56:171
Operations Research
Fall 1999

Homework

Dept of Mechanical & Industrial Engineering
University of Iowa

56:171 Operations Research Homework #1 - Due Wednesday, September 8, 1999

In each case below, you must formulate a linear programming model that will solve the problem. Be sure to define the meaning of your variables! Then use LINDO (or other appropriate software) to find the optimal solution. State the optimal objective value, and describe in "layman's terms" the optimal decisions.

1. Bisco's new sugar-free, fat-free chocolate squares are so popular that the company cannot keep up with demand. Regional demands shown in the following table total 2000 cases per week, but Bisco can produce only 60% of that number.

	Northeast	Southeast	Midwest	West
Demand (cases)	620	490	510	380
Profit/case	1.60	1.40	1.90	1.20

The table also shows the different profit levels per case experienced in the region due to competition and consumer tastes. Bisco wants to find a maximum profit plan that fulfills between 50% and 70% of each region's demand.

2. Cattle feed can be mixed from oats, corn, alfalfa, and peanut hulls. The following table shows the current cost per ton (in dollars) of each of these ingredients, together with the percentage of recommended daily allowances for protein, fat, and fiber that a serving of it fulfills.

	Oats	Corn	Alfalfa	Peanut hulls
% protein	60	80	55	40
% fat	50	70	40	100
% fiber	90	30	60	80
Cost $/ton	200	150	100	75

We want to find a minimum cost way to produce feed that satisfies at least 60% of the daily allowance for protein and fiber while not exceeding 60% of the fat allowance.

3. "Mama's Kitchen" serves from 5:30 a.m. each morning until 1:30 p.m. in the afternoon. Tables are set and cleared by busers working 4-hour shifts beginning on the hour from 5:00 a.m. through 10:00 a.m. Most are college students who hate to get up in the morning, so Mama's pays $9 per hour for the 5:00, 6:00, and 7:00 a.m. shifts, and $7.50 per hour for the others. (That is, a person works a shift consisting of 4 consecutive hours, with the wages equal to 4x$9 for the three early shifts, and 4x$7.50 for the 3 later shifts.) The manager seeks a minimum cost staffing plan that will have at least the number of busers on duty each hour as specified below:

	5 am	6 am	7 am	8 am	9 am	10am	11am	Noon	1 pm
#reqd	2	3	5	5	3	2	4	6	3

4. a. Draw the feasible region of the following LP:

$$\begin{array}{ll} \text{Maximize} & 3X_1 + \ \ 2X_2 \\ \text{subject to} & 4X_1 + 7X_2 \leq 28 \\ & \ \ X_1 + \ \ X_2 \leq 6 \\ & 3X_1 + \ \ X_2 \leq 9 \\ & X_1 \geq 0,\ X_2 \geq 0 \end{array}$$

b. Use the simplex algorithm to find the optimal solution of the above LP. *(Show the initial and each succeeding tableau.)*

c. On the sketch of the feasible region in (a), indicate the initial basic solution and the basic solution at each succeeding iteration.

56:171 Operations Research
Homework #2 - Due Wednesday, September 15, 1999

1. ***Solving Systems of Equations***. Use the Gauss-Jordan method to determine whether each of the following linear systems has no solution, a unique solution, or an infinite number of solutions. Indicate all of the solutions (if any exist).

System A:

$$\begin{array}{ccccccc} X_1 & & & +X_4 & = & 5 \\ & X_2 & & +2X_4 & = & 5 \\ & & X_3 & +0.5X_4 & = & 1 \\ & & 2X_3 & +X_4 & = & 3 \end{array}$$

System B:

$$\begin{array}{cccccc} X_1 & +X_2 & +X_3 & & = & 1 \\ & X_2 & +2X_3 & +X_4 & = & 2 \\ & & & X_4 & = & 3 \end{array}$$

2. ***Simplex Algorithm:*** Use the simplex algorithm to find the optimal solution to the following LP:

$$\begin{array}{lll} \text{Min } z = & 4x_1 - x_2 & \\ \text{subject to} & 2x_1 + x_2 & 8 \\ & x_2 & 5 \\ & x_1 - x_2 & 4 \\ & x_1\, 0,\ x_2\, 0 & \end{array}$$

Show the initial tableau, each intermediate tableau, and the final tableau. Explain how you have decided on the location of each pivot and how you have decided to stop at the final tableau.

3. ***LP Model Formulation***. (Exercise 12, page 114, of O.R. text by Winston) For a telephone survey, a marketing research group needs to contact at least 150 wives, 120 husbands, 100 single adult males, and 110 single adult females. It costs \$2 to make a daytime call and (because of higher labor costs) \$5 to make an evening call. The table below lists the results. Because of limited staff, at most half of all phone calls can be evening calls. *Assume that the determined number of daytime calls and evening calls must all be completed before the results are tabulated (i.e., the staff cannot stop when the required number of individuals in each category has been contacted.)* Formulate an LP to minimize the cost of completing the survey, and solve it with LINDO.

Person responding	% of daytime calls	% of evening calls
Wife	30	30
Husband	10	30
Single male	10	15
Single female	10	20
None	40	5

4. Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. *Also answer the question accompanying each classification, if any.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*

(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element. Would the objective improve with this pivot?*

(**C**) Unique nondegenerate optimum.

(**D**) Optimal tableau, with alternate optimum. *State the values of the basic variables. Circle a pivot element which would lead to another optimal basic solution. Which variable will enter the basis, and at what value?*

(**E**) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal

Warning: Some of these classifications might be used for more than one tableau, while others might not be used at all!

(i) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	-3	0	1	1	0	0	2	3	-45	
0	0	0	-4	0	0	1	0	0	9	_______
0	-6	0	3	-2	1	0	2	3	5	
0	4	1	2	-5	0	0	1	1	8	

(ii) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	-2	-45	
0	0	0	-4	0	0	1	3	0	9	_________
0	-4	1	2	-5	0	0	-2	1	0	
0	-6	0	3	-2	1	0	-4	3	5	

(iii) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	1	0	0	3	5	-45	
0	0	0	-4	0	0	1	3	0	3	_________
0	4	1	2	-5	0	0	2	1	7	
0	-6	0	3	-2	1	0	-4	3	15	

(iv) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	-3	0	0	2	0	-45	
0	0	0	-1	0	0	1	3	0	9	_________
0	4	1	-4	-5	0	0	2	1	3	
0	-6	0	3	-2	1	0	-4	3	5	

(v) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	0	1	0	0	0	12	-45	
0	0	0	-4	0	0	1	3	0	9	_______
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	5	

(vi) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	_________
0	-6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

(vii)-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	1	0	0	-2	0	-45	
0	4	1	2	-5	0	0	2	1	5	__________
0	-6	0	3	2	1	0	-4	3	0	
0	0	0	-4	0	0	1	3	0	9	

(viii)-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	2	0	-1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	__________
0	6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

(ix) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	4	0	0	-2	2	-45	
0	0	0	-4	0	0	1	-3	0	3	__________
0	4	1	2	-5	0	0	2	1	-8	
0	-6	0	3	-2	1	0	-4	3	15	

56:171 Operations Research Homework #3 - Due Wednesday, September 22, 1999

1. ***LP Duality*** Find the LP dual of each of the following LP problems:

(a) Max $z = 4X_1 - X_2 + 2X_3$

s.t. $X_1 + X_2 \leq 5$

$2X_1 + X_2 \leq 7$

$2X_2 + X_3 \geq 6$

$X_1 + X_3 = 4$

$X_1 \geq 0$ (X_2 & X_3 unrestricted in sign)

(b) Min $w = 4Y_1 + 2Y_2 - Y_3$

s.t. $Y_1 + 2Y_2 \leq 6$

$Y_1 - Y_2 + 2Y_3 = 8$

$Y_1 \geq 0$, $Y_2 \geq 0$ (Y_3 unrestricted in sign)

2. ***Sensitivity Analysis*** Using the LINDO output in figure 5.19, answer the questions (a) through (d) in problem #3 ("Wivco" product mix problem) , pages 225-226 of the text *Operations Research*, by W. Winston..

Wivco produced product 1 and product 2 by processing raw material. Up to 90 lb of raw material may be purchased at a cost of \$10/lb. One pound of raw material may be used to produce either 1 lb of product 1 or 0.33 lb of product 2. Using a pound of raw material to produce a pound of product 1 requires 2 hours of labor or 3 hours to produce 0.33 lb of product 2. A total of 200 hours of labor are available, and at most 40 pounds of product 2 can be sold. Product 1 sells for \$13/lb and product 2 for \$40/lb. Let

RM = pounds of raw material processed
P1 = pounds of raw material used to produce product 1
P2 = pounds of raw material used to produce product 2

To maximize profit, Wivco should solve the following LP:

Max z = $13P1 + 40(0.33)P2 - 10RM$

s.t. $RM \geq P1 + P2$

$2P1 + 3P2 \leq 200$

$RM \leq 90$

$0.33P2 \leq 40$

$P1, P2, RM \geq 0$

Use LINDO output to answer the following questions:

a) If only 87 lb of raw material could be purchased, what would be Wivco's profits?
b) If product 2 sold for \$39.50/lb, what would be the new optimal solution to Wivco's problem?
c) What is the most that Wivco should be willing to pay for another pound of raw material?
d) What is the most that Wivco should be willing to pay for another hour of labor?

3. Sensitivity Analysis (Exercise 27, page 331 of Operations Research, by W. Winston) The following question concerns the Rylon example discussed in Section 3.9. After defining

RB = ounces of Regular Brute produced annually
LB = ounces of Luxury Brute produced annually
RC = ounces of Regular Chanelle produced annually
LC = ounces of Luxury Chanelle produced annually
RM = pounds of raw material purchased annually

The LINDO output below was obtained for this problem. Use this output to answer the following questions:

a) Interpret the shadow price of each constraint.
b) If the price of Regular Brute were to increase by 50 cents per ounce, what would be the new optimal solution to the Rylon problem?
c) If 8000 laboratory hours were available each year, but only 2000 lb of raw material were available each year, would Rylon's profits increase or decrease? (Hint: Use the 100% Rule to show that the current basis remains optimal. Then use reasoning analogous to (34)-(37) to determine the new objective function value.)
d) Rylon is considering expanding its laboratory capacity. Two options are under consideration:

   Option 1: For a cost of $10,000 (incurred at the present time), annual laboratory capacity can be increased by 1000 hours.

