56:171 Operations Research
Sample Quizzes, Fall 1992

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Quiz #1

Indicate whether each statement is **true** or **false**.

______ a. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot column, the resulting tableau is infeasible.

______ b. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.

______ c. The number of basic variables in an LP is equal to the number of rows, including the objective function row.

______ d. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

______ e. When you enter an LP formulation into LINDO, you must include any nonnegativity constraints.

______ f. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

______ g. A "pivot" in a nonbasic column of a tableau will make it a basic column.

______ h. A "pivot" in row i of the column for variable X_j will increase the number of basic variables.

______ i. If a slack variable S_i for row i is basic in the optimal solution, then variable X_i cannot be basic.

______ j. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you must pivot in row i.

______ k. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you cannot pivot in row i.

______ l. If a zero appears on the right-hand-side of row i of an LP tableau, then the tableau is called "degenerate".

______ m. If an LP model is of the form Ax•b, x•0, and b is nonnegative, then there is no need for artificial variables.

______ n. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

______ o. A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.

______ p. A basic solution of the problem "minimize cx subject to Ax•b, x•0" corresponds to a corner of the feasible region.

______ q. It may happen that an LP problem has (exactly) two optimal solutions.

______ r. The feasible region is the set of all points that satisfy at least one constraint.

______ s. Adding constraints to an LP may improve the optimal objective function value.

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Quiz 1 Solutions

False If, when solving an LP by the simplex method, you make a mistake in choosing a pivot column, the resulting tableau is infeasible. *The objective will worsen rather than improve.*

True If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.

True The number of basic variables in an LP is equal to the number of rows, including the objective function row.

False When you enter an LP formulation into LINDO, you must first convert all inequalities to equations. *This is done automatically by LINDO.*

False When you enter an LP formulation into LINDO, you must include any nonnegativity constraints. *This is always assumed by LINDO.*

True When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

True A "pivot" in a nonbasic column of a tableau will make it a basic column.

False A "pivot" in row i of the column for variable X_j will increase the number of basic variables. *The number of basic variables remains constant... the variable Xi will replace another variable in the basis.*

False If a slack variable S_i for row i is basic in the optimal solution, then variable X_i cannot be basic.

False If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you must pivot in row i. *Will not pivot in row i if the substitution rate in the pivot column in row i is not positive.*

False If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you cannot pivot in row i. *Row i will be included in the minimum ratio test if the substitution rate in the pivot column is positive, in which case the ratio is zero and it will "win".*

True If a zero appears on the right-hand-side of row i of an LP tableau, then the tableau is called "degenerate".

True If an LP model is of the form Ax•b, x•0, and b is nonnegative, then there is no need for artificial variables.

True If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

True A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.

True A basic solution of the problem "minimize cx subject to Ax•b, x•0" corresponds to a corner of the feasible region.

False It may happen that an LP problem has (exactly) two optimal solutions. *If there is more than one optimal solution, there must be an infinite number.... every point on the line segment between two optimal basic solutions is optimal.*

False The feasible region is the set of all points that satisfy at least one constraint. *The feasible region is the set of all points that satisfy ALL constraints.*

False Adding constraints to an LP may improve the optimal objective function value. *Further restricting the solution set from which you choose the optimum can never result in an improved solution.*

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Quiz #2

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:
X1 = number of STANDARD golf bags manufactured per quarter
X2 = number of DELUXE golf bags manufactured per quarter
Four operations are required, with the time per golf bag as follows:

	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

LINDO provides the following output:

```
MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END

 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE
        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110           .000000
        X2       251.997800           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000          4.375086
```

3)	111.602000	.000000
4)	.000000	6.937440
5)	18.000232	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	10.000000	3.500135	3.700000
X2	9.000000	5.285715	2.333400

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	630.000000	52.364582	134.400000
3	600.000000	INFINITY	111.602000
4	708.000000	192.000010	128.002800
5	135.000000	INFINITY	18.000232

THE TABLEAU

ROW	(BASIS)	X1	X2	SLK 2	SLK3	SLK 4	SLK 5	
1	ART	.00	.00	4.375	.00	6.937	.00	7668.012
2	X2	.00	1.00	1.875	.00	-1.312	.00	251.998
3	SLK 3	.00	.00	-1.000	1.00	.200	.00	111.602
4	X1	1.00	.00	-1.250	.00	1.875	.00	540.003
5	SLK 5	.00	.00	-.344	.00	.141	1.00	18.000

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If an additional 100 hours were available in the cut-&-dye department, PAR would be able to obtain an additional $________ in profits.

b. If an additional 100 hours were available in the sewing department, PAR would be able to obtain an additional $________ in profits.

c. If the variable "SLK 2" were increased, this would have the effect of

___ increasing the hours used in the cut-&-dye department
___ decreasing the hours used in the cut-&-dye department
___ increasing the hours used in the sewing department
___ decreasing the hours used in the sewing department
___ none of the above

d. If the variable "SLK 2" were increased by 10, this would (__increase/__decrease) X1 by ________ STANDARD golf bags/quarter.

e. If a pivot were to be performed to enter the variable SLK2 into the basis, the value of SLK2 in the resulting basic solution would be approximately

__ 252/1.875 __ 111.6 ___ 540/1.25 __ 18/0.344
__ not sufficient information

f. If the profit on STANDARD bags were to decrease from $10 each to $7 each, the number of STANDARD bags would (__increase/__decrease/__remain the same/__not sufficient info.)

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Quiz 2 Solutions

- If an additional 100 hours were available in the cut-&-dye department, PAR would be able to obtain an additional $NSI (229.1= 4.375x52.3645) in profits.
- If an additional 100 hours were available in the sewing department, PAR would be able to obtain an additional $ zero in profits.
- If an additional 100 hours were available in the finishing department, PAR would be able to obtain an additional $692.74 in profits.

- If an additional 100 hours were available in the sewing department, PAR would be able to obtain an additional $_zero__ in profits.
- If the variable "SLK 2" were increased, this would have the effect of

  ___ increasing the hours used in the cut-&-dye department
  _X_ decreasing the hours used in the cut-&-dye department
  ___ increasing the hours used in the sewing department
  ___ decreasing the hours used in the sewing department
  ___ none of the above
- If the variable "SLK 4" were increased, this would have the effect of

  ___ increasing the hours used in the cut-&-dye department
  ___increasing the hours used in the cut-&-dye department
  _X_ increasing the hours used in the sewing department (since SLK4 "substitutes" for SLK3)
  ___ decreasing the hours used in the sewing department
  ___ none of the above
- If the variable "SLK 2" were increased by 10, this would (_X increase/__decrease) X1 by __12.5__ STANDARD golf bags/quarter.
- If a pivot were to be performed to enter the variable SLK2 into the basis, the value of SLK2 in the resulting basic solution would be approximately

  _X_ 252/1.875 __ 111.6 ___ 540/1.25 __ 18/0.344
  __ not sufficient information
- If the variable "SLK 4" were increased by 10, this would (__increase/_X decrease) X1 by _18.75_ STANDARD golf bags/quarter.
- If a pivot were to be performed to enter the variable SLK4 into the basis, the value of SLK4 in the resulting basic solution would be approximately

  __ 252/1.312 __ 111.6/0.2 ___ 540/1.875 _X 18/0.141
  __ not sufficient information
- If the profit on STANDARD bags were to decrease from $10 each to $7 each, the number of STANDARD bags would (__increase/__decrease/_X remain the same/__not sufficient info.)
- If the profit on STANDARD bags were to increase from $10 each to $15 each, the number of STANDARD bags would (_X increase/__decrease/__remain the same/__not sufficient info.)

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Quiz #3

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:
X1 = number of STANDARD golf bags manufactured per quarter
X2 = number of DELUXE golf bags manufactured per quarter
Four operations are required, with the time per golf bag as follows:

	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.

Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

LINDO provides the following output:

```
MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END

 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE
        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110           .000000
        X2       251.997800           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000          4.375086
        3)       111.602000           .000000
        4)          .000000          6.937440
        5)        18.000232           .000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE         ALLOWABLE
                   COEF          INCREASE          DECREASE
       X1       10.000000         3.500135          3.700000
       X2        9.000000         5.285715          2.333400
```

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	630.000000	52.364582	134.400000
3	600.000000	INFINITY	111.602000
4	708.000000	192.000010	128.002800
5	135.000000	INFINITY	18.000232

THE TABLEAU

ROW	(BASIS)	X1	X2	SLK 2	SLK3	SLK 4	SLK 5	
1	ART	.00	.00	4.375	.00	6.937	.00	7668.012
2	X2	.00	1.00	1.875	.00	-1.312	.00	251.998
3	SLK 3	.00	.00	-1.000	1.00	.200	.00	111.602
4	X1	1.00	.00	-1.250	.00	1.875	.00	540.003
5	SLK 5	.00	.00	-.344	.00	.141	1.00	18.000

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If the profit on DELUXE bags were to decrease from $9 each to $7 each, the number of DELUXE bags to be produced would (__increase/__decrease/__remain the same/__not sufficient info.)

b. The LP problem above has
___ exactly one optimal solution
___ exactly two optimal solutions
___ an infinite number of optimal solutions

c. If an additional 10 hours were available in the sewing department, PAR would be able to obtain an additional $________ in profits.

d. If an additional 10 hours were available in the finishing department, PAR would be able to obtain an additional $________ in profits.

e. If the variable "SLK 4" were increased, this would be equivalent to
___ increasing the hours used in the cut-&-dye department
___ decreasing the hours used in the cut-&-dye department
___ increasing the hours used in the sewing department
___ decreasing the hours used in the sewing department
___ none of the above

f. If the variable "SLK 4" were increased by 10, this would (__increase/__decrease) X1 by ________ STANDARD golf bags/quarter.

g. If the variable "SLK 4" were increased by 10, this would (__increase/__decrease) X2 by ________ DELUXE golf bags/quarter.

h. If a pivot were to be performed to enter the variable SLK4 into the basis, the value of SLK4 in the resulting basic solution would be approximately

__ $252/1.312$ __ $0.2/111.6$ __ $540/1.875$ __ $0.141/18$

__ $1.312/252$ __ $111.6/0.2$ __ $1.875/540$ __ $18/0.141$

__ not sufficient information

i. If the variable SLK 4 were to enter the basis, then the variable __________ will leave the basis.

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Quiz 3 Solutions

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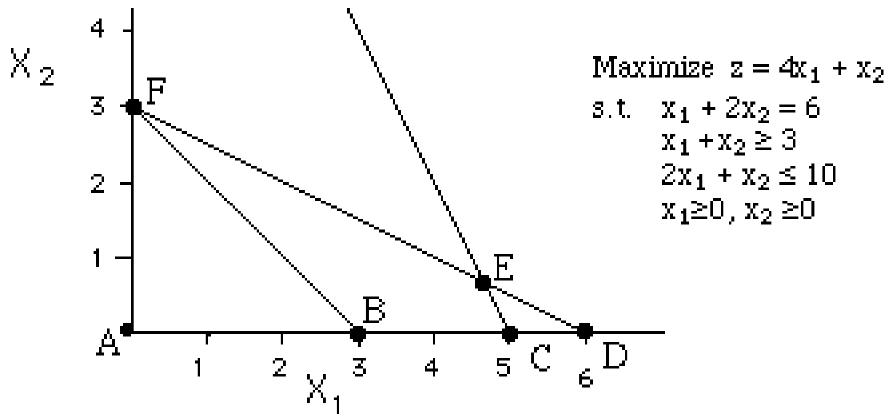
Quiz #4



a. Which of the points are feasible solutions in the LP problem above?
(Circle one or more): A B C D E F

b. Which of the points are basic solutions?
(Circle one or more): A B C D E F

c. Which of the points are degenerate solutions?
(Circle one or more): A B C D E F

d. In order to formulate the LP using only equality & nonnegativity constraints, 2 additional variables (x_3 and x_4) were defined. Insert the correct sign (+ or -) in each constraint below:

$$-z + 4x_1 + x_2 = 0$$
$$x_1 + 2x_2 = 6$$
$$x_1 + x_2 \ \_\_\_ x_3 = 3$$
$$2x_1 + x_2 \ \_\_\_ x_4 = 10$$

e. Indicate (by X) which variables are basic (in addition to -z)

.... at point C: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4

.... at point E: _X_(-z) ___x_1, ___x_2, _____x_3, _____x_4

f. Which of the following does the dual LP for the original LP (with 2 variables) above?
Indicate (by X) all that apply: Objective type: ___ Minimize _____ Maximize

Objective function: ___ $6y_1 + 3y_2 + 10y_3$ ___ $6y_1 - 3y_2 + 10y_3$ ___$6y_1 + 3y_2 - 10y_3$

Subject to:

___ $y_1 + y_2 + 2y_3$ • 4	___ $y_1 + y_2 + 2y_3$ • 4	_____ $y_1 + y_2 + 2y_3 = 4$
___ $2y_1 + y_2 + y_3$ • 1	___ $2y_1 + y_2 + y_3$ • 1	_____ $2y_1 + y_2 + y_3 = 1$
___ y_1 • 0	___ y_2 • 0	_____ y_3 • 0
___ y_1 • 0	___ y_2 • 0	_____ y_3 • 0

___ $y_1 = 0$ ___ $y_2 = 0$ ____ $y_3 = 0$

g. It happens that point E on the graph above is optimal. What does the Complementary Slackness theorem tell us about the optimal solution y* of the dual problem? (Check each that *must* be true, according to the theorem)

___ $y_1^* + y_2^* + 2y_3^* = 4$ ___ $2y_1^* + y_2^* + y_3^* = 1$

___ $y_1^* = 0$ ___ $y_2^* = 0$ ___ $y_3^* = 0$

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Quiz 4 Solutions

The initial tableau for this LP (after including surplus & slack variables as in (d) below) is

-z	X_1	X_2	X_3	X_4	rhs
1	4	1	0	0	0
0	1	2	0	0	6
0	1	1	-1	0	3
0	2	1	0	1	10

In a basic solution, there will be (in addition to -z) three basic variables and one nonbasic variable. Therefore, there will be four possible basic solutions, each found by letting one of the four variables be nonbasic (& therefore zero). The four tableaus are:

(i.) x_1, x_2, & x_3 basic, x_4 nonbasic:

-z	X_1	X_2	X_3	X_4	rhs
1	0	0	0	¯2.33	¯19.3
0	1	0	0	0.667	4.67
0	0	1	0	¯0.333	0.667
0	0	0	1	0.333	2.33

(point E)

(ii.) x_1, x_2, & x_4 basic, x_3 nonbasic:

-z	X_1	X_2	X_3	X_4	rhs
1	0	0	7	0	¯3
0	1	0	¯2	0	0
0	0	1	1	0	3
0	0	0	3	1	7

(point F)

(iii.) x_1, x_3, & x_4 basic, x_2 nonbasic:

-z	X_1	X_2	X_3	X_4	rhs
1	0	¯7	0	0	¯24
0	1	2	0	0	6
0	0	1	1	0	3
0	0	¯3	0	1	¯2

(point D, which is infeasible with $X_4<0$)

(iv.) x_2, x_3, & x_4 basic, $x_1$nonbasic:

-z	X_1	X_2	X_3	X_4	rhs
1	3.5	0	0	0	¯3
0	0.5	1	0	0	3
0	¯0.5	0	1	0	0
0	1.5	0	0	1	7

(point F, AGAIN!)

a. *Which of the points are feasible solutions in the LP problem above?* Points E & F (Note that the feasible region is the line segment EF!)
b. *Which of the points are basic solutions?* The points D, E, &F are basic sol'ns (see above). Point D is a basic nonfeasible solution.
c. *Which of the points are degenerate solutions?* Only F is degenerate. At point F, two variables are nonzero (X_2>0 and X_4>0), but X_1=X_3=0. There should be (in addition to -z) three basic variables, so one of the basic variables must have the value zero. If we choose either X_1 or X_3 to be basic, as in tableaus (ii) and (iv) above, we get the same basic solution X=(0,3,0,7)
d. *In order to formulate the LP using only equality & nonnegativity constraints, 2 additional variables (x_3 and x_4) were defined:*

$$\begin{aligned} -z + 4x_1 + x_2 &= 0 \\ x_1 + 2x_2 \quad &= 6 \\ x_1 + x_2 - x_3 &= 3 \\ 2x_1 + x_2 + x_4 &= 10 \end{aligned}$$

e. *Indicate (by X) which variables are basic (in addition to -z)*
.... at point C: _*IGNORE THIS QUESTION!*
.... at point E: _X_(-z) _X_x_1, _X_x_2, __X__x_3, _____x_4
f. *Which of the following is true of the dual LP for the original LP (with 2 variables) above? Indicate (by X) all that apply:* Objective type: _X_ Minimize _____ Maximize
Objective function: _X_ $6y_1 + 3y_2 + 10y_3$ ___ $6y_1 - 3y_2 + 10y_3$ ___$6y_1 + 3y_2 - 10y_3$
Subject to:

