56:171
Operations Research
Homework Fall 2000

Dennis L. Bricker
Dept. of Industrial Engineering
The University of Iowa

56:171 Operations Research
Homework #1 - Due Wednesday, August 30, 2000

In each case below, you must formulate a linear programming model that will solve the problem. Be sure to define the meaning of your variables! Then use LINDO (or other appropriate software) to find the optimal solution. State the optimal objective value, and describe in "layman's terms" the optimal decisions.

1. Walnut Orchard has two farms that grow wheat and corn. Because of differing soil conditions, there are differences in the yields and costs of growing crops on the two farms. The yields and costs are shown in the table below. Farm #1 has 100 acres available for cultivation, while Farm #2 has 150 acres. The farm has contracted to grow 11,000 bushels of corn and 6000 bushels of wheat. Determine a planting plan that will minimize the cost of meeting these contracts.

	Farm #1	Farm #2
Corn yield/acre	100 bushels	120 bushels
Cost/acre of corn	$90	$115
Wheat yield/acre	40 bushels	35 bushels
Cost/acre of wheat	$90	$80

Note: We are assuming that the costs and yields are known with certainty, which is not the case in the "real world"!

2. A firm manufactures chicken feed by mixing three different ingredients. Each ingredient contains four key nutrients: protein, fat, vitamin A, and vitamin B. The amount of each nutrient contained in 1 kilogram of the three basic ingredients is summarized in the table below:

Ingredient	Protein (grams)	Fat (grams)	Vitamin A (units)	Vitamin B (units)
1	25	11	235	12
2	45	10	160	6
3	32	7	190	10

The costs per kg of Ingredients 1, 2, and 3 are $0.55, $0.42, and $0.38, respectively. Each kg of the feed must contain at least 35 grams of protein, a minimum of 8 grams (and a maximum of 10 grams) of fat, at least 200 units of vitamin A and at least 10 units of vitamin B. Formulate an LP model for finding the feed mix that has the minimum cost per kg.
--*revised 8/28/00*

3. "Mama's Kitchen" serves from 5:30 a.m. each morning until 1:30 p.m. in the afternoon. Tables are set and cleared by busers working 4-hour shifts beginning on the hour from 5:00 a.m. through 10:00 a.m. Most are college students who hate to get up in the morning, so Mama's pays $9 per hour for the 5:00, 6:00, and 7:00 a.m. shifts, and $7.50 per hour for the others. (That is, a person works a shift consisting of 4 consecutive hours, with the wages equal to 4x$9 for the three early shifts, and 4x$7.50 for the 3 later shifts.) The manager seeks a minimum cost staffing plan that will have at least the number of busers on duty each hour as specified below:

	5 am	6 am	7 am	8 am	9 am	10am	11am	Noon	1 pm
#reqd	2	3	5	5	3	2	4	6	3

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Homework #2 -- Due Wednesday, Sept. 6

The Diet Problem. "The goal of the diet problem is to find the cheapest combination of foods that will satisfy all the daily nutritional requirements of a person." Go to the URL:

http://www-fp.mcs.anl.gov/otc/Guide/CaseStudies/diet/index.html

and click on *"Give it a try."* Then on the next page select *"Edit the constraints"* and click on *"Go on"*.

a. What are the restrictions on calories in the default set of requirements?

Go back to the previous page, where approximately 100 foods are listed for your selection. Choose "Default requirements", and select 15 foods which you think would provide an economical menu meeting the requirements. Then click on "Go on" again.

b. What is the minimum-cost menu meeting the nutritional requirements using the foods you indicated? Indicate the solution in the left 2 columns of the table below.

Change the default upper limit on calories to 1500/day and solve the problem again. (Be sure that the lower bound ≤ upper bound!)

c. What is the minimum-cost menu meeting the nutritional requirements using the foods you indicated? Indicate the solution in the right 2 columns of the table below.

Quantity (# servings)	Cost	Food (& serving size)	Quantity (# servings)	Cost
		1.		
		2.		
		3.		
		4.		
		5.		
		6.		
		7.		
		8.		
		9.		
		10.		
		11.		
		12.		
		13.		
		14.		
		15.		
Total Cost:	$	<><><><><><><><>	Total Cost:	$

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Homework #3 -- Due Wednesday, Sept. 13

1. Simplex Algorithm: Use the simplex algorithm to find the optimal solution to the following LP:

$$\text{Maximize } z = 4x_1 + x_2$$

$$\text{subject to } \begin{cases} 2x_1 + x_2 \le 9 \\ x_2 \le 5 \\ x_1 - x_2 \le 4 \\ x_1 \ge 0, x_2 \ge 0 \end{cases}$$

Show the initial tableau, each intermediate tableau, and the final tableau. Explain how you have decided on the location of each pivot and how you have decided to stop at the final tableau.

2. Below are several ***simplex tableaus***. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. *Also answer the question accompanying each classification, if any.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*

(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element. Would the objective improve with this pivot?*

(**C**) Unique nondegenerate optimum.