   Option 2: For a cost of $200,000 (incurred at present time), annual laboratory capacity can be increased by 10,000 hours.

   Suppose that all other aspects of the problem remain unchanged and that future profits are discounted, with the interest rate being 11 1/9 % per year. Which option, if any, should Rylon choose?
e) Rylon is considering purchasing a new type of raw material. Unlimited quantities can be purchased at $8/lb. It requires 3 laboratory hours to process a pound of the new raw material. Each processed pound yields 2 oz of Regular Brute and 1 oz of Regular Chanelle. Should Rylon purchase any of the new material? *(Show how this question can be answered without revising the model and re-running LINDO.)*

```
MAX     7 RB + 14 LB + 6 RC + 10 LC - 3 RM
  SUBJECT TO
         2)   RM <=   4000
         3)   3 LB + 2 LC + RM <=   6000
         4)   RB + LB - 3 RM =    0
         5)   RC + LC - 4 RM =    0
  END

        OBJECTIVE FUNCTION VALUE

        1)      172666.7

  VARIABLE        VALUE          REDUCED COST
        RB     11333.333008          0.000000
        LB       666.666687          0.000000
        RC     16000.000000          0.000000
        LC         0.000000          0.666667
        RM      4000.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         39.666668
        3)         0.000000          2.333333
        4)         0.000000          7.000000
        5)         0.000000          6.000000

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                               OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT         ALLOWABLE         ALLOWABLE
                   COEF           INCREASE          DECREASE
       RB        7.000000         1.000000         11.900001
       LB       14.000000       119.000000          1.000000
       RC        6.000000         INFINITY          0.666667
       LC       10.000000         0.666667          INFINITY
       RM       -3.000000         INFINITY         39.666668
```

```
                              RIGHTHAND SIDE RANGES
     ROW         CURRENT         ALLOWABLE         ALLOWABLE
                   RHS           INCREASE          DECREASE
       2     4000.000000       2000.000000       3400.000000
       3     6000.000000      33999.996094       2000.000000
       4        0.000000          INFINITY      11333.333008
       5        0.000000          INFINITY      16000.000000
```

4. (A modification of Exercise 3, page 317, of Operations Research, by W. Winston). You have been assigned to evaluate the efficiency of the Port Charles Police Department. Seven precincts are to be evaluated. The inputs and outputs for each precinct are as follows:
   Inputs:
      Number of policemen
      Number of vehicles used
   Outputs:
      Number of patrol units responding to service requests (thousands/year)
      Number of convictions obtained each year (hundreds/year)

You are given the following data:

Precinct	# policemen	# vehicles	# responses	# convictions
A	200	60	6	8
B	250	65	5.5	9
C	300	90	8	9.5
D	400	120	10	11
E	350	100	9.5	9
F	300	80	5	7.5
G	275	85	9	8

Use this information to determine which precincts, if any, are inefficient. For any inefficient precincts, determine the nature of the inefficiency.

56:171 Operations Research
Homework #4 - Due Wednesday, September 29, 1999

1. ***Linear Programming sensitivity.*** SunCo processes oil into aviation fuel and heating oil. It costs $40 to purchase each barrel of oil, which is then distilled and yields 0.5 barrel of aviation fuel and 0.5 barrel of heating oil. Output from the distillation may be sold directly or processed in the catalytic cracker. If sold after distillation without further processing, aviation fuel sells for $60/barrel. If sold after distillation without further processing, heating oil sells for $40/barrel. It takes 1 hour to process 1000 barrels of aviation fuel in the catalytic cracker, and these can be sold for $130/barrel. It takes 45 minutes to process 1000 barrels of heating oil in the cracker, and these can be sold for $90/barrel. Each day, at most 20,000 barrels of oil can be purchased, and 8 hours of catalytic cracker time are available. Formulate an LP to maximize SunCo's profits.

   Define the decision variables
   - **OIL** = # of barrels of oil purchased
   - **HSOLD** = # of barrels of heating oil sold
   - **HCRACK** = # of barrels of heating oil processed in catalytic cracker
   - **ASOLD** = # of barrels of aviation fuel sold
   - **ACRACK** = # of barrels of aviation fuel processed in catalytic cracker

The LP model to maximize profit is

```
Maximize 40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK - 40 OIL
subject to
        OIL = 20000                            (available supply)
        0.5 OIL = ASOLD + ACRACK          (aviation fuel & heating oil
        0.5 OIL = HSOLD + HCRACK           each constitute 50% of
                                            product of distilling)
        0.001ACRACK + 0.00075 HCRACK =  8       (avail. time for cracker)
            OIL = 0, ASOLD=0, ACRACK=0, HSOLD=0, HCRACK=0
```

The fourth constraint (5th row) imposes the available time limitation of the catalytic cracker:

Note: 1. Because the coefficients of this constraint are very small relative to those in other rows, LINDO will display a warning, and advise you to scale your problem, defining your variables to all have units of thousands of barrels, for example, so that your constraints would appear as:

```
OIL = 20                              (available supply)
0.5 OIL = ASOLD + ACRACK              (aviation fuel & heating oil
0.5 OIL = HSOLD + HCRACK                    each constitute 50% of
                                            product of distilling)
 ACRACK + 0.75HCRACK = 8.
```

and your objective function would have units of $1000's. This will avoid inaccuracies in the computation which might be caused by round-off errors. Because this problem is so small, this should not be a difficulty here, so in obtaining the LINDO output below, I have not scaled my input this way.

2. Other formulations are OK. For example, one could eliminate the variable OIL above by substituting (2ASOLD+2ACRACK) everywhere it appears.
3. When entering the model into LINDO, all variables must be on the left side of a constraint. Furthermore, it is not necessary to define a row for each non-negativity constraint (nonnegativity of every variable is assumed by LINDO).

The output of LINDO follows:

```
MAX     40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK - 40 OIL
SUBJECT TO
       2)   OIL <=   20000
       3) - ASOLD - ACRACK + 0.5 OIL =    0
```

```
       4) - HSOLD - HCRACK + 0.5 OIL =    0
       5)   0.00075 HCRACK + 0.001 ACRACK <=   8
 END

 LP OPTIMUM FOUND

       OBJECTIVE FUNCTION VALUE

       1)    760000.000

  VARIABLE        VALUE          REDUCED COST
     HSOLD     10000.000000           .000000
    HCRACK          .000000          2.500000
     ASOLD      2000.000300           .000000
    ACRACK      8000.000000           .000000
       OIL     20000.000000           .000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000        10.000000
        3)          .000000       -60.000000
        4)          .000000       -40.000000
        5)          .000000     70000.000000
```

The optimal solution, then, is to purchase all the available oil (20,000 barrels), which will produce 10,000 barrels of heating oil and 10,000 barrels of aviation fuel. All of the heating oil is sold without further processing (HSOLD), while 8000 barrels of the aviation fuel is processed in the catalytic cracker (ACRACK). The remaining 2000 barrels of aviation fuel is sold without further processing (ASOLD). This plan should generate a profit of $760,000.

LINDO will also display the following information, which is useful in sensitivity analysis:

```
 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
    HSOLD       40.000000         INFINITY         2.500000
   HCRACK       90.000000         2.500000         INFINITY
    ASOLD       60.000000         3.333333        20.000000
   ACRACK      130.000000         INFINITY         3.333333
      OIL      -40.000000         INFINITY        10.000000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2    20000.000000         INFINITY      4000.001000
        3         .000000      2000.000300         INFINITY
        4         .000000     10000.000000         INFINITY
        5        8.000000         2.000000         8.000000

THE TABLEAU
      ROW  (BASIS)      HSOLD   HCRACK  ASOLD  ACRACK  OIL    SLK 2
        1      ART       .000    2.500   .000   .000   .000   10.000
        2      OIL       .000     .000   .000   .000  1.000    1.000
        3    ASOLD       .000    -.750  1.000   .000   .000     .500
        4    HSOLD      1.000    1.000   .000   .000   .000     .500
        5   ACRACK       .000     .750   .000  1.000   .000     .000


      ROW     SLK 5
```

```
1  0.70E+05  0.76E+06
2      .000 20000.000
3 -1000.000  2000.000
4      .000 10000.000
5  1000.000  8000.000
```

Using the LINDO output above, answer the following questions:

a. Suppose that only 18,000 barrels of oil is available for purchase. What reduction in profit will result?
b. If the selling price of (unprocessed) aviation fuel were to drop by 10%, will the optimal solution change?
c. Suppose that the catalytic cracker must be shut down for 15 minutes. What loss of profit will result? How will the optimal solution change? (That is, what will be the values of each of the variables?)

2. ***Linear Programming sensitivity.*** A paper recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are:

Input type	Cost $/ton	Pulp content
Box board	5	15%
Tissue paper	6	20%
Newsprint	8	30%
Book paper	10	40%

Two methods, de-inking and asphalt dispersion, can be used to process the four inputs into pulp. It costs $20 to de-ink a ton of any input. The process of de-inking removes 10% of the input's pulp. It costs $15 to apply asphalt dispersion to a ton of material. The asphalt dispersion process removes 20% of the input's pulp. At most 3000 tons of input can be run through the asphalt dispersion process or the de-inking process. Grade 1 paper can only be produced with newsprint or book paper pulp; grade 2 paper, only with book paper, tissue paper, or box board pulp; and grade 3 paper, only with newsprint, tissue paper, or box board pulp. To meet its current demands, the company needs 500 tons of pulp for grade 1 paper, 500 tons of pulp for grade 2 paper, and 600 tons of pulp for grade 3 paper. The LP model below was formulated to minimize the cost of meeting the demands for pulp.