___ $y_1 + y_2 + 2y_3 \bullet 4$	_X_ $y_1 + y_2 + 2y_3 \bullet 4$	_____ $y_1 + y_2 + 2y_3 = 4$
___ $2y_1 + y_2 + y_3 \bullet 1$	_X_ $2y_1 + y_2 + y_3 \bullet 1$	_____ $2y_1 + y_2 + y_3 = 1$
___ $y_1 \bullet 0$	___ $y_2 \bullet 0$	__X__ $y_3 \bullet 0$
___ $y_1 \bullet 0$	_X_ $y_2 \bullet 0$	_____ $y_3 \bullet 0$
___ $y_1 = 0$	___ $y_2 = 0$	_____ $y_3 = 0$

Note that y_1 is unrestricted in sign because of the equality in the corresponding primal constraint. If you were to first express the primal in "normal" form, i.e., with "•" constraints, then this will yield an equivalent dual problem, but with y_2•0 and the signs of the coefficients of y_2 reversed in the objective and two constraints.
g. *It happens that point E on the graph above is optimal. What does the Complementary Slackness theorem tell us about the optimal solution y* of the dual problem?*
Since X_1^*>0 & X_2^*>0, the corresponding dual constraints must be "tight":

_X $y_1^* + y_2^* + 2y_3^* = 4$ __X $2y_1^* + y_2^* + y_3^* = 1$

Since the second primal constraint is not tight at point E, the corresponding dual variable must be zero:

__X $y_2^* = 0$ (since second primal constraint is not tight)

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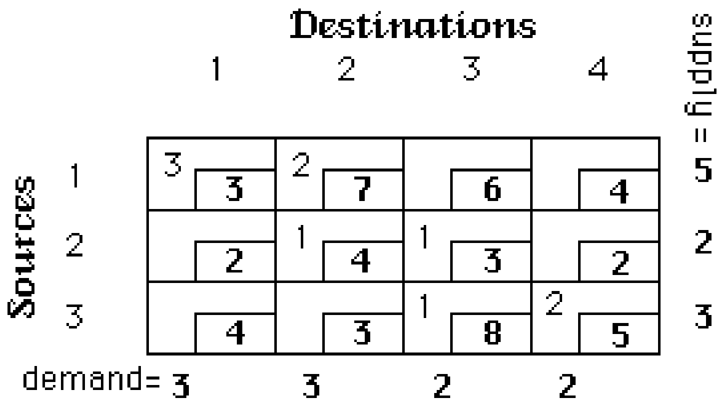
Quiz #5



Circle "T" or "F" to indicate whether true or false:

a.) T or F : The shipments indicated in the above table are a feasible solution to this transportation problem.
b.) T or F : The above transportation problem is "balanced".
c.) T or F : An assignment problem may be considered to be a special case of a transportation problem with all "transportation" costs equal to 1.
d.) T or F : If one unit were to be shipped from source #3 to destination #1, the result would be a reduction in the total cost .
e.) T or F : The Hungarian method might be used to solve the above transportation problem.
f.) T or F : If X_{32} were made a basic variable, then the solution becomes degenerate.
g.) T or F : The shipments indicated in the above table are a degenerate solution to this transportation problem.
h.) T or F : The above transportation problem will have 4 dual variables.
i.) T or F : If dual variable U_1 is zero, then dual variable U_2 must be 2.
j.) T or F : The transportation problem above is a special case of a linear programming problem.

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Quiz 5 Solutions

a.) True : If X_{32} were made a basic variable, then the solution becomes degenerate.
b.) True : The above transportation problem is "balanced".
c.) False: The above transportation problem will have 4 dual variables. *(This problem will have m+n=3+4=7 dual variable.)*
d.) True If one unit were to be shipped from source #3 to destination #1, the result would be a reduction in the total cost .
e.) False: The Hungarian method might be used to solve the above transportation problem. *(The Hungarian method is used to solve the assignment problem.)*
f.) True The transportation problem above is a special case of a linear programming problem.

g.) False: The shipments indicated in the above table are a degenerate solution to this transportation problem. *(This problem will have m+n-1 = 6 basic variables, and so all 6 are positive in this solution.)*

h.) False: An assignment problem may be considered to be a special case of a transportation problem with all "transportation" costs equal to 1. *(It is a transportation problem with each supply equal to 1 and each demand equal to 1.)*

i.) False: If dual variable U_1 is zero, then dual variable U_2 must be 2. *(V_2 would be 7, so that $U_1+V_2=C_{12}$, and then U_2 must have the value -3 in order that $U_2+V_2=C_{22}$.)*

j.) False: If X_{21} were made a basic variable, then the solution becomes degenerate.

k.) True The shipments indicated in the above table are a feasible solution to this transportation problem.

l.) True If X_{13} were made a basic variable, then the solution becomes degenerate.

m.) True The shipments indicated in the above table are a basic solution to this transportation problem.

n.) False: If X_{13} were made a basic variable, then X_{33} would become nonbasic. *(X_{33} would be unchanged, while X23 would be reduced to zero and become nonbasic.)*

o.) False: If one unit were to be shipped from source #2 to destination #1, the result would be a reduction in the total cost . *(There would be an increase of $2.)*

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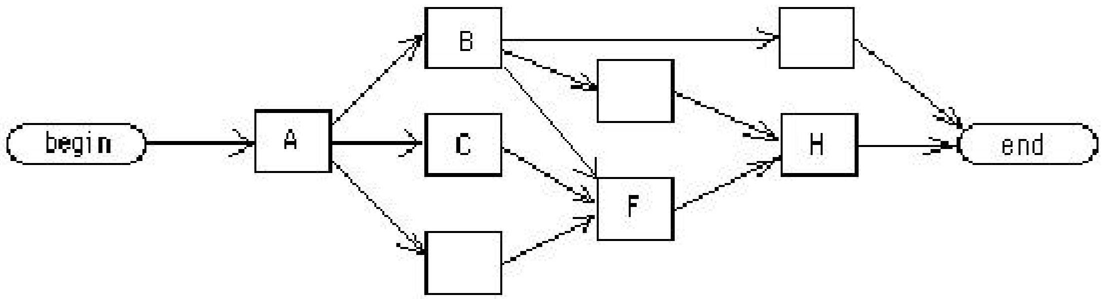
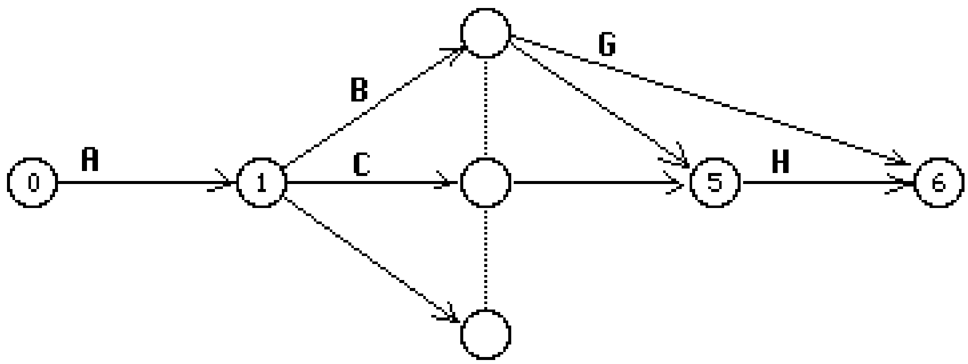
Quiz #6

Activity	Description	Predecessor(s)	Duration (days)
A	Clear & level site	none	2
B	Erect building	A	6
C	Install generator	A	4
D	Install maintenance equipment	B	4
E	Install water tank	A	2
F	Connect generator & tank to building	B,C,E	5
G	Paint & finish work on building	B	3
H	Facility test & checkout	D,F	2

Complete the AON network by labeling the nodes:



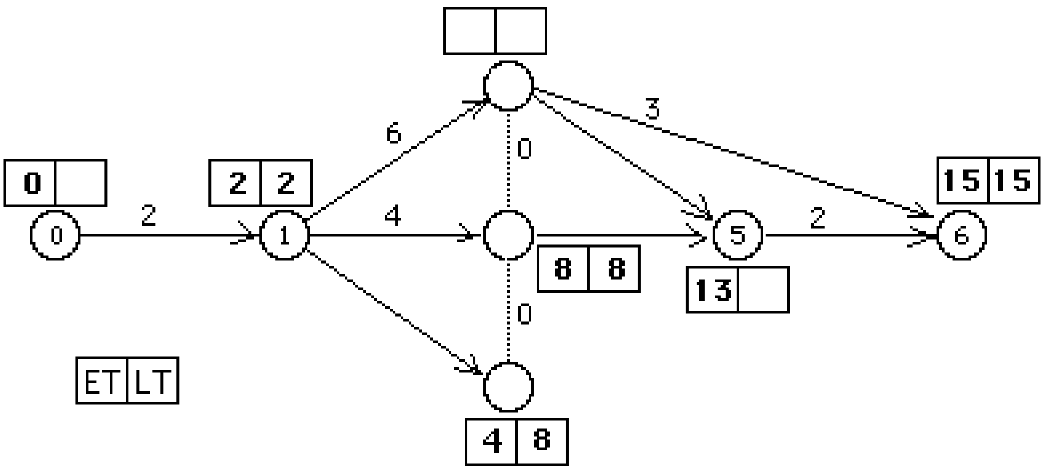
b. Complete the AOA network by labeling the arrows (three arrows are unlabeled, not including those for the "dummy" activities):



c. Two "dummy" activities in the AOA network above have no directions indicated. Add directions to these two arrows.