(**D**) Optimal tableau, with alternate optimum. *State the values of the basic variables. Circle a pivot element which would lead to another optimal basic solution. Which variable will enter the basis, and at what value?*

(**E**) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible basic solution.

Warning: Some of these classifications might be used for more than one tableau, while others might not be used at all!

(i) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	-3	0	1	1	0	0	2	3	-45	
0	0	0	-4	0	0	1	0	0	9	______
0	-6	0	3	-2	1	0	2	3	5	
0	4	1	2	-5	0	0	1	1	8	

(ii) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	-2	-45	
0	0	0	-4	0	0	1	3	0	9	________
0	-4	1	2	-5	0	0	-2	1	0	
0	-6	0	3	-2	1	0	-4	3	5	

(iii)-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	1	0	0	3	5	-45	
0	0	0	-4	0	0	1	3	0	3	________
0	4	1	2	-5	0	0	2	1	7	
0	-6	0	3	-2	1	0	-4	3	15	

(iv)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	-3	0	0	2	0	-45	
0	0	0	-1	0	0	1	3	0	9	________
0	4	1	-4	-5	0	0	2	1	3	
0	-6	0	3	-2	1	0	-4	3	5	

(v)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	0	1	0	0	0	12	-45	
0	0	0	-4	0	0	1	3	0	9	______
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	5	

(vi)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	________
0	-6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

(vii)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	1	0	0	-2	0	-45	
0	4	1	2	-5	0	0	2	1	5	________
0	-6	0	3	2	1	0	-4	3	0	
0	0	0	-4	0	0	1	3	0	9	

(viii)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	2	0	-1	3	0	0	2	0	-45	
0	0	0	-4	0	0	1	3	0	9	________
0	6	0	3	-2	1	0	-4	3	5	
0	4	1	2	-5	0	0	2	1	8	

(ix)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	4	0	0	-2	2	-45	
0	0	0	-4	0	0	1	-3	0	3	________
0	4	1	2	-5	0	0	2	1	-8	
0	-6	0	3	-2	1	0	-4	3	15	

3. ***LP Model Formulation*** (from *Operations Research*, by W. Winston (3rd edition), page 191): Carco uses robots to manufacture cars. The following demands for cars must be met (not necessarily on time, but all demands must be met by end of quarter 4):

Quarter #	1	2	3	4
Demand	600	800	500	400

At the beginning of the first quarter, Carco has two robots. Robots can be purchased at the beginning of each quarter, but a maximum of two per quarter can be purchased. Each robot can build up to 200 cars per quarter. It costs $5000 to purchase a robot. Each quarter, a robot incurs $500 in maintenance costs (even if it is not being used to build any cars). Robots can also be sold at the beginning of each quarter for $3000. At the end of each quarter, a holding cost of $200 for each car in inventory is incurred. If any demand is backlogged, a cost of $300 per car is incurred for each quarter the customer must wait. At the end of quarter 4, Carco must have at least two robots.

a. Formulate an LP to minimize the total cost incurred in meeting the next four quarters' demands for cars. Be sure to define your variables (including units) clearly! (Ignore any integer restrictions.)

b. Use *LINDO* (or other LP solver) to find the optimal solution and describe it briefly in "plain English". Are integer numbers of robots bought & sold?

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Homework #4 -- Due Wednesday, Sept. 20

1. ***LP Duality:*** Write the dual of the following LP:

$$Min\ 3x_1 + 2x_2 - 4x_3$$

$$\text{subject to } \begin{cases} 5x_1 - 7x_2 + x_3 \ge 12 \\ x_1 - x_2 + 2x_3 = 18 \\ 2x_1 - x_3 \le 6 \\ x_2 + 2x_3 \ge 10 \\ x_j \ge 0,\ j=1,2,3 \end{cases}$$

2. Consider the following primal LP problem:

$$Max\ x_1 + 2x_2 - 9x_3 + 8x_4 - 36x_5$$

$$\text{subject to } \begin{cases} 2x_2 - x_3 + x_4 - 3x_5 \le 40 \\ x_1 - x_2 + 2x_4 - 2x_5 \le 10 \\ x_j \ge 0,\ j=1,2,3,4,5 \end{cases}$$

   a. Write the dual LP problem
   b. Sketch the feasible region of the dual LP in 2 dimensions, and use it to find the optimal solution.
   c. Using complementary slackness conditions,
      - write equations which must be satisfied by the optimal primal solution x*
      - which primal variables must be zero?
   d. Using the information in (c.), determine the optimal solution x*.

3. ***Sensitivity Analysis*** (based on LP model Homework #3 from *Operations Research*, by W. Winston (3rd edition), page 191): Carco uses robots to manufacture cars. The following demands for cars must be met (not necessarily on time, but all demands must be met by end of quarter 4):

Quarter #	1	2	3	4
Demand	600	800	500	400

At the beginning of the first quarter, Carco has two robots. Robots can be purchased at the beginning of each quarter, but a maximum of two per quarter can be purchased. Each robot can build up to 200 cars per quarter. It costs $5000 to purchase a robot. Each quarter, a robot incurs $500 in maintenance costs (even if it is not being used to build any cars). Robots can also be sold at the beginning of each quarter for $3000. At the end of each quarter, a holding cost of $200 for each car in inventory is incurred. If any demand is backlogged, a cost of $300 per car is incurred for each quarter the customer must wait. At the end of quarter 4, Carco must have at least two robots.