Hint:: *you may wish to define the variables*

BOX = tons of purchased boxboard
TISS = tons of purchased tissue
NEWS = tons of purchased newsprint
BOOK = tons of purchased book paper
BOX1 = tons of boxboard sent through de-inking
TISS1 = tons of tissue sent through de-inking
NEWS1 = tons of newsprint sent through de-inking
BOOK1 = tons of book paper sent through de-inking
BOX2 = tons of boxboard sent through asphalt dispersion
TISS2 = tons of tissue sent through asphalt dispersion
NEWS2 = tons of newsprint sent through asphalt dispersion
BOOK2 = tons of book paper sent through asphalt dispersion
PBOX = tons of pulp recovered from boxboard
PTISS = tons of pulp recovered from tissue
PNEWS= tons of pulp recovered from newsprint
PBOOK = tons of pulp recovered from book paper
PBOX1 = tons of boxboard pulp used for grade 1 paper,
PBOX2 = tons of boxboard pulp used for grade 2 paper, etc.
...
PBOOK3 = tons of book paper pulp used for grade 3 paper.

The LP model using these variables is:

```
MIN  5 BOX +6 TISS +8 NEWS +10 BOOK +20 BOX1 +20 TISS1 +20 NEWS1
          +20 BOOK1 +15 BOX2 +15 TISS2 +15 NEWS2 +15 BOOK2
 SUBJECT TO
        2) - BOX + BOX1 + BOX2 <=   0
        3) - TISS + TISS1 + TISS2 <=   0
        4) - NEWS + NEWS1 + NEWS2 <=   0
        5) - BOOK + BOOK1 + BOOK2 <=   0
        6)   0.135 BOX1 + 0.12 BOX2 - PBOX =    0
        7)   0.18 TISS1 + 0.16 TISS2 - PTISS =    0
        8)   0.27 NEWS1 + 0.24 NEWS2 - PNEWS =    0
        9)   0.36 BOOK1 + 0.32 BOOK2 - PBOOK =    0
       10) - PBOX + PBOX2 + PBOX3 <=   0
       11) - PTISS + PTISS2 + PTISS3 <=   0
       12) - PNEWS + PNEWS1 + PNEWS3 <=   0
       13) - PBOOK + PBOOK1 + PBOOK2 <=   0
       14)   PNEWS1 + PBOOK1 >=   500
       15)   PBOX2 + PTISS2 + PBOOK2 >=   500
       16)   PBOX3 + PTISS3 + PNEWS3 >=   600
       17)   BOX1 + TISS1 + NEWS1 + BOOK1 <=   3000
       18)   BOX2 + TISS2 + NEWS2 + BOOK2 <=   3000
 END
```

- Rows 2-5 state that only the supply of each input material which is purchased can be processed, either by de-inking or asphalt dispersion.
- Row 6 states that the recovered pulp in the boxboard is 90% of that in the boxboard which is processed by de-inking, i.e., (0.90)(0.15)BOX1, since boxboard is 15% pulp, plus 80% of that in the boxboard which is processed by asphalt dispersion, i.e., (0.80)(0.15)BOX2.
- Rows 7-9 are similar to row 6, but for different input materials.
- Rows 10-13 state that no more than the pulp which is recovered from each input may be used in making paper (grades 1, 2, &/or 3). Note that some variables are omitted, e.g., PBOX1, since boxboard cannot be used in Grade 1 paper.
- Rows 14-16 state that the demand for each grade of paper must be satisfied.
- Rows 17-18 state that each process (de-inking & asphalt dispersion) has a maximum throughput of 3000 tons. (The problem statement is unclear here, and might be interpreted as stating the total throughput of both processes cannot exceed 3000 tons, in which case rows 17&18 would be replaced by

```
17)   BOX1 + TISS1 + NEWS1 + BOOK1
    + BOX2 + TISS2 + NEWS2 + BOOK2 <=   3000
```

The solution found by LINDO is as follows:

```
 LP OPTIMUM FOUND AT STEP     25
       OBJECTIVE FUNCTION VALUE

       1)      140000.0

 VARIABLE        VALUE          REDUCED COST
      BOX         0.000000          0.000000
     TISS         0.000000          6.000000
     NEWS      2500.000000          0.000000
     BOOK      2833.333252          0.000000
     BOX1         0.000000         11.124999
    TISS1         0.000000          1.499999
    NEWS1         0.000000          0.249999
    BOOK1      2333.333252          0.000000
     BOX2         0.000000          9.333334
    TISS2         0.000000          0.222223
    NEWS2      2500.000000          0.000000
    BOOK2       500.000000          0.000000
     PBOX         0.000000          0.000000
```

PTISS	0.000000	0.000000
PNEWS	600.000000	0.000000
PBOOK	1000.000000	0.000000
PBOX2	0.000000	19.444445
PBOX3	0.000000	0.000000
PTISS2	0.000000	19.444445
PTISS3	0.000000	0.000000
PNEWS1	0.000000	19.444445
PNEWS3	600.000000	0.000000
PBOOK1	500.000000	0.000000
PBOOK2	500.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	5.000000
3)	0.000000	0.000000
4)	0.000000	8.000000
5)	0.000000	10.000000
6)	0.000000	-102.777779
7)	0.000000	-102.777779
8)	0.000000	-102.777779
9)	0.000000	-83.333336
10)	0.000000	102.777779
11)	0.000000	102.777779
12)	0.000000	102.777779
13)	0.000000	83.333336
14)	0.000000	-83.333336
15)	0.000000	-83.333336
16)	0.000000	-102.777779
17)	666.666687	0.000000
18)	0.000000	1.666667

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
BOX	5.000000	INFINITY	5.000000
TISS	6.000000	INFINITY	6.000000
NEWS	8.000000	0.333334	4.666667
BOOK	10.000000	6.000000	1.999989
BOX1	20.000000	INFINITY	11.124999
TISS1	20.000000	INFINITY	1.499999
NEWS1	20.000000	INFINITY	0.249999
BOOK1	20.000000	0.249999	0.750001
BOX2	15.000000	INFINITY	9.333333
TISS2	15.000000	INFINITY	0.222222
NEWS2	15.000000	0.222221	4.666667
BOOK2	15.000000	0.666667	0.222221
PBOX	0.000000	INFINITY	77.777779
PTISS	0.000000	INFINITY	1.388890
PNEWS	0.000000	1.388890	19.444443
PBOOK	0.000000	19.444443	83.333336
PBOX2	0.000000	INFINITY	19.444443
PBOX3	0.000000	19.444443	77.777779
PTISS2	0.000000	INFINITY	19.444443
PTISS3	0.000000	19.444443	1.388890
PNEWS1	0.000000	INFINITY	19.444443
PNEWS3	0.000000	1.388890	19.444443
PBOOK1	0.000000	19.444443	83.333336
PBOOK2	0.000000	19.444443	83.333336

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE

2	0.000000	0.000000	INFINITY
3	0.000000	INFINITY	0.000000
4	0.000000	2500.000000	INFINITY
5	0.000000	2833.333252	INFINITY
6	0.000000	0.000000	600.000000
7	0.000000	0.000000	600.000000
8	0.000000	120.000008	600.000000
9	0.000000	240.000015	840.000000
10	0.000000	600.000000	0.000000
11	0.000000	600.000000	0.000000
12	0.000000	600.000000	120.000008
13	0.000000	840.000000	240.000015
14	500.000000	240.000015	500.000000
15	500.000000	240.000015	500.000000
16	600.000000	120.000008	600.000000
17	3000.000000	INFINITY	666.666687
18	3000.000000	2625.000000	500.000000

THE TABLEAU

ROW	(BASIS)	BOX	TISS	NEWS	BOOK	BOX1	TISS1
1	ART	0.000	6.000	0.000	0.000	11.125	1.500
2	BOOK	0.000	0.000	0.000	1.000	-0.062	-0.083
3	SLK 3	0.000	-1.000	0.000	0.000	0.000	1.000
4	SLK 17	0.000	0.000	0.000	0.000	0.500	0.333
5	BOOK1	0.000	0.000	0.000	0.000	0.500	0.667
6	PBOX	0.000	0.000	0.000	0.000	-0.135	0.000
7	PTISS	0.000	0.000	0.000	0.000	0.000	-0.180
8	PNEWS	0.000	0.000	0.000	0.000	0.135	0.180
9	PBOOK	0.000	0.000	0.000	0.000	0.000	0.000
10	PBOX3	0.000	0.000	0.000	0.000	-0.135	0.000
11	PTISS3	0.000	0.000	0.000	0.000	0.000	-0.180
12	PNEWS3	0.000	0.000	0.000	0.000	0.135	0.180
13	PBOOK2	0.000	0.000	0.000	0.000	0.000	0.000
14	PBOOK1	0.000	0.000	0.000	0.000	0.000	0.000
15	NEWS2	0.000	0.000	0.000	0.000	0.562	0.750
16	NEWS	0.000	0.000	1.000	0.000	0.562	0.750
17	BOX	1.000	0.000	0.000	0.000	-1.000	0.000
18	BOOK2	0.000	0.000	0.000	0.000	-0.562	-0.750

ROW	NEWS1	BOOK1	BOX2	TISS2	NEWS2	BOOK2	PBOX
1	0.250	0.000	9.333	0.222	0.000	0.000	0.000
2	-0.125	0.000	0.056	0.037	0.000	0.000	0.000
3	0.000	0.000	0.000	1.000	0.000	0.000	0.000
4	0.000	0.000	0.444	0.296	0.000	0.000	0.000
5	1.000	1.000	-0.444	-0.296	0.000	0.000	0.000
6	0.000	0.000	-0.120	0.000	0.000	0.000	1.000
7	0.000	0.000	0.000	-0.160	0.000	0.000	0.000
8	0.000	0.000	0.120	0.160	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	-0.120	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	-0.160	0.000	0.000	0.000
12	0.000	0.000	0.120	0.160	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	1.125	0.000	0.500	0.667	1.000	0.000	0.000
16	0.125	0.000	0.500	0.667	0.000	0.000	0.000
17	0.000	0.000	-1.000	0.000	0.000	0.000	0.000
18	-1.125	0.000	0.500	0.333	0.000	1.000	0.000