d. Three nodes in the AOA network above are not labeled. Label them.

e. Complete the computation of the earliest & latest times for the events (indicated in the boxes below). *There are four values to be computed!*



f. Indicate whether acitivity F is critical, and for activity G, compute:

ES = earliest start time LS = latest start time
EF = earliest finish time LF = latest finish time
TF = total float (slack)

Activity	Duration	ES	LS	EF	LF	TF	Critical?
A	2	0	0	2	2	0	Yes
B	6	2	2	8	8	0	Yes
C	4	2	4	6	8	2	No
D	4	8	9	12	13	1	No
E	2	2	6	4	8	4	No
F	5	8	8	13	13	0	___
G	3	___	___	___	___	___	No
H	2	13	13	15	15	0	Yes

g. What is the earliest completion time for the project? ___________

❒❒❒❒❒❒❒❒❒❒❒❒❒❒

Quiz 6 Solutions

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Quiz #7

Consider the following variation of the situation in the 2nd homework problem: As a price on a TV game show, you are given a choice of two boxes which look identical, but with different contents:

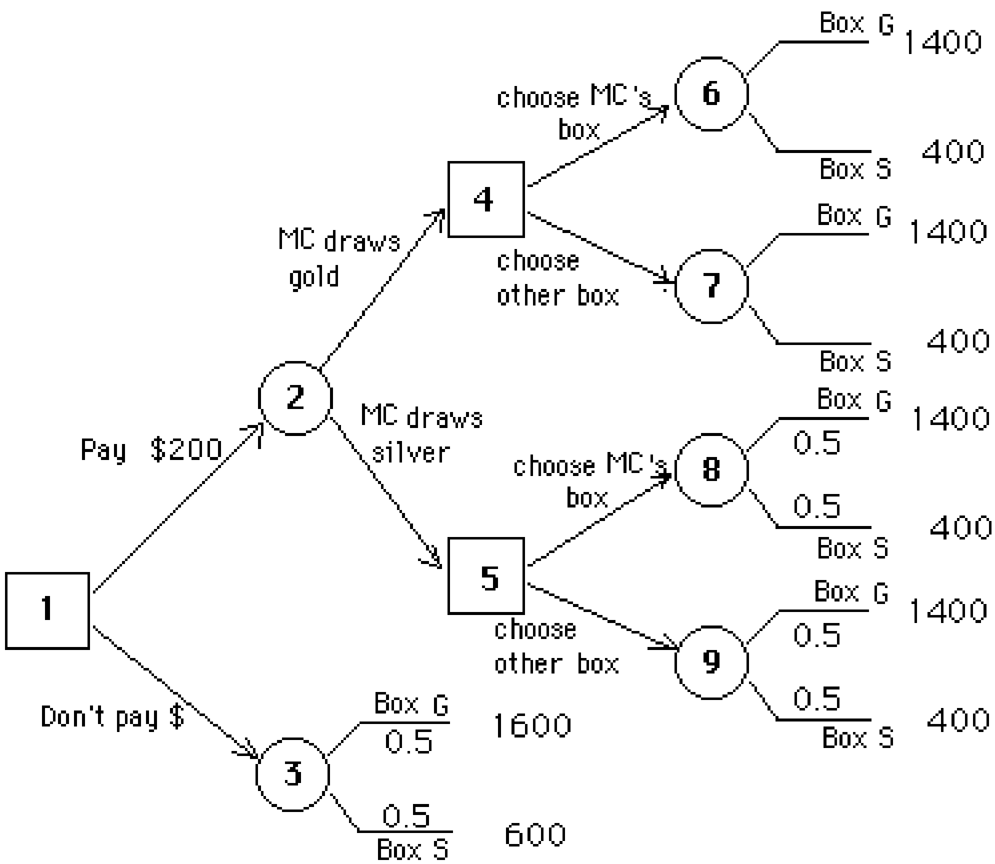
- one (box "G"), containing 4 coins: **3** gold coins and **1** silver coin,
- another (box "S"), containing 2 coins: **1** gold coin and **1** silver coin.

(Note that the 2 boxes contain different numbers of coins.) Gold coins have a value of $500 each, and silver coins a value of $100. If you give the MC (master of ceremonies) $200, he will select a coin at random (each of the six coins having equal likelihood, which means that he is twice as likely to select box "G"), and will show you the coin and also indicate the box from which it came. (He then returns the coin which he selected to its original box.)

That is, you must first decide whether to buy the information offered. If not, then you arbitrarily pick one of the boxes (with equal probabilities), and receive your payoff. If you pay for the information, the MC selects a coin, showing you the coin and pointing out the box from which he had selected the coin (which we will call "MC's box"). You must then select either the "MC's box" or the "other box". To help you to solve this problem, I have drawn the following decision tree, without the probabilities specified. (Note that the $200 has been already subtracted from each payoff where appropriate.)

We wish to know

- the optimal strategy
- the expected value of sample information (EVSI), i.e., the value of the information which the MC is offering you.



a. What is the probability that the MC selects a gold coin to show you?______ (Use this to label the branches at node #2).

b. According to ***Bayes' rule:***

$$P\{\text{MC's box is box G} \mid \text{MC draws gold}\} = \frac{P\{A \mid B\}P\{C\}}{P\{D\}}$$

Match the letters (A,B, C, & D) in this formula with the events below. (Some events may be indicated by more than one letter, and others by no letter.)

___: MC draws a coin
___: MC's box is box "G"
___: MC's box is box "S"
___: MC's coin is from box "G"
___: MC's coin is from box "S"
___: MC draws a gold coin
___: MC draws a silver coin
___: You toss a nickel to select the box

c. If the MC selects a gold coin, what is the probability that it came from box "G" (having 3 gold and 1 silver coin)? ________ (Use this to label the branches at nodes #6 and #7.)

d. Label each of the other branches from a chance node with its probabilities.

e. Fold back the nodes of the tree, indicating each branch selected at a decision node.

f. What is your optimal strategy? (Mark with an X)

(at node 1) ____ Pay MC $200 ____ Do NOT pay MC
(at node 4) ____ If MC draws gold coin, select his box
____ If MC draws gold coin, select other box
(at node 5) ____ If MC draws silver coin, select his box
____ If MC draws silver coin, select other box

g. What is your expected payoff using this strategy? ____________

h. What is EVSI, i.e., the expected value of the information the MC is offering you? __________

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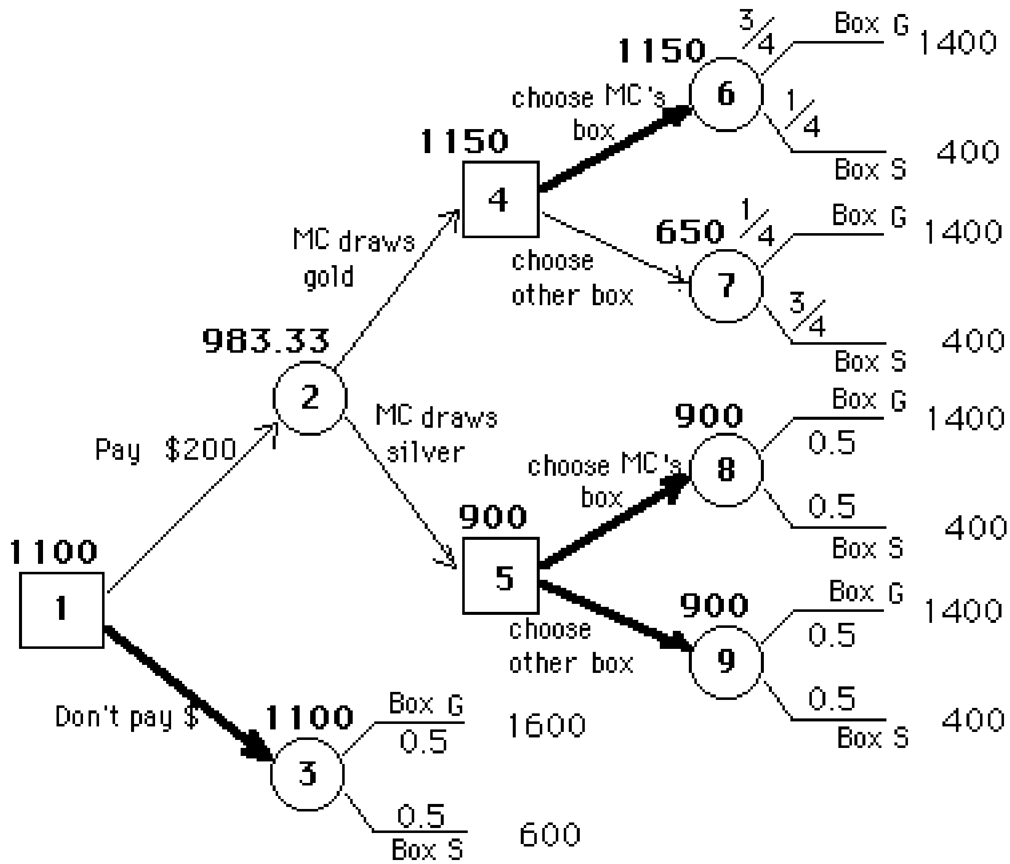
Quiz 7 Solutions



