70	0.91	0.973
11	0.70	0.91	0.973
12	0.70	0.91	0.973
13	0.70	0.91	0.973
14	0.70	0.91	0.973

Stage 2

s \ x:	1	2	3
3	0.6300	-999.9999	-999.999
4	0.8190	-999.9999	-999.999
5	α	0.6930	-999.999
6	0.8757	0.9009	-999.999
7	0.8757	β	0.699
8	0.8757	0.9633	0.909
9	0.8757	0.9633	0.972
10	0.8757	0.9633	0.972
11	0.8757	0.9633	0.972
12	0.8757	0.9633	0.972
13	0.8757	0.9633	0.972
14	0.8757	0.9633	0.972

Stage 3

s \ x:	1	2	3
5	0.5040	-999.9999	-999.9999
6	0.6552	-999.9999	-999.9999
7	0.7006	0.6048	-999.9999
8	δ	0.7862	-999.9999
9	0.7706	0.8407	0.6250
10	0.7706	0.8649	0.8124
11	0.7776	0.9247	0.8687
12	0.7776	0.9247	0.8937
13	0.7776	0.9331	0.9556
14	0.7776	0.9331	0.9556

Stage 4

s \ x:	1	2	3
8	0.4788	-999.9999	-999.9999
9	0.6224	-999.9999	-999.9999
10	0.6655	-999.9999	-999.9999
11	0.7469	0.5027	-999.9999
12	0.7986	0.6536	-999.9999
13	0.8216	0.6988	-999.9999
14	0.8785	0.7843	0.5039

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
8	0.4788	1	5
9	0.6224	1	6
10	0.6655	1	7
11	0.7469	1	8
12	0.7986	1	9
13	0.8216	1	10
14	0.8785	1	11

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
5	0.5040	1	3
6	0.6552	1	4
7	0.7006	1	5
8	0.7862	γ	ε
9	0.8407	2	5
10	0.8649	2	6
11	0.9247	2	7
12	0.9247	2	8
13	0.9556	3	7
14	0.9556	3	8

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
1	0.7000	1	0
2	0.9100	2	0
3	0.9730	3	0
4	0.9730	3	1
5	0.9730	3	2
6	0.9730	3	3
7	0.9730	3	4
8	0.9730	3	5
9	0.9730	3	6
10	0.9730	3	7
11	0.9730	3	8
12	0.9730	3	9
13	0.9730	3	10
14	0.9730	3	11

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
3	0.6300	1	1
4	0.8190	1	2
5	0.8757	1	3
6	ϕ	2	2
7	0.9633	2	3
8	0.9633	2	4
9	0.9720	3	3
10	0.9720	3	4
11	0.9720	3	5
12	0.9720	3	6
13	0.9720	3	7
14	0.9720	3	8