ROW	PTISS	PNEWS	PBOOK	PBOX2	PBOX3	PTISS2	PTISS3

1	0.000	0.000	0.000	19.444	0.000	19.444	0.000
2	0.000	0.000	0.000	3.241	0.000	3.241	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.926	0.000	0.926	0.000
5	0.000	0.000	0.000	-0.926	0.000	-0.926	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	1.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	1.000	0.000	-1.000	0.000	-1.000	0.000
9	0.000	0.000	1.000	1.000	0.000	1.000	0.000
10	0.000	0.000	0.000	1.000	1.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	1.000	1.000
12	0.000	0.000	0.000	-1.000	0.000	-1.000	0.000
13	0.000	0.000	0.000	1.000	0.000	1.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	-4.167	0.000	-4.167	0.000
16	0.000	0.000	0.000	-4.167	0.000	-4.167	0.000
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	4.167	0.000	4.167	0.000

ROW	PNEWS1	PNEWS3	PBOOK1	PBOOK2	SLK 2	SLK 3	SLK 4
1	19.444	0.000	0.000	0.000	5.000	0.000	8.000
2	3.241	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	1.000	0.000
4	0.926	0.000	0.000	0.000	0.000	0.000	0.000
5	-0.926	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	-1.000	0.000	0.000	0.000	0.000	0.000	0.000
9	1.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	1.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	1.000	0.000	0.000	0.000
14	1.000	0.000	1.000	0.000	0.000	0.000	0.000
15	-4.167	0.000	0.000	0.000	0.000	0.000	0.000
16	-4.167	0.000	0.000	0.000	0.000	0.000	-1.000
17	0.000	0.000	0.000	0.000	-1.000	0.000	0.000
18	4.167	0.000	0.000	0.000	0.000	0.000	0.000

ROW	SLK 5	SLK 10	SLK 11	SLK 12	SLK 13	SLK 14	SLK 15
1	10.000	102.778	102.778	102.778	83.333	83.333	83.333
2	-1.000	0.463	0.463	0.463	-2.778	-2.778	-2.778
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	3.704	3.704	3.704	2.778	2.778	2.778
5	0.000	-3.704	-3.704	-3.704	-2.778	-2.778	-2.778
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	-1.000	-1.000	-1.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	-1.000	-1.000	-1.000
10	0.000	1.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	1.000	0.000	0.000	0.000	0.000
12	0.000	-1.000	-1.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.000	0.000	-1.000
14	0.000	0.000	0.000	0.000	0.000	-1.000	0.000
15	0.000	-4.167	-4.167	-4.167	0.000	0.000	0.000
16	0.000	-4.167	-4.167	-4.167	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	4.167	4.167	4.167	0.000	0.000	0.000

ROW	SLK 16	SLK 17	SLK 18	RHS

```
 1  0.10E+03  0.00E+00   1.7     -0.14E+06
 2     0.463     0.000     0.111  2833.333
 3     0.000     0.000     0.000     0.000
 4     3.704     1.000     0.889   666.667
 5    -3.704     0.000    -0.889  2333.333
 6     0.000     0.000     0.000     0.000
 7     0.000     0.000     0.000     0.000
 8    -1.000     0.000     0.000   600.000
 9     0.000     0.000     0.000  1000.000
10     0.000     0.000     0.000     0.000
11     0.000     0.000     0.000     0.000
12    -1.000     0.000     0.000   600.000
13     0.000     0.000     0.000   500.000
14     0.000     0.000     0.000   500.000
15    -4.167     0.000     0.000  2500.000
16    -4.167     0.000     0.000  2500.000
17     0.000     0.000     0.000     0.000
18     4.167     0.000     1.000   500.000
```

a. *Complete the following statement:* the optimal solution is to purchase only newsprint and book paper, process ______ tons of the book paper and _____ tons of the newsprint by asphalt dispersion, and the remaining book paper by de-inking. This yields ______ tons of pulp from the newsprint and ______ tons of pulp from the book paper. One-half of the pulp from book paper is used in each of grades 1 & 2 paper, and the newsprint is used in grade 3 paper. This plan will use ______% of the de-inking capacity and ______% of the asphalt dispersion capacity. (Note that BOX is a basic variable, but has a value of zero, categorizing this solution as ______________.

b. How much must tissue drop in price in order that it would enter the solution? ________

c. If tissue were to enter the solution (e.g., because of the drop in price you determined in (b)), how much would be purchased? ________________ *(Hint: use the minimum ratio test!)*

d. How much would the cost decrease if 10 additional tons of pulp for grade 1 paper were required? ________________

e. If ten additional tons of pulp for grade 1 paper were required, how would the quantities of raw materials (boxboard, newsprint and book paper) change?_______________ *(Hint: if the surplus variable for the row stating the requirement were to increase, what effect would it have on BOX, NEWS, and BOOK?)*

3. ***Transportation problem***. A bank has two sites at which checks are processed. Site 1 can process 10,000 checks per day and site 2 can process 6000 checks per day. The bank processes three types of checks: vendor checks, salary checks, and personal checks. Each day 5000 checks of each type must be processed. The processing cost per check depends on the site at which the check is processed:

Type of check	Site #1	Site #2
Vendor checks	5¢	3¢
Salary checks	4¢	4¢
Personal checks	2¢	5¢

(a.) Formulate a balanced transportation problem to minimize the daily cost of processing checks. (That is, provide the transportation tableau for the problem.) What is the number of basic variables in any basic solution to this problem?

(b.) Use both the Northwest-Corner and Vogel's Approximation Method to find a basic feasible solution to the problem. Compute the total cost for each solution.
(c.) Starting with the Northwest-Corner solution, perform the simplex algorithm to find the optimal solution to this problem. At each iteration, state the values of the dual variables and reduced costs of each nonbasic "shipment".

(d.) How far from optimal (as a % of the optimal cost) was the solution found by Vogel's Approximation Method?

4. Paltry Properties has just acquired four rental homes. Paltry wishes to have the houses painted within the next week so that all can be available for the prime rental season. This means that each house will have to be painted by a different contractor. The following table shows the bids (in $K) received from four contractors on the four houses.

House	Painter #1	Painter #2	Painter #3	Painter #4
A	2.5	1.3	3.6	1.8
B	2.9	1.4	5.0	2.2
C	2.2	1.6	3.2	2.4
D	3.1	1.8	4.0	2.5

Paltry wants to decide which bids to accept in order to paint all houses at minimum total cost.

5. Tube Steel Incorporated (TSI) is optimizing production at its 4 hot mills. TSI makes 8 types of tubular products which are either solid or hollow, and come in 4 diameters. The following two tables show production costs (in dollars) per tube of each product at each mill and the extrusion times (in minutes) for each allowed combination. Missing values indicate product-mill combinations that are not feasible.

Unit Cost (dollars)

Product	Mill 1	Mill 2	Mill 3	Mill 4
0.5 inch solid	0.10	0.10	---	0.15
1 inch solid	0.15	0.18	--	0.20
2 inch solid	0.25	0.15	--	0.30
4 inch solid	0.55	0.50	--	--
0.5 inch hollow	--	0.20	0.13	0.25
1 inch hollow	--	0.30	0.18	0.35
2 inch hollow	--	0.50	0.28	0.55
4 inch hollow	--	1.00	0.60	--

Unit Time (minutes)

Product	Mill 1	Mill 2	Mill 3	Mill 4
0.5 inch solid	0.5	0.5	--	0.1
1 inch solid	0.6	0.6	--	0.6
2 inch solid	0.8	1.0	--	0.6
4 inch solid	1.0	1.0	--	--
0.5 inch hollow	--	1.0	0.5	0.5
1 inch hollow	--	1.2	0.6	0.6
2 inch hollow	--	1.6	0.8	0.8
4 inch hollow	--	2.0	1.0	--

Yearly minimum requirements for the solid sizes (in thousands) are 250, 150, 150, and 80, respectively. For the hollow sizes they are 190, 190, 160, and 150. The mills can operate up to three 40-hour shifts per week, 50 weeks per year. Present policy is that each mill must operate at least one shift.

(a) Formulate a linear programming model to meet demand and shift requirements at minimum total cost using the decision variables:

X_{pm} = thousands of units of product *p* produced annually at mill *m*

(b) Solve your LP model using LINDO (or other LP solver).

(c) What are the marginal costs of producing each of the eight products?

(d) Based upon the computer output, explain why the policy of operating all mills at least one shift is costing the company money.

(e) Two options being considered would open mills 3 or 4 on weekends (i.e., add up to 16 extra house to each of 3 shifts over 50 weeks). Taking each option separately, determine or bound as well as possible from your computer results the impact these changes would have on total production cost.

(f) Another option being considered is to hire young industrial engineers to find ways of reducing the unit costs of production at high-cost mill #4. For each of the 6 products there taken separately, use your computer results to determine to what level unit costs would have to be reduced before three could be any change in the optimal production plan.

(g) A final pair of options being considered is to install equipment to produce 4-inch solid and 4-inch hollow tubes at mill #4. The new equipment would produce either product in 1 minute per unit. Taking each product separately, determine the unit production cost that would have to be achieved to make it economical to use the new facilities.

56:171 Operations Research
Homework #5 - Due Wednesday, October 6, 1999

1. The campus bookstore must decide how many textbooks to order for a freshman economics course to be offered next semester. The bookstore believes that either seven, eight, nine, or ten sections of the course will be offered, each section consisting of 40 students. The publisher is offering bookstores a discount if they place their orders early. If the bookstore orders too few texts and runs out, the publisher will air express additional books at the bookstore's expense. If it orders too many texts, the store can return unsold texts to the publisher for a partial credit. The bookstore is considering ordering either 280, 320, 360, or 400 textbooks in order to get the discount. Taking into account the discounts, air express expenses, and credits for returned texts, the bookstore manager estimates the following resulting profits.