Note that the original version of this quiz had several errors: the probabilities assigned to the branches at nodes #6 & #7.

a. *What is the probability that the MC selects a gold coin to show you?*__2/3_ *(Use this to label the branches at node #2).*

b. *According to* ***Bayes' rule:***

$$P\{\text{MC's box is box G} \mid \text{MC draws gold}\} = \frac{P\{A \mid B\}P\{C\}}{P\{D\}}$$

Match the letters (A,B, C, & D) in this formula with the events below. (Some events may be indicated by more than one letter, and others by no letter.)

___: MC draws a coin
B,C: MC's box is box "G"
___: MC's box is box "S"
___: MC's coin is from box "G"
___: MC's coin is from box "S"
A,D: MC draws a gold coin
___: MC draws a silver coin
___: You toss a nickel to select the box

c. *If the MC selects a gold coin, what is the probability that it came from box "G" (having 3 gold and 1 silver coin)?*

P{MC's box is box G | MC draws gold}

$$= \frac{P\{\text{MC draws gold } | \text{ MC's box is box G}\}P\{\text{MC's box is box G}\}}{P\{\text{ MC draws gold }\}}$$

$$= \frac{\frac{3}{4} \times \frac{2}{3}}{\frac{2}{3}} = \frac{3}{4}$$

(Use this to label the branches at nodes #6 and #7.)

d. *Label each of the other branches from a chance node with its probabilities.* See tree diagram above.

e. *Fold back the nodes of the tree, indicating each branch selected at a decision node.* See tree diagram above.

f. *What is your optimal strategy? (Mark with an* X*)*

(at node 1) ____ *Pay MC $200* _X_ *Do NOT pay MC*

(at node 4) __X_ *If MC draws gold coin, select his box*

____ *If MC draws gold coin, select other box*

(at node 5) ____ *If MC draws silver coin, select his box*

__X_ *If MC draws silver coin, select other box*

g. *What is your expected payoff using this strategy?* __$1100__

h. *What is EVSI, i.e., the expected value of the information the MC is offering you?* __$83.33_

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Quiz #8

At the beginning of each year, January 1, my car is inspected, and judged to be in (1) *good*, (2) *fair*, or (3) *broken-down* condition. If it is in broken-down condition, I will replace the car, and will keep it another year otherwise.

Assume that

- my operating cost for the next year depends upon the condition of the car (after replacement, if any) at the beginning of the year
- if the car breaks down, it does so immediately before January 1, i.e., it will not be replaced until January 1.

The probability distribution for the condition of the car next Jan. 1 depends upon the condition of the car on Jan. 1 of the current year:

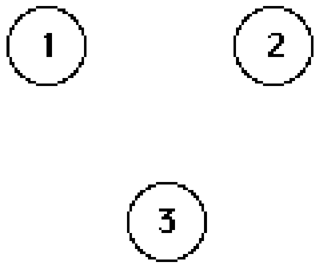
- A *good* car will be *good* at the beginning of the next year with probability 80%, *fair* , with probability 15%, and *broken-down* with probability 5%.
- A *fair* car will be a *fair* car at the beginning of the next year with probability 75% and *broken-down* with probability 25%.

Replacement costs: It costs $8000 to purchase a *good* car; a *broken-down* car has no trade-in value.

Operating costs: It costs $1000 per year to operate a *good* car and $1500 to operate a *fair* car.

I wish to model my car as a Markov chain. (That is, the state of the system is the condition of my current car and its replacements in future years.) *(Assume that the costs and probabilities above remain valid throughout my lifetime.)*

Consult the computer output to answer the questions below where appropriate. In this output, • = steady state probabilities, μ = mean first passage times, P = transition probabilities, and F = first passage probabilities.



i	π_i
1	0.5
2	0.375
3	0.125

$$M = \begin{bmatrix} 2 & 6.6666667 & 8 \\ 6 & 2.6666667 & 4 \\ 2 & 6.6666667 & 8 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0 & 0.75 & 0.25 \\ 0.8 & 0.15 & 0.05 \end{bmatrix} \qquad F = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0 & 0.75 & 0.25 \\ 0.8 & 0.15 & 0.05 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.68 & 0.24 & 0.08 \\ 0.2 & 0.6 & 0.2 \\ 0.68 & 0.24 & 0.08 \end{bmatrix} \qquad F^2 = \begin{bmatrix} 0.04 & 0.1275 & 0.0775 \\ 0.2 & 0.0375 & 0.1875 \\ 0.04 & 0.1275 & 0.0775 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.608 & 0.294 & 0.098 \\ 0.32 & 0.51 & 0.17 \\ 0.608 & 0.294 & 0.098 \end{bmatrix} \qquad F^3 = \begin{bmatrix} 0.032 & 0.108375 & 0.090125 \\ 0.16 & 0.031875 & 0.140625 \\ 0.032 & 0.108375 & 0.090125 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.5648 & 0.3264 & 0.1088 \\ 0.392 & 0.456 & 0.152 \\ 0.5648 & 0.3264 & 0.1088 \end{bmatrix} \qquad F^4 = \begin{bmatrix} 0.0256 & 0.09211875 & 0.09319375 \\ 0.128 & 0.02709375 & 0.10546875 \\ 0.0256 & 0.09211875 & 0.09319375 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.53888 & 0.34584 & 0.11528 \\ 0.4352 & 0.4236 & 0.1412 \\ 0.53888 & 0.34584 & 0.11528 \end{bmatrix} \qquad F^5 = \begin{bmatrix} 0.02048 & 0.078300938 & 0.090375313 \\ 0.1024 & 0.023029687 & 0.079101563 \\ 0.02048 & 0.078300938 & 0.090375313 \end{bmatrix}$$

(1.) Indicate the transitions and transition probabilities on the diagram above.

(2.) If last January 1 (1992) my car was in *good* condition, what is the probability that on January 1, 1995 (i.e., 3 years later) my car will be in *fair* condition? Select ***nearest*** value:

a. 0.13 b. 0.29
c. 0.2 d. 0.32

(3.) What fraction of the future years will my car be in good condition, under steady state conditions?

a. 4/5 b. 1/2
c. 3/8 d. 1/8
e. none of the above

(4.) What will be the average cost/year during future years, if the system is in steady state?

a. less than \$2000 | b. between \$2000 and \$2300
c. between \$2300 and \$2600 | d. between \$2600 and \$2900

(5.) Which of the equations below (<u>one or more</u>) must be solved to compute the steady-state probabilities?

a. $0.8\bullet_1 + 0.15\bullet_2 + 0.05\bullet_3 = 1$ | b. $\bullet_2 = 0.15\bullet_1 + 0.75\bullet_2 + 0.15\bullet_3$
c. $\bullet_1 = 0.8\bullet_1 + 0.15\bullet_2 + 0.05\bullet_3$ | d. $0.75\bullet_2 + 0.25\bullet_3 = \bullet_2$
e. $\bullet_1 + \bullet_2 + \bullet_3 = 1$ | f. $0.05\bullet_1 + 0.25\bullet_2 + 0.05\bullet_3 = 1$

(6.) What is the probability that the car I have on Jan. 1, 1992, if in good condition, will first need replacement in Jan. 1996 (i.e., 4 years later)?

a. less than 10% | b. between 10% and 12%
c. between 12 and 15% | d. greater than 15%

(7.) What is the expected time until the next replacement, if the car is in *good* condition? Select ***nearest*** value:

a. 5 years | b. 6 years
c. 7 years | d. 8 years

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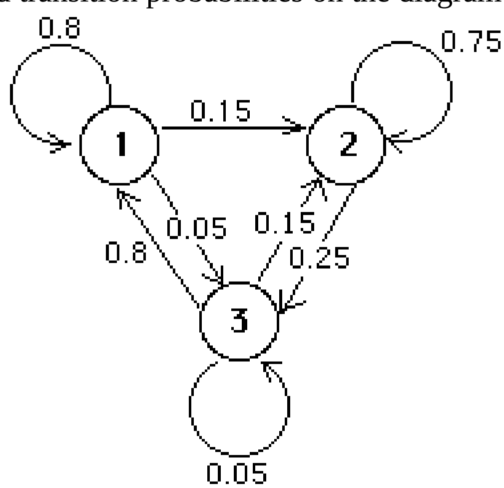
Quiz 8 Solutions

(1.) Indicate the transitions and transition probabilities on the diagram above.



(2.) If last January 1 (1992) my car was in *good* condition, what is the probability that on January 1, 1995 (i.e., 3 years later) my car will be in *fair* condition? Select ***nearest*** value: $p^3_{1,2} = 0.294$ = *(approximately)*

a. 0.13 | ->**b. 0.29**
c. 0.2 | d. 0.32

(3.) What fraction of the future years will my car be in good condition, under steady state conditions?

a. 4/5 | ->**b. 1/2**
c. 3/8 | d. 1/8
e. none of the above

(4.) What will be the average cost/year during future years, if the system is in steady state?

a. less than \$2000 ->**b. between \$2000 and \$2300**
c. between \$2300 and \$2600 d. between \$2600 and \$2900
$\$1000\pi_1 + \$1500\,\pi_2 + (\$8000+\$1000)\pi_3 = \$2187.50$ *(Note that the cost, when the car is determined to be in state 3 (broken-down) includes the replacement cost and the operating cost of the new car for the following year.)*

(5.) Which of the equations below (<u>one or more</u>) must be solved to compute the steady-state probabilities?

a. $0.8\bullet_1 + 0.15\bullet_2 + 0.05\bullet_3 = 1$ ->**b.** $\mathbf{\bullet_2 = 0.15\bullet_1 + 0.75\bullet_2 + 0.15\bullet_3}$
c. $\bullet_1 = 0.8\bullet_1 + 0.15\bullet_2 + 0.05\bullet_3$ d. $0.75\bullet_2 + 0.25\bullet_3 = \bullet_2$
->**e.** $\mathbf{\bullet_1 + \bullet_2 + \bullet_3 = 1}$ f. $0.05\bullet_1 + 0.25\bullet_2 + 0.05\bullet_3 = 1$

(6.) What is the probability that the car I have on Jan. 1, 1992, if in good condition, will first need replacement in Jan. 1996 (i.e., 4 years later)?

->**a. less than 10%** b. between 10% and 12%
c. between 12 and 15% d. greater than 15%
First-Passage probability $f^4_{1,3} = 0.09319375$

(7.) What is the expected time until the next replacement, if the car is in *good* condition? Select ***nearest*** value:

a. 5 years b. 6 years
c. 7 years d. 8 years
Mean First Passage time $= m_{1,3} = 8.$

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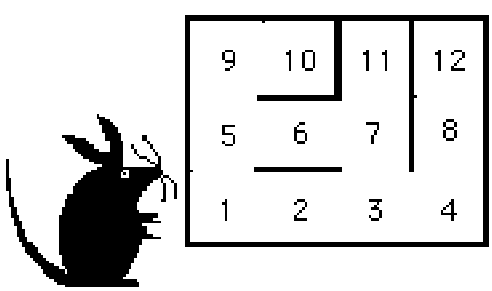
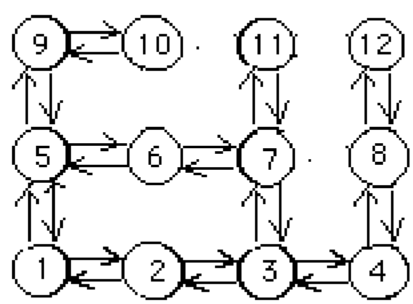
Quiz #9

PART ONE: A rat is placed in location #1 of a maze shown below on the left. A Markov chain model has been built where the state of the "system" is the location of the rat after he leaves his current location.





Steady State Distribution

i	π_i
1	0.0833
2	0.0833
3	0.125
4	0.0833
5	0.125
6	0.0833
7	0.125
8	0.0833
9	0.0833
10	0.0417
11	0.0417
12	0.0417

In assigning transition probabilities, it is assumed that the rat is equally likely to leave a location by any of the available paths:

P =

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.5	0	0	0.5	0	0	0	0	0	0	0
2	0.5	0	0.5	0	0	0	0	0	0	0	0	0
3	0	0.333	0	0.333	0	0	0.333	0	0	0	0	0
4	0	0	0.5	0	0	0	0	0.5	0	0	0	0
5	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0
6	0	0	0	0	0.5	0	0.5	0	0	0	0	0
7	0	0	0.333	0	0	0.333	0	0	0	0	0.333	0
8	0	0	0	0.5	0	0	0	0	0	0	0	0.5
9	0	0	0	0	0.5	0	0	0	0	0.5	0	0
10	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0	0

(If he arrives at a "dead end", he will retrace his last move with probability 1.)

M =

	1	2	3	4	5	6	7	8	9	10	11	12
1	12	9.33	12.7	31.7	8.67	15.3	16	52.7	29.7	52.7	39	75.7
2	10.7	12	7.33	26.3	15.3	18	14.7	47.3	36.3	59.3	37.7	70.3
3	19.3	12.7	8	19	20	18.7	11.3	40	41	64	34.3	63
4	24.3	17.7	5	12	25	23.7	16.3	21	46	69	39.3	44
5	11.3	16.7	16	35	8	10.7	15.3	56	21	44	38.3	79
6	16.7	18	13.3	32.3	9.33	12	8.67	53.3	30.3	53.3	31.7	76.3
7	20	17.3	8.67	27.7	16.7	11.3	8	48.7	37.7	60.7	23	71.7
8	27.3	20.7	8	3	28	26.7	19.3	12	49	72	42.3	23
9	14.3	19.7	19	38	3	13.7	18.3	59	12	23	41.3	82
10	15.3	20.7	20	39	4	14.7	19.3	60	1	24	42.3	83
11	21	18.3	9.67	28.7	17.7	12.3	1	49.7	38.7	61.7	24	72.7
12	28.3	21.7	9	4	29	27.7	20.3	1	50	73	43.3	24

1. On the diagram (above right) indicate the transition probabilities into and out of state #1 only.

2. If we record the rat's location over a period of several days, which location do you expect to be visited most frequently by the rat?

a. all equally often
b. location 7 more often than others
c. locations 3 & 5 equally often
d. locations 3, 5, &7 equally often
e. locations 1, 3, 5, &7 equally often

3. The number of transient states in this Markov chain model is

a. 0 b. 6 c. 9
d. 10 e. 12 f. none of the above

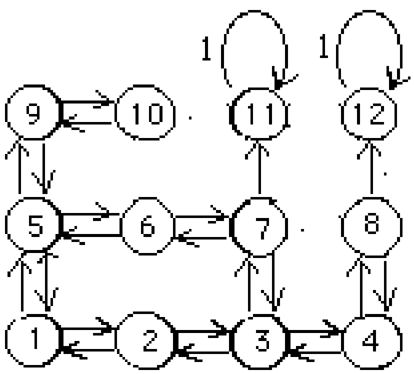
4. If the rat begins in location #1, what is the expected number of moves required to reach location #12?

a. five b. between 5 and 20 c. between 20 and 50
d. between 50 and 75 e. between 75 and 100 f. over 100

5. If the rat begins in location #1, how many locations will the rat visit before returning to his starting point?

a. five b. between 5 and 20 c. between 20 and 50
d. between 50 and 75 e. between 75 and 100 f. over 100

PART TWO: Suppose that locations 11 and 12 contain food, so that the rat does not leave when he finds it. States 11 & 12 then become absorbing states, and the Markov chain model becomes:



The matrices A and E for this Markov chain are:

A =

	11	12
1	0.69	0.31
2	0.655	0.345
3	0.621	0.379
4	0.414	0.586
5	0.724	0.276
6	0.759	0.241
7	0.793	0.207
8	0.207	0.793
9	0.724	0.276
10	0.724	0.276