# books ordered	7 sections	8 sections	9 sections	10 sections
280	$2800	$2720	$2640	$2480
320	$2600	$3200	$3040	$2880
360	$2400	$3000	$3600	$3440
400	$2200	$2800	$3400	$4000

(a) What is the decision if the manager uses the maximax criterion?
(b) What is the decision if the manager uses the maximin criterion?
(c) What is the decision if the manager uses the minimax regret criterion?

Suppose now that, based upon conversations held with the chairperson of the economics department, the bookstore manager believes the following probabilities hold:

P{7 sections offered} = 10%
P{8 sections offered} = 30%
P{9 sections offered} = 40%
P{10 sections offered} = 20%

(d) Using the expected value criterion, determine how many books the manager should purchase in order to maximize the store's expected profit.
(e) Based upon the probabilities given, determine the expected value of perfect information and interpret its meaning.

2. John Deere is making a special offer on its model 603 riding mower. If ordered prior to May 1, the mower will cost Adams Hardware $820; after May 1 the cost rises to $920. Adams sells the mowers for $1150 and it incurs $100 in sales expenses on each mower. Any mower left in inventory at the end of the summer can be sold at Adams's end-of-season clearance sale and will net the firm only $750. Adams expects to sell between one and three of these mowers this summer. Management estimates that the probability of selling one mower is twice the probability of selling two mowers, and the probability of selling three mowers is 40%. How many mowers should Adams purchase prior to May 1 in order to maximize its expected profits?

3. T. Bone Puckett, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants located in South Carolina. Three alternative decisions are being considered:

- Expand the plant and produce light-weight, durable materials for possible sales to the military, a market with little foreign competition.

- Maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition.

- Sell the plant now.

If one the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends upon foreign market conditions, including the status of a trade embargo bill in Congress. The following payoff table describes this decision situation.

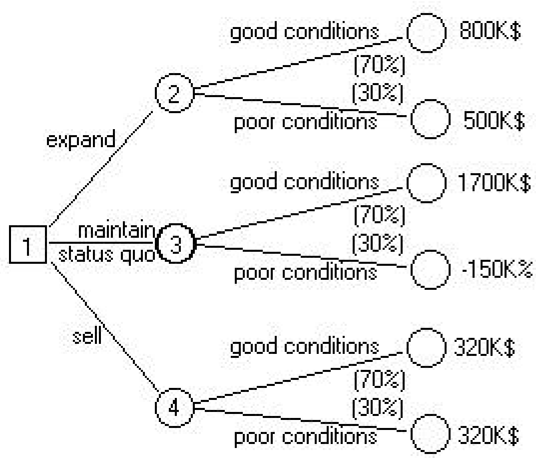
Decision	Good foreign competitive conditions	Poor foreign competitive conditions
Expand	$800,000	$500,000
Maintain status quo	$1,300,000	- $150,000
Sell now	$320,000	$320,000

(a.) Determine the best decision using the following decision criteria:
- ❑ Maximax
- ❑ Maximin
- ❑ Minimax regret

(b.) Assume it is now possible to estimate a probability of 70% that good foreign competitive conditions will exist and a probability of 30% that poor conditions will exist. Determine the best decision using expected value and expected opportunity loss.

(c.) Compute the expected value of perfect information.

(d.) Fold back the decision tree below:



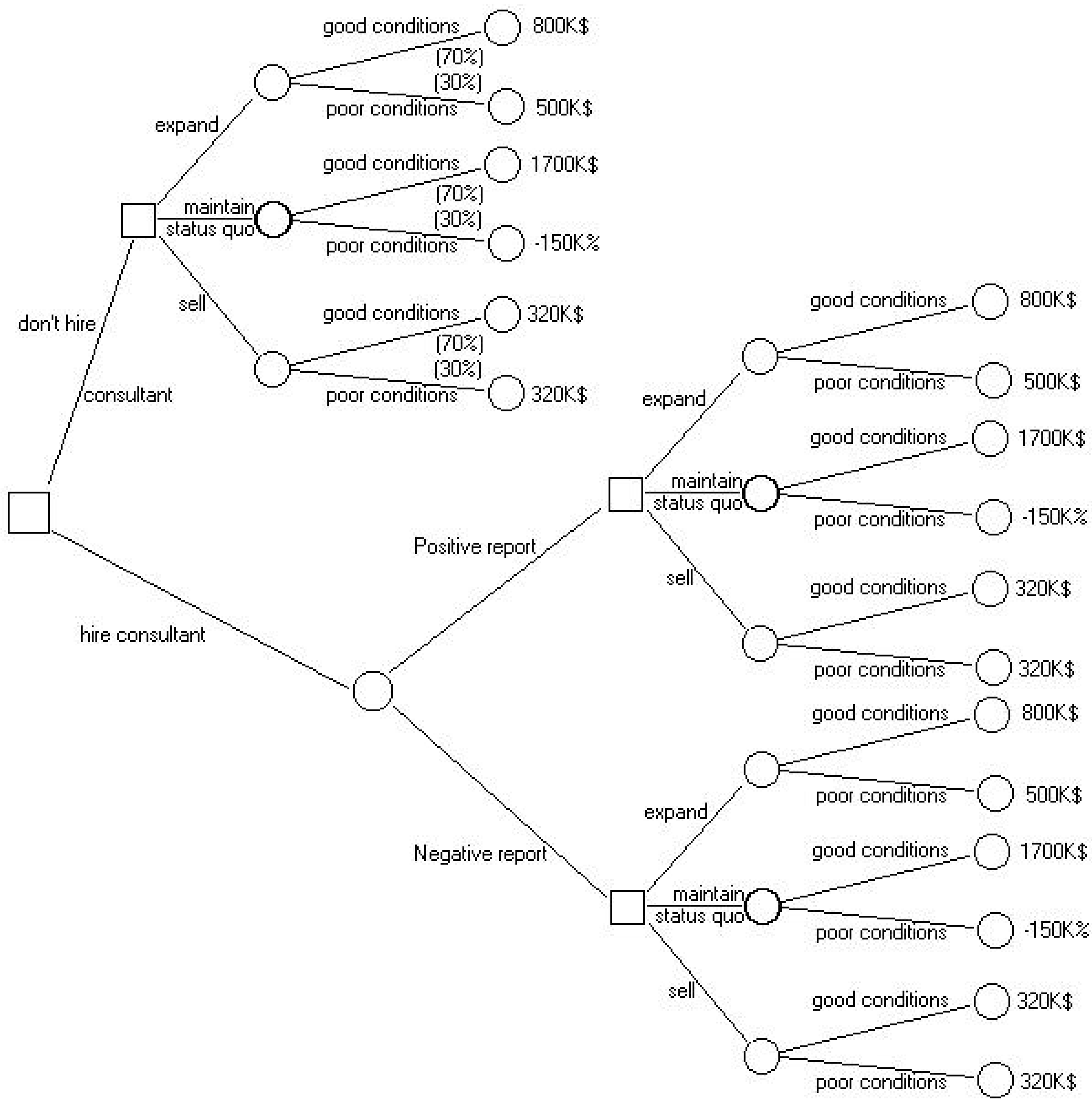
Puckett has hired a consulting firm to provide a report on future political and market situations. The report will be positive (P) or negative (N), indicating either a good (g) or poor (p) future foreign competitive situation. The conditional probability of each report outcome given each state of nature is

P{P|g} = 70%
P{N|g} = 30%
P{P|p} = 20%
P{N|p} = 80%

(e.) Determine the posterior probabilities using Bayes' rule:

P{g| P} = _______ %
P{p| P} = _______ %
P{g| N} = _______ %
P{p| N} = _______ %

(f.) Perform a decision tree analysis using the posterior probabilities that you have just computed.


good conditions
800K$
(70%)
(30%)
poor conditions
500K$
expand
good conditions
1700K$
maintain
status quo
(70%)
(30%)
poor conditions
-150K%
sell
good conditions
320K$
(70%)
(30%)
poor conditions
320K$
don't hire
consultant
hire consultant
Positive report
Negative report
good conditions
800K$
poor conditions
500K$
expand
good conditions
1700K$
maintain
status quo
poor conditions
-150K%
sell
good conditions
320K$
poor conditions
320K$
good conditions
800K$
poor conditions
500K$
expand
good conditions
1700K$
maintain
status quo
poor conditions
-150K%
sell
good conditions
320K$
poor conditions
320K$

56:171 Operations Research
Homework #6 - Due Friday, October 22, 1999

1. **Integer Programming Formulation** *(#5, page 547, of O.R. text by W. Winston)* The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS). The cost of signing each pitcher and the number of victories each pitcher will add to the Cubs are shown below.

Pitcher	Cost of signing ($million)	Right- or Left-handed?	Victories added to Cubs
RS	$6	Right	6
BS	$4	Right	5
DE	$3	Right	3
ST	$2	Left	3
TS	$2	Right	2

Subject to the following restrictions, the Cubs want to sign the pitchers who will add the most victories to the team.

- ❑ At most $12 can be spent.
- ❑ If DE and ST are signed, then BS cannot be signed.
- ❑ At most two right-handed pitchers can be signed.
- ❑ The cubs cannot sign both BS and RS.

Formulate an integer LP to help the Cubs determine whom they should sign. Solve the problem, using LINDO (or equivalent) software.

2. **Integer Programming Formulation.** *(#4, p. 547, O.R. text, W. Winston)* A court decision has stated that the enrollment of each high school in Metropolis must be at least 20% black. The numbers of black and white high school students in each of the city's five school districts are shown in the table below.

District	White students	Black students
1	80	30
2	70	5
3	90	10
4	50	40
5	60	30

The distance (in miles) that a student in each district must travel to each high school is:

District	HS #1	HS #2
1	1	2
2	0.5	1.7
3	0.8	0.8
4	1.3	0.4
5	1.5	0.6

School board policy requires that all the students in a given district attend the same school. Assuming that each school must have an enrollment of at least 150 students, formulate an integer LP that will minimize the total distance that Metropolis students must travel to high school. Find the solution, using LINDO (or equivalent) software.

3. Assignment Problem (#2, p. 387, O.R. text, W. Winston) Five workers are available to perform four jobs. The time (in hours) it takes each worker to perform each job is given in the table below:

Worker #	Job 1	Job 2	Job 3	Job 4
1	10	15	10	15
2	12	8	20	16
3	12	9	12	18
4	6	12	15	18
5	16	12	8	12

The goal is to assign workers to jobs so as to minimize the total time required to perform the four jobs. Use the Hungarian method to solve the problem.

56:171 Operations Research
Homework #7 - Due Friday, October 29, 1999

1. Integer LP Formulation [Exercise 35, §9-2, page 499 of Winston text] A power plant has 3 boilers. If a given boiler is operated, it can be used to produce a quantity of steam (in tons) between the minimum and maximum amounts given in the table below:

Boiler #	Minimum steam (T.)	Maximum steam (T.)	Cost/ton ($)
1	500	1000	10
2	300	900	8
3	400	800	6

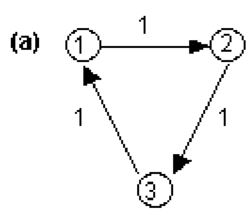
Steam from the boilers is used to produce power on three turbines. If operated, each turbine can process an amount of steam (in tons) between the minimum and maximum given below. (The cost of processing a ton of steam and the power produced by each turbine is also given.)

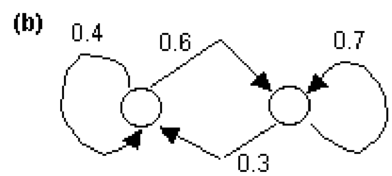
Turbine number	Minimum steam input	Maximum steam input	Kwh per ton of steam	Processing cost ($) per ton of steam
1	300	600	4	2
2	500	800	5	3
3	600	900	6	4

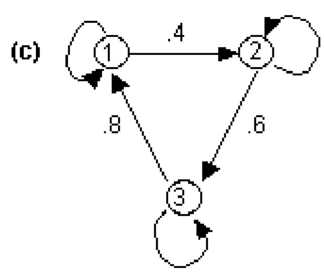
a. Formulate an integer linear programming model that can be used to minimize the cost of producing 8000 kwh of power. The model should specify
- which boilers are to be used to produce steam
- the steam to be produced by each boiler
- which turbines are to be used to generate the power
- the steam to be processed by each turbine

b. Use LINDO to solve the problem.

2. Discrete-time Markov chains Write the complete transition probability matrices for the Markov chains having the following diagrams. (Note that in (c), some of the transition probabilities are missing in the diagram.)







3. Discrete-time Markov chains Let X_n denote the quality of the n^{th} item produced by a production system, with $X_n=1$ meaning "good" and $X_n=2$ meaning "defective". Suppose that $\{X_n: n=0,1,2,...\}$ is a Markov chain whose transition probability matrix is

$$P = \begin{bmatrix} 0.99 & .01 \\ .15 & .85 \end{bmatrix}$$

That is, if the previous item was "good", the probability of producing a defective item is 1%, but if the previous item was defective, there is an 85% probability that the next item will also be defective.

a. Draw and label the transition diagram for this Markov chain.

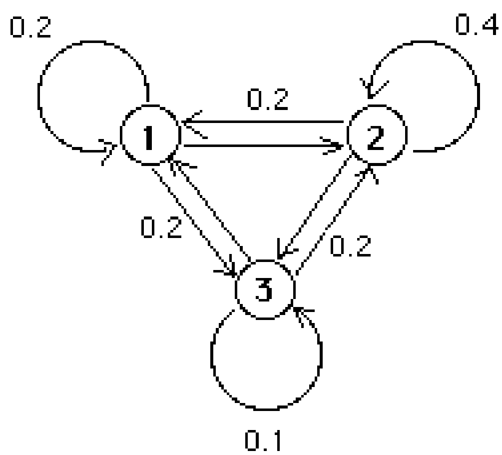
b. What is the probability that, if the first item is good, the second is defective?

c. What is the probability that, if the first item is defective, the second is defective?

d. What is the probability that, if the first two items are defective, the third is defective?

e. What is the probability that, if the first item is good, the third is defective?

(c.)



56:171 Operations Research
Homework #8 – Due Wednesday, 3 November, 1999

1. Markov Chains. *(Based upon Exercise 4, §19.5, page 982 of text by W. Winston)*
At the beginning of each year, my car is in good, fair, or broken-down condition. A good car will be good at the beginning of next year with probability 85%, fair with probability 10%, or broken-down with probability 5%. A fair car will be fair at the beginning of the next year with probability 75%, or broken-down with probability 25%. It costs \$9000 to purchase a good car; a fair car can be traded in for \$2500; and a broken-down car has no trade-in value and must immediately be replaced by a good car. It costs \$1000 per year to operate a good car and \$1500 to operate a fair car. Should I replace my car as soon as it becomes a fair car, or should I drive my car until it breaks down? Assume that the cost of operating a car during a year depends on the type of car on hand at the *beginning* of the year (after a new car, if any, arrives).

Define a Markov chain model with three states (Good, Fair, & Broken-down). Assume, as implied by the problem statement, that break-down occurs only at the *end* of a year, and then (at the beginning of the next year) the broken-down car "must immediately be replaced". For each of the two replacement policies mentioned, answer the following:

a. Draw a diagram of the Markov chain and write down the transition probability matrix.

b. Write down the equations which could be solved to obtain the steadystate probabilities.

c. Solve the equations, either manually or using appropriate computer software.

d. Compute the average cost per year for the replacement policy.

e. What is the expected time between break-downs?

f. What replacement policy do you recommend?

Note: assume that ***state 1= Good, state 2= Fair, and state 3= Broken-down****.*

56:171 Operations Research
Homework #9 – Due Wednesday, 10 November, 1999

1. ***Manufacturing System with Inspection & Rework***: Consider a system in which there are three machining operations, each followed by an inspection. Relevant data are:

OPERATION	TIME RQMT. (man-hrs)	OPERATING COST ($/hr.)	SCRAP RATE %	%SENT BACK FOR REWORK
Machine A	1.5	20.00	12	
Inspection A	0.25	8.00	4	9
Machine B	1.0	16.00	5	
Inspection B	0.25	8.00	3	6
Machine C	1.5	20.00	4	
Inspection C	.5	8.00	5	8
Pack & Ship	0.25	8.00		

The raw materials (blanks) cost $75.00 per part, and scrap value recovered is $10.00 per part. An order for 25 completed parts must be filled.

(a.) Sketch the diagram for a Markov chain model of this system. Identify the absorbing states.

(b.) What percent of the parts which are started are successfully completed?

(c.) What is the **expected** number of blanks which are required to fill the order for 25 parts?

(d.) Suppose that we machine 5 more than the expected number of required blanks which you found in part (c) (rounded up to the next integer). What is the expected number of successfully completed parts? What is the standard deviation of the number of successfully completed parts*? (Hint: the number of successfully completed parts will have the binomial distribution. What are the mean & standard deviation of the binomial distribution?)*

(e.) Assuming that the probability distribution of the number of successfully completed parts in (d) is *approximately* normal, with the mean and standard deviation which you have computed, consult tables for the normal distribution to estimate the probability that **at least** 25 parts are successfully completed?

(f.) What is the probability that a part which passes inspection B will ultimately be scrapped?

(g.) What are the estimated man-hour requirements on Machine C to complete this order?

(h.) What is the expected total of materials cost and operations cost, minus scrap value recovered, per completed part?

See the attached computer output analyzing this Markov chain to answer the questions above.

Transition Probability Matrix

from	1	2	3	4	5	6	7	8
1	0	0.85	0	0	0	0	0	0.15
2	0.08	0	0.88	0	0	0	0	0.04
3	0	0	0	0.94	0	0	0	0.06
4	0	0	0.05	0	0.91	0	0	0.04
5	0	0	0	0	0	0.95	0	0.05
6	0	0	0	0	0.09	0	0.84	0.07
7	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	1

A = Absorption Probabilities

from	7	8
1	0.62861	0.37139
2	0.739541	0.260459
3	0.783241	0.216759
4	0.833235	0.166765
5	0.872608	0.127392
6	0.918535	0.0814653

E = Expected No. Visits to Transient States

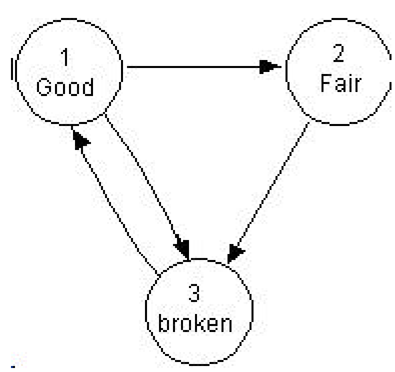
from	1	2	3	4	5	6
1	1.07296	0.912017	0.842156	0.791627	0.787732	0.748345
2	0.0858369	1.07296	0.990772	0.931326	0.926743	0.880406
3	0	0	1.04932	0.986359	0.981505	0.93243
4	0	0	0.0524659	1.04932	1.04415	0.991947
5	0	0	0	0	1.09349	1.03882
6	0	0	0	0	0.0984144	1.09349

2. Continuous-time Markov Chains. Consider the replacement problem in Homework #8:

At the beginning of each year, my car is in good, fair, or broken-down condition.

- A good car will be good at the beginning of next year with probability 85%, fair with probability 10%, or broken-down with probability 5%.
- A fair car will be fair at the beginning of the next year with probability 75%, or broken-down with probability 25%.