E =

	1	2	3	4	5	6	7	8	9	10
1	4.07	2.97	2.79	1.24	4.76	2.28	2.07	0.621	3.17	1.59
2	2.97	3.52	3.1	1.38	3.62	1.86	1.97	0.69	2.41	1.21
3	1.86	2.07	3.41	1.52	2.48	1.45	1.86	0.759	1.66	0.828
4	1.24	1.38	2.28	2.34	1.66	0.96	1.24	1.17	1.1	0.552
5	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	3.93	1.97
6	2.28	1.86	2.17	0.96	4.03	3.1	2.28	0.483	2.69	1.34
7	1.38	1.31	1.86	0.82	2.17	1.52	2.38	0.414	1.45	0.724
8	0.62	0.69	1.14	1.17	0.82	0.48	0.62	1.59	0.552	0.276
9	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	2.97
10	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	3.97

1. If the rat begins at location #1, the probability that the rat finds the food at location #11 first (before the food at #12) is (nearest to)

a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

2. The expected number of times that the rat returns to his initial location before finding food is

a. less than 4 b. between 4 and 9 c. between 9 and 25

d. between 25 and 40 e. between 40 and 80 f. more than 80

3. If the rat manages to reach location #7 before finding food, the probability that he first finds the food at location #11 is

a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

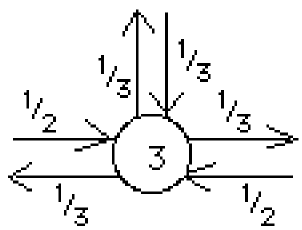
4. The number of transient states in this Markov chain model is

a. 0 b. 6 c. 9
d. 10 e. 12 f. none of the above

❒❒❒❒❒❒❒❒❒❒❒❒❒❒

Quiz 9 Solutions

1. On the diagram (above right) indicate the transition probabilities into and out of state #3 only.



2. If we record the rat's location over a period of several days, which location do you expect to be visited most frequently by the rat? *Based upon the steady-state probability 0.125.*

a. all equally often b. location 7 more often than others
c. locations 3 & 5 equally often **->d. locations 3, 5, &7 equally often**
e. locations 1, 3, 5, &7 equally often

3. The number of transient states in this Markov chain model is

->a. 0 b. 6 c. 9

d. 10 e. 12 f. none of the above

4. If the rat begins in location #1, what is the expected number of moves required to reach location #12?

a. five b. between 5 and 20 c. between 20 and 50

d. between 50 and 75 **->e. between 75 and 100** f. over 100

Mean First Passage Time = $m_{1,12}$ = *75.7.*

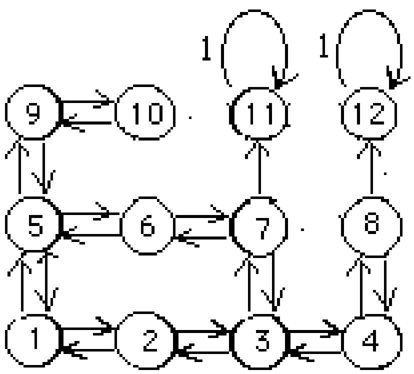
5. If the rat begins in location #1, how many locations will the rat visit before returning to his starting point?

a. five **->b. between 5 and 20** c. between 20 and 50

d. between 50 and 75 e. between 75 and 100 f. over 100

Mean First Passage Time = $m_{1,1}$ = *12*

PART TWO: Suppose that locations 11 and 12 contain food, so that the rat does not leave when he finds it. States 11 & 12 then become absorbing states, and the Markov chain model becomes:



The matrices A and E for this Markov chain are:

$A =$

	11	12
1	0.69	0.31
2	0.655	0.345
3	0.621	0.379
4	0.414	0.586
5	0.724	0.276
6	0.759	0.241
7	0.793	0.207
8	0.207	0.793
9	0.724	0.276
10	0.724	0.276

$E =$

	1	2	3	4	5	6	7	8	9	10
1	4.07	2.97	2.79	1.24	4.76	2.28	2.07	0.621	3.17	1.59
2	2.97	3.52	3.1	1.38	3.62	1.86	1.97	0.69	2.41	1.21
3	1.86	2.07	3.41	1.52	2.48	1.45	1.86	0.759	1.66	0.828
4	1.24	1.38	2.28	2.34	1.66	0.96	1.24	1.17	1.1	0.552
5	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	3.93	1.97
6	2.28	1.86	2.17	0.96	4.03	3.1	2.28	0.483	2.69	1.34
7	1.38	1.31	1.86	0.82	2.17	1.52	2.38	0.414	1.45	0.724
8	0.62	0.69	1.14	1.17	0.82	0.48	0.62	1.59	0.552	0.276
9	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	2.97
10	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	3.97

1. If the rat begins at location #1, the probability that the rat finds the food at location #11 first (before the food at #12) is (nearest to)

a. 50% b. 60% **->c. 70%**

d. 80% e. 90% f. 95%

Absorption probability $a_{1,12}$ = *0.69.*

2. The expected number of times that the rat returns to his initial location (#1) before finding food is

->a. less than 4 b. between 4 and 9 c. between 9 and 25
d. between 25 and 40 e. between 40 and 80 f. more than 80

Expected # of visits = $e_{1,1}$ = 4.07 *(including beginning visit), so that expected # returns is 3.07.*

3. If the rat manages to reach location #7 before finding food, the probability that he first finds the food at location #11 is
a. 50% b. 60% c. 70%
->d. 80% e. 90% f. 95%

Absorption probability $a_{7,11}$ = *0.793.*

4. The number of transient states in this Markov chain model is
a. 0 b. 6 c. 9
->d. 10 e. 12 f. none of the above

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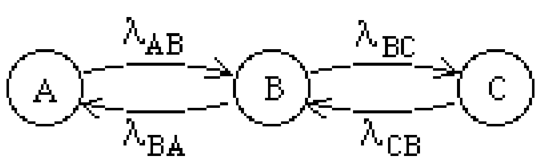
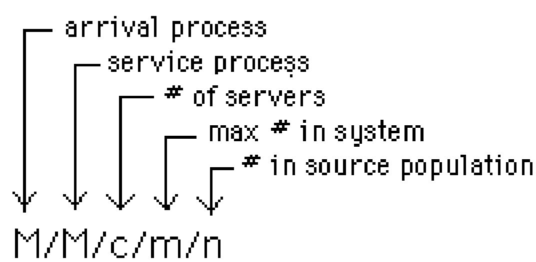
Quiz #10

A repairman is responsible for maintaining two machines in working condition. When both are in good condition, they operate simultaneously. However, a machine operates for an average of only 1 hour, when it fails and repair begins. Repair of a machine requires an average of 30 minutes. (Only one machine at a time can be repaired.) Define a continuous-time Markov chain with states:

A. Both machines have failed, with repair in progress on one machine
B. One machine is operable, and the other is being repaired
C. Both machines are in operating condition





(1.) In this model, the probability distribution of the time required to repair a machine is assumed to be:
a. Uniform b. Markov c. Poisson
d. Normal e. exponential f. None of the above

(2.) The transition rate l_{AB} is
a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BA}$ e. λ_{BA} f. None of the above

(3.) The transition rate l_{CB} is
a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{CB}$ e. λ_{BC} f. None of the above

(4.) The repair time will be less than t with probability

a. e^{-2t} b. $1 - e^{-2t}$ c. $1- e^{2t}$

d. $1- 2e^{-t}$ e. $2e^{t}$ f. None of the above

(5.) The transition diagram for this Markov chain (if we identify state A=0, B=1, and C=2) is identical to that of a queueing system of type:

a. M/M/1 b. M/M/2 c. M/M/1/2

d. M/M/2/2 e. M/M/2/1 f. M/M/1/2/2

where the arrival rate is 2/hour and the service rate is 1/hour.

(6.) The steady-state probability distribution must satisfy the equation(s) *(one or more):*

a. $\bullet_A + \bullet_B + \bullet_C = 1$ b. $\lambda_{AB}\bullet_A = \lambda_{BA}\bullet_B$

c. $\lambda_{BA}\bullet_A = \lambda_{AB}\bullet_B$ d. $\bullet_A = \lambda_{AB}\bullet_A + (\lambda_{BA} + \lambda_{BC})\bullet_B + \lambda_{CB}\bullet_C$

e. $\lambda_{BC}\bullet_B = \lambda_{CB}\bullet_C$ f. $\bullet_B = \lambda_{AB}\bullet_A + (\lambda_{BA} + \lambda_{BC})\bullet_B + \lambda_{CB}\bullet_C$

(7.) The average utilization of these machines in steady state (i.e., the fraction of maximum capacity at which they will operate), is:

a. $\bullet_B+\bullet_C$ b. $0.5(\bullet_B+\bullet_C)$ c. $\bullet_B+2\bullet_C$

d. $\bullet_A+\bullet_B+\bullet_C$ e. $2(\bullet_B+\bullet_C)$ f. $0.5(\bullet_B+2\bullet_C)$

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Quiz 10 Solutions

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Quiz #11

✠✺✠✺✠ PART ONE ✠✺✠✺✠

For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

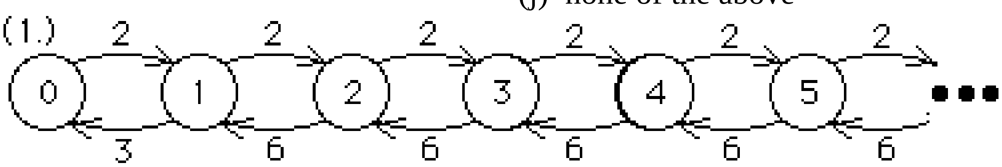
(a) M/M/1 (b) M/M/2 (c) M/M/1/4

(d) M/M/4 (e) M/M/2/4 (f) M/M/2/4/4

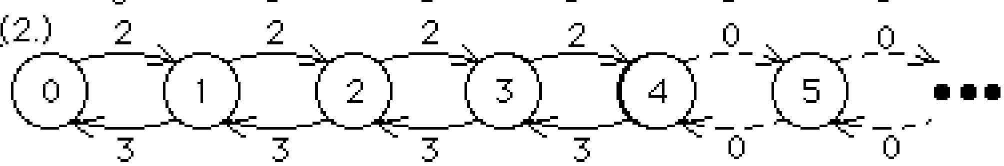
(g) M/M/1/2/4 (h) M/M/4/2 (i) M/M/4/4

(j) none of the above

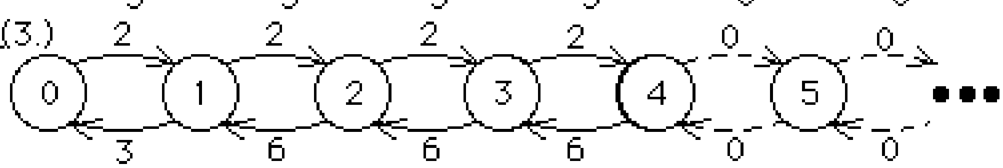
___(1.)



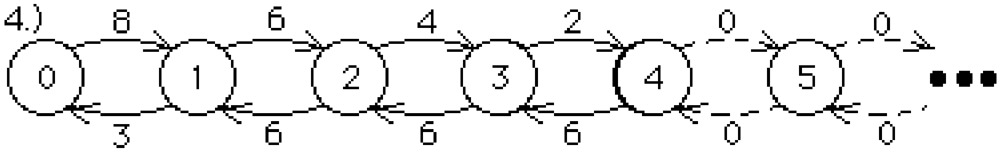
___(2.)



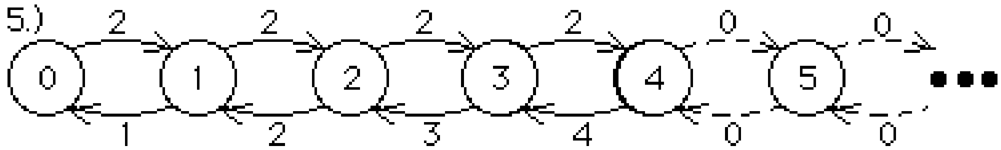
___(3.)



__(4.)



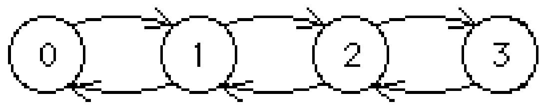
__(5.)



✠✺✠✺✠ PART TWO ✠✺✠✺✠

Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of 4 hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it. When both work on the same car, the average repair time for that car is only 3 hours (exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when tthe mechanics are idle, but one every 4 hours when the mechanics are busy. While 3 cars are in the shop, however, no cars arrive.

1. Label the transition diagram below with transition rates:



2. Which equation is used to compute the steady-state probability π_0? *(Note: The arithmetic is correct!)*

(a.) $\dfrac{1}{\pi_0} = 1 + \dfrac{1/2}{1/3} + \dfrac{1/4}{1/2} + \dfrac{1/4}{1/2} = \dfrac{1}{0.2857}$

(b.) $\dfrac{1}{\pi_0} = 1 + \dfrac{1/2}{1/3} + \dfrac{1/2 \times 1/4}{1/3 \times 1/2} + \dfrac{1/2 \times 1/4 \times 1/4}{1/3 \times 1/2 \times 1/2} = \dfrac{1}{0.2759}$

(c.) $\dfrac{1}{\pi_0} = 1 + \dfrac{1/3}{1/2} + \dfrac{1/3 \times 1/2}{1/2 \times 1/4} + \dfrac{1/3 \times 1/2 \times 1/2}{1/2 \times 1/4 \times 1/4} = \dfrac{1}{0.1765}$

(d.) $\dfrac{1}{\pi_0} = 1 + \dfrac{1/2}{1/3} + \left(\dfrac{1/4}{1/2}\right)^2 + \left(\dfrac{1/4}{1/2}\right)^3 = \dfrac{1}{0.3478}$

3. What fraction of the day will both mechanics be idle? *(Choose nearest answer.)*

a. 20% b. 25% c. 30%
d. 35% e. 40% f. 45%

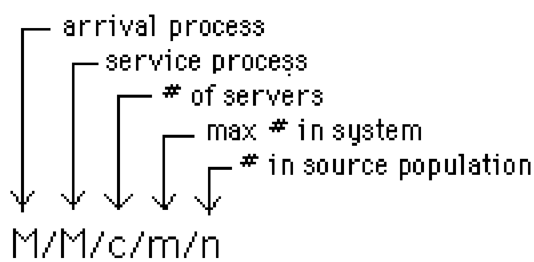
4. What fraction of the day will both mechanics be working on the same car? *(Choose nearest answer.)*

a. 20% b. 25% c. 30%
d. 35% e. 40% f. 45%

5. The average number of cars in the shop is 1.14 and the average time between arrivals is 3.41 hours. What is the average turnaround time (i.e., total time both waiting and being repaired) of a car in the shop? *(Choose nearest answer.)*

a. 3 hours b. 4 hours c. 5 hours
d. 6 hours e. 7 hours f. 8 hours

Note: Kendall's notation:



Quiz 11 Solutions

✠✺✠✺✠ PART ONE ✠✺✠✺✠

(1.) is M/M/2 queueing system (with infinite capacity). There are 2 servers, each with service rate 3/unit time.

(2.) is M/M/1/4 queueing system.

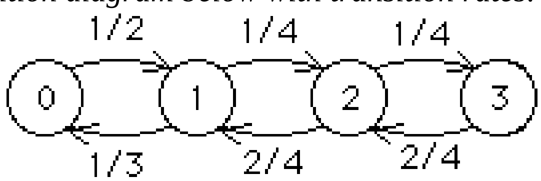
(3.) is M/M/2/4 queueing system.

(4.) is M/M/2/4/4 queueing system. That is, there is a finite source population of size 4, each "person" having an arrival rate of 2/unit time, and there are 2 servers, each with service rate 3/unit time.

(5.) is M/M/4/4 queueing system. There are 4 servers, each with service rate 1/unit time.

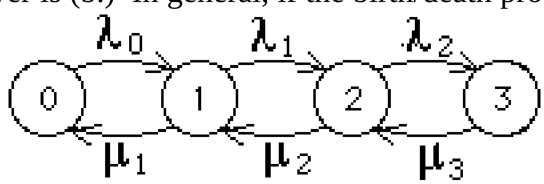
✠✺✠✺✠ PART TWO ✠✺✠✺✠

1. *Label the transition diagram below with transition rates:*



(rates are /hour)

2. The correct answer is (b.) In general, if the birth/death process has the diagram



then π_0 is computed by

$$\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2} + \frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}\times\frac{\lambda_2}{\mu_3}$$

3. What fraction of the day will both mechanics be idle? **c. 30%

According to the answer to (2), $\pi_0 = 27.59\%$.

4. What fraction of the day will both mechanics be working on the same car? *(Choose nearest answer.)* **e. 40%

$\pi_1 = (\lambda_0/\mu_1)\pi_0 = 1.5\pi_0 = 41.38\%$, i.e., the second term in the series in (2) times π_0.

5. The average number of cars in the shop is 1.14 and the average time between arrivals is 3.41 hours. What is the average turnaround time (i.e., total time both waiting and being repaired) of a car in the shop? *(Choose nearest answer.)*
*m***b. 4 hours

Little's Law says that $L=\underline{\lambda}W$, where L=average number in system = 1.14, and $\underline{\lambda}$ = average arrival rate = 1/3.41hr. Therefore, $W=L/\underline{\lambda}= (1.14)(3.41) = 3.89$ hours.