It costs $9000 to purchase a good car; a fair car can be traded in for $2500; and a broken-down car has no trade-in value and must immediately be replaced by a good car. It costs $1000 per year to operate a good car and $1500 to operate a fair car. Should I replace my car as soon as it becomes a fair car, or should I drive my car until it breaks down? Assume that the cost of operating a car during a year depends on the type of car on hand at the *beginning* of the year (after a new car, if any, arrives).



In Homework #8, the Markov chain model assumed that break-down occurs only at the *end* of a year, and then (at the beginning of the next year) the broken-down car "must immediately be replaced" with a car in good condition. In fact, of course, the change in condition can occur at any time during the year, and a continuous-time Markov chain model would be a closer representation of reality. Let's assume that when my car breaks down, it takes me an average of 0.02 years (about 1 week) to find and purchase a replacement car (and that this delay has exponential distribution.) Again define a Markov chain model with three states (Good, Fair, & Brokendown).

a. What should be the transition rates, so that the probability of a change of condition during a one-year period is in agreement with the probabilities given in Homework #8?

> *Hint: The cdf of the exponential distribution is*
>
> $$F(t) = P\left\{\text{time to next event } \leq t\right\} = 1 - e^{-\lambda t}$$
>
> *Since there is a 15% probability that the system has changed states during the next year, the transition rate λ_1 should therefore satisfy*
>
> $$F(1) = 1 - e^{-\lambda} = 0.15$$
>
> *The value of λ_{12} should be twice the value of λ_{13} (since the transition probabilities p_{12} and p_{13} were 10% and 5%, respectively), and $\lambda_1 = \lambda_{12} + \lambda_{13}$, so $\lambda_{12} = 2/3\ \lambda_1$ and $\lambda_{13} = 1/3\ \lambda_1$. To get the transition rate λ_{31}, observe that the expected value of the length of time required to replace my broken-down car is $1/\lambda_{31} = 0.02$ years.*

b. Write the matrix of transition rates.

c. Write the set of equations that must be solved for a steadystate distribution.

d. Find the steadystate distribution.

e. What does this model predict will be my average operating cost/year (not including replacement costs)?

To compute the average replacement costs per year is not quite so simple. (We must multiply the replacement costs by the expected number of replacements/year, not by π_3 (the fraction of the year spent in state 3). Let T = average time between replacements. Then

$$\pi_3 = \frac{\text{average time from breakdown to replacement}}{\text{average length of time between replacements}} = \frac{0.02 \text{ year}}{T}$$

What then is T? The number of replacements per year should then be 1/T.

f. What average replacement cost per year is predicted by this model?

56:171 Operations Research
Homework #10 – Due Wednesday, November 17, 1999

1. Barges arrive at the La Crosse lock on the Mississippi River at an average rate of one every 1.5 hours. Assume a Poisson (memoryless) arrival process, i.e., time between arrivals has exponential distribution. It requires an average of 30 minutes to move a barge through the lock.
Assuming that the arrival process and service process are memoryless (i.e., the time between and the time to move the barge through the lock have exponential distributions), find:

a. The average number of barges in the system, i.e., either using or waiting to use the lock.
b. The average time spent by a barge at the lock.
c. The fraction of the time that the lock is busy.
d. The standard deviation of the time to move a barge through the lock.

2. In a particular manufacturing cell, one repairman has the responsibility of maintaining four machines. For the machines, the time between breakdowns is exponentially distributed with an average of 4 hours. On the average, it takes half an hour to fix a machine (exponentially distributed).

a. What is the steady-state probability distribution of the number of machines which are broken down?
b. What fraction of the time will the repairman be busy?
c. What is the average number of machines in need of repair (including those in the process of being repaired)?
d. What is the average time between a machine breakdown and that machine being restored to operating condition?

3. Customers arrive at a service center with two servers at the rate 10/hour. Average time for serving a customer is 10 minutes (exponentially distributed). Compare the average customer waiting times and the server utilization of two alternative systems:

a. Each server has a queue, and customers are equally likely to enter either queue (so that in effect, the arrival rate for each queue is 5/hour).
b. All customers enter the same queue, and a server selects the next customer to be served from the head of this queue.

Note that in (a), it is assumed that customers may not jump from one queue to the other if the other server becomes available!

56:171 Operations Research
Homework #11 – Due Wednesday, December 1, 1999

1. ***(Extension of example presented in class)*** Suppose that a new car costs $10,000 and that the annual operating cost & trade-in value are as follows

Age of car (years)	Trade-in value	Operating cost in previous year
1	$7000	$300
2	$6000	$500
3	$4000	$800
4	$3000	$1200
5	$2000	$1600
6 or more	$1000	$2200

Starting with a new car, what is the replacement policy that minimizes the net cost of owning and operating a car for the next ten years? (Do not include the cost of the initial car.)

As in the class notes, define:
G(t) = minimum total cost incurred from time t until the end of the planning period, if a new car has just been purchased. *(Note: this does not include the cost of purchasing this initial new car.)*
X*(t) = optimal replacement time for a car which has been purchased at the beginning of period t.

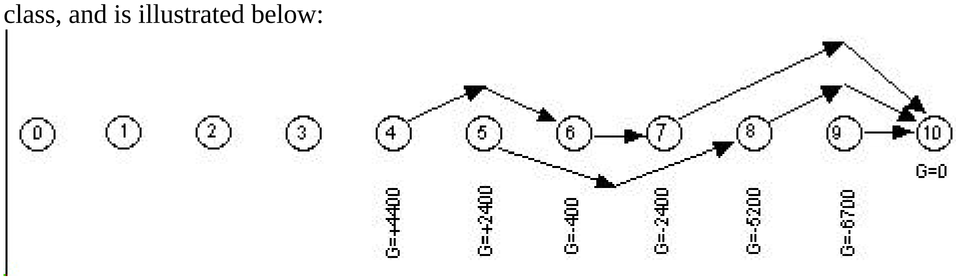
The optimal value function G(t) is defined recursively by

$$G(t) = \underset{t+1 \le x \le T}{\text{minimum}} \left\{ \sum_{i=1}^{x-t} C_i - S_{x-t} + P_x + G(x) \right\}$$

where

P_t = purchase price of a new car at time t
C_i = cost of operation & maintenance of a car in its ith year.
S_j = trade-in value of a car of age j

The computation of G(4) through G(10), i.e., for the final 6 years, was done in the example presented in class, and is illustrated below:



That is, if a new car has been obtained at t=4, the minimum total cost of owning a car until t=10 will be $4400, and the optimal decision would be to keep the car for 2 years before trading it in (4→6), then keeping the next car for 1 year (6→7), and keeping that car for 3 years (7→10).

Complete the computation in order to compute G(0). What are the optimal times at which cars are to be replaced?

2. ***Optimal Reliability by means of redundancy.*** A system consists of six components, each of which is necessary for the operation of the system. The weight and the reliability of each component, i.e., the probability that the component survives for the designed lifetime, is shown in the table below:

Component	Weight (kg)	Reliability (%)
1	1	80
2	2	75
3	1	90
4	3	85
5	2	80
6	2	90

The total weight of the system is to be no more than 18 kg. How many redundant units of each component should be included in order to maximize the reliability of the system?

As in the class notes, the problem is solved by dynamic programming, where the stages correspond to the components, the state of the system is the available capacity, and the decision at a stage is the number of units of the corresponding component to be included. We impose a sequential decision structure, in which component 6 is first to be considered (at which time the state of the system is 18), then component 5, followed by component 4, etc. The optimal value function

$f_n(s)$ = maximum reliability that can be obtained for the subsystem consisting of components n, n-1, n-2, … 2, 1 if the weight allowed for these components is s.

where

$f_0(s) = 100\%$

and we wish to determine $f_6(18)$. The maximum number of units of a component has been arbitrarily set a the value 4, i.e., the possible decisions are x=1, 2, 3, or 4. The tables below show the computations. *(Note that values of -¥ in the table indicate that the corresponding combination of state and decision aren't feasible—for example, if at stage 1 only 2 kg of capacity remains to be filled, then since the weight of component is 1 kg, it would be infeasible to include more than 2 units of this component.)*

---Stage 1---

s \ x:	1	2	3	4
1	0.8000	$-\infty$	$-\infty$	$-\infty$
2	0.8000	0.9600	$-\infty$	$-\infty$
3	0.8000	0.9600	0.9920	¯999.9999
4	0.8000	0.9600	0.9920	0.9984
5	0.8000	0.9600	0.9920	0.9984
6	0.8000	0.9600	0.9920	0.9984
7	0.8000	0.9600	0.9920	0.9984
8	0.8000	0.9600	0.9920	0.9984
9	0.8000	0.9600	0.9920	0.9984
10	0.8000	0.9600	0.9920	0.9984
11	0.8000	0.9600	0.9920	0.9984
12	0.8000	0.9600	0.9920	0.9984
13	0.8000	0.9600	0.9920	0.9984
14	0.8000	0.9600	0.9920	0.9984
15	0.8000	0.9600	0.9920	0.9984
16	0.8000	0.9600	0.9920	0.9984
17	0.8000	0.9600	0.9920	0.9984
18	0.8000	0.9600	0.9920	0.9984

---Stage 2---

s \ x:	1	2	3	4
3	0.6000	$-\infty$	$-\infty$	$-\infty$
4	0.7200	$-\infty$	$-\infty$	$-\infty$
5	0.7440	0.7500	$-\infty$	$-\infty$
6	0.7488	0.9000	$-\infty$	$-\infty$

7	0.7488	0.9300	0.7875	$-\infty$
8	0.7488	0.9360	0.9450	$-\infty$
9	0.7488	0.9360	0.9765	0.7969
10	0.7488	0.9360	0.9828	0.9563
11	0.7488	0.9360	0.9828	0.9881
12	0.7488	0.9360	0.9828	0.9945
13	0.7488	0.9360	0.9828	0.9945
14	0.7488	0.9360	0.9828	0.9945
15	0.7488	0.9360	0.9828	0.9945
16	0.7488	0.9360	0.9828	0.9945
17	0.7488	0.9360	0.9828	0.9945
18	0.7488	0.9360	0.9828	0.9945

---Stage 3---

s \ x:	1	2	3	4
4	0.5400	$-\infty$	$-\infty$	$-\infty$
5	0.6480	0.5940	$-\infty$	$-\infty$
6	0.6750	0.7128	0.5994	$-\infty$
7	0.8100	0.7425	0.7193	0.5999
8	0.8370	0.8910	0.7493	0.7199
9	0.8505	0.9207	0.8991	0.7499
10	0.8789	0.9356	0.9291	0.8999
11	0.8845	0.9667	0.9441	0.9299
12	0.8893	0.9730	0.9755	0.9449
13	0.8951	0.9782	0.9818	0.9764
14	0.8951	0.9846	0.9871	0.9827
15	0.8951	0.9846	0.9935	0.9880
16	0.8951	0.9846	0.9935	0.9944
17	0.8951	0.9846	0.9935	0.9944
18	0.8951	0.9846	0.9935	0.9944

---Stage 4---

s \ x:	1	2	3	4
7	0.4590	$-\infty$	$-\infty$	$-\infty$
8	0.5508	$-\infty$	$-\infty$	$-\infty$
9	0.6059	$-\infty$	$-\infty$	$-\infty$
10	0.6885	0.5279	$-\infty$	$-\infty$
11	0.7573	0.6334	$-\infty$	$-\infty$
12	0.7826	0.6968	$-\infty$	$-\infty$
13	0.7952	0.7918	0.5382	$-\infty$
14	0.8217	0.8710	0.6458	$-\infty$
15	0.8292	0.9000	0.7104	$-\infty$
16	0.8345	0.9145	0.8073	0.5397
17	0.8391	0.9450	0.8880	0.6477
18	0.8445	0.9536	0.9176	0.7124

---Stage 5---

s \ x:	1	2	3	4
9	0.3672	$-\infty$	$-\infty$	$-\infty$
10	0.4406	$-\infty$	$-\infty$	$-\infty$
11	0.4847	0.4406	$-\infty$	$-\infty$
12	0.5508	0.5288	$-\infty$	$-\infty$

13	0.6059	0.5816	0.4553	—∞
14	0.6261	0.6610	0.5464	—∞
15	0.6362	0.7271	0.6010	0.4583
16	0.6968	0.7513	0.6830	0.5499
17	0.7200	0.7634	0.7513	0.6049
18	0.7316	0.8361	0.7763	0.6874

---Stage 6---

s \ x:	1	2	3	4
11	0.3305	—∞	—∞	—∞
12	0.3966	—∞	—∞	—∞
13	0.4362	0.3635	—∞	—∞
14	0.4957	0.4362	—∞	—∞
15	0.5453	0.4799	0.3668	—∞
16	0.5949	0.5453	0.4402	—∞
17	0.6544	0.5998	0.4842	0.3672
18	0.6762	0.6544	??????	0.4406

a. Compute the missing value above, for the combination of s=18, x=3 at stage 6.

Based upon the above computations, the following tables give the maximum reliability for each state in each stage. *For example, at stage 6, if 15 kg of capacity remain, a reliability of 54.53% can be achieved, using 1 unit of component #6, leaving (since the weight of component #6 is 2 kg.) 13 kg of capacity entering the next stage (stage 5).*

Stage 6:

State	Optimal Values	Optimal Decisions	Resulting State
11	0.3305	1	9
12	0.3966	1	10
13	0.4362	1	11
14	0.4957	1	12
15	0.5453	1	13
16	0.5949	1	14
17	0.6544	1	15
18	0.6762	1	16

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
9	0.3672	1	7
10	0.4406	1	8
11	0.4847	1	9
12	0.5508	1	10
13	0.6059	1	11
14	0.6610	2	10
15	0.7271	2	11
16	0.7513	2	12
17	0.7634	2	13
18	0.8361	2	14

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
7	0.4590	1	4
8	0.5508	1	5
9	0.6059	1	6

10	0.6885	1	7
11	0.7573	1	8
12	0.7826	1	9
13	0.7952	1	10
14	0.8710	2	8
15	0.9000	2	9
16	0.9145	2	10
17	0.9450	2	11
18	0.9536	2	12

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
4	0.5400	1	3
5	0.6480	1	4
6	0.7128	2	4
7	0.8100	1	6
8	0.8910	2	6
9	0.9207	2	7
10	0.9356	2	8
11	0.9667	2	9
12	0.9755	3	9
13	0.9818	3	10
14	0.9871	3	11
15	0.9935	3	12
16	0.9944	4	12
17	0.9944	4	13
18	0.9944	4	14

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
3	0.6000	1	1
4	0.7200	1	2
5	0.7500	2	1
6	0.9000	2	2
7	0.9300	2	3
8	0.9450	3	2
9	0.9765	3	3
10	0.9828	3	4
11	0.9881	4	3
12	0.9945	4	4
13	0.9945	4	5
14	0.9945	4	6
15	0.9945	4	7
16	0.9945	4	8
17	0.9945	4	9
18	0.9945	4	10

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
1	0.8000	1	0
2	0.9600	2	0
3	0.9920	3	0
4	0.9984	4	0
5	0.9984	4	1
6	0.9984	4	2
7	0.9984	4	3
8	0.9984	4	4

```
         9   0.9984         4              5
        10   0.9984         4              6
        11   0.9984         4              7
        12   0.9984         4              8
        13   0.9984         4              9
        14   0.9984         4             10
        15   0.9984         4             11
        16   0.9984         4             12
        17   0.9984         4             13
        18   0.9984         4             14
++++++++++++++++++++++++++++++++++++++++++++++++++++
                  Optimal System Reliability Using Redundancy

*** Optimal value is 0.67616208 ***

                             Optimal Solution

                        stage       state       decision
                           6          18           1
                           5          16           2
                           4          12           1
                           3           9           2
                           2           7           2
                           1           3           3
                           0           0
++++++++++++++++++++++++++++++++++++++++++++++++++++
```

Suppose that, instead of 18 kg, the capacity is limited to 17 kg.

b. What is the maximum reliability that can be achieved?

c. Specify the optimal solution, i.e., the number of units of each component to be included.

56:171 Operations Research
HW#12 – Due December 8, 1999

1. ***Production Planning*** We wish to plan production of an expensive, low-demand item for the next three months (January, February, & March).

- the cost of production is \$5 for setup, plus \$5 per unit produced, up to a maximum of 4 units.
- the storage cost for inventory is \$2 per unit, based upon the level at the beginning of the month.
- a maximum of 3 units may be kept in inventory at the end of each month; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each month:

demand d	0	1	2
P{D=d}	0.2	0.5	0.3

- there is a penalty of \$25 per unit for any demand which cannot be satisfied. Backorders are not allowed.
- the initial inventory (i.e., the inventory at the end of December) is 1.
- a salvage value of \$4 per unit is received for any inventory remaining at the end of the last month (March)

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 3 = January, stage 2 = February, etc.** (i.e., n = # months remaining in planning period.)

a. What is the optimal production quantity for January? _______

b. What is the total expected cost for the three months? __________

c. If, during January, the demand is 1 unit, what should be produced in February? ______

d. Three values have been blanked out in the computer output, What are they?

i. the optimal value $f_2(1)$ ___________

ii. the optimal decision $x_2^*(1)$ ___________

iii. the cost associated with the decision to produce 1 unit in February when the inventory is 0 at the end of January. ____________

---Stage 1 (March)---

s \ x:	0	1	2	3	4
0	27.50	16.70	11.40	12.40	14.20
1	8.70	8.40	9.40	11.20	15.00
2	−0.40	6.40	8.20	12.00	17.00
3	−1.60	5.20	9.00	14.00	19.00

State	Optimal Values	Optimal Decision
0	11.40	2
1	8.40	1
2	−0.40	0
3	−1.60	0

---Stage 2 (February)---

s \ x:	0	1	2	3	4
0	38.90	$\boxed{?????}$	22.70	22.40	24.00
1	20.30	19.70	19.40	21.00	25.40
2	11.70	16.40	18.00	22.40	27.40
3	8.40	15.00	19.40	24.40	29.40

State	Optimal Values	Optimal Decision
0	22.40	3
1	$\boxed{????}$	$\boxed{??}$
2	11.70	0
3	8.40	0

---Stage 3 (January)---

s \ x:	0	1	2	3	4
0	49.90	39.30	33.76	33.35	34.39
1	31.30	30.76	30.35	31.39	35.40
2	22.76	27.35	28.39	32.40	37.40
3	19.35	25.39	29.40	34.40	39.40

State	Optimal Values	Optimal Decision
0	33.35	3
1	30.35	2
2	22.76	0
3	19.35	0

2. ***Markov chains*** The Green Valley Christmas Tree Farm owns a plot of land with 5000 evergreen trees. Each year they allow individuals to select and cut Christmas trees. However, they protect small trees (usually less than 4 feet tall) so that they will grow and be available for sale in future years. Currently 1500 trees are classified as protected trees, while the remaining 3500 are available for cutting. However, even though a tree is available for cutting in a given year, it possibly might not be selected for cutting until future years. While most trees not cut in a given year live until the next year (protected or unprotected), approximately 20% are lost to disease. Each year, approximately 50% of the unprotected trees are cut, and 35% of the protected trees surviving from the previous year have matured sufficiently to be made available for cutting.

(a.) Define a Markov chain model of the system consisting of a **single** tree. Sketch the transition diagram and write down the probability matrix.

(b.) What are the absorbing states of this model?

(c.) What is the probability that a tree which is protected is eventually sold? that it eventually dies of disease?

(d.) How many of the farm's 5000 trees are expected to be sold eventually, and how many will be lost to disease?

(e.) If a tree is initially protected, what is the expected number of years until it is either sold or dies?