Decision Variables :

Rt : robots available during quarter t (after robots are bought or sold for the quarter)
Bt : robots bought during quarter t
St : robots sold during quarter t
It : cars in inventory at end of quarter t
Ct : cars produced during quarter t
Dt : backlogged demand for cars at end of quarter t

Using the LINDO output below, answer the following questions:

a. During the first quarter, a one-time offer of 20% discount on robots is offered. Will this change the optimal solution shown below?
b. In the optimal solution, is any demand backlogged?

c. Suppose that the penalty for backlogging demand is $250 per month instead of $300. Will this change the optimal solution? *Note: this change applies to all quarters simultaneously!*

d. If the demand in quarter #3 were to increase by 100 cars, what would be the change in the objective function?

e. Suppose that we know in advance that demand for 10 cars must be backlogged in quarter #2. Using the substitution rates found in the tableau, describe how this would change the optimal solution.

```
MIN      500 R1 + 500 R2 + 500 R3 + 500 R4 + 200 I1 + 200 I2 + 200 I3
     + 200 I4 + 5000 B1 + 5000 B2 + 5000 B3 + 5000 B4 - 3000 S1 - 3000 S2
     - 3000 S3 - 3000 S4 + 300 D1 + 300 D2 + 300 D3 + 300 D4
SUBJECT TO
        2)   R1 - B1 + S1 =    2
        3) - R1 + R2 - B2 + S2 =    0
        4) - R2 + R3 - B3 + S3 =    0
        5) - R3 + R4 - B4 + S4 =    0
        6)   I1 - D1 - C1 =  - 600
        7) - I1 + I2 + D1 - D2 - C2 =  - 800
        8) - I2 + I3 + D2 - D3 - C3 =  - 500
        9) - I3 + I4 + D3 - D4 - C4 =  - 400
       10)   R4 >=   2
       11) - 200 R1 + C1 <=   0
       12) - 200 R2 + C2 <=   0
       13) - 200 R3 + C3 <=   0
       14) - 200 R4 + C4 <=   0
       15)   D4 =    0
END
SLB       R4        2.00000
SUB       B1        2.00000
SUB       B2        2.00000
SUB       B3        2.00000
SUB       B4        2.00000

      OBJECTIVE FUNCTION VALUE
      1)      9750.000

VARIABLE        VALUE          REDUCED COST
      R1         3.000000          0.000000
      R2         4.000000          0.000000
      R3         2.500000          0.000000
      R4         2.000000       3500.000000
      I1         0.000000        190.000000
      I2         0.000000        210.000000
      I3         0.000000        202.500000
      I4         0.000000        200.000000
      B1         1.000000          0.000000
      B2         1.000000          0.000000
      B3         0.000000       2000.000000
      B4         0.000000       2000.000000
      S1         0.000000       2000.000000
      S2         0.000000       2000.000000
      S3         1.500000          0.000000
      S4         0.500000          0.000000
      D1         0.000000        310.000000
      D2         0.000000        290.000000
      D3         0.000000        297.500000
      D4         0.000000        300.000000
      C1       600.000000          0.000000
      C2       800.000000          0.000000
      C3       500.000000          0.000000
      C4       400.000000          0.000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000       5000.000000
      3)         0.000000       5000.000000
      4)         0.000000       3000.000000
      5)         0.000000       3000.000000
      6)         0.000000          2.500000
      7)         0.000000         12.500000
      8)         0.000000          2.500000
      9)         0.000000          0.000000
```

10)	0.000000	0.000000
11)	0.000000	2.500000
12)	0.000000	12.500000
13)	0.000000	2.500000
14)	0.000000	0.000000
15)	0.000000	0.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
R1	500.000000	62000.000000	500.000000
R2	500.000000	38000.000000	2500.000000
R3	500.000000	42000.000000	500.000000
R4	500.000000	INFINITY	3500.000000
I1	200.000000	INFINITY	190.000000
I2	200.000000	INFINITY	210.000000
I3	200.000000	INFINITY	202.500000
I4	200.000000	INFINITY	200.000000
B1	5000.000000	62000.000000	500.000000
B2	5000.000000	500.000000	2000.000000
B3	5000.000000	INFINITY	2000.000000
B4	5000.000000	INFINITY	2000.000000
S1	-3000.000000	INFINITY	2000.000000
S2	-3000.000000	INFINITY	2000.000000
S3	-3000.000000	500.000000	2000.000000
S4	-3000.000000	3500.000000	500.000000
D1	300.000000	INFINITY	310.000000
D2	300.000000	INFINITY	290.000000
D3	300.000000	INFINITY	297.500000
D4	300.000000	INFINITY	300.000000
C1	0.000000	310.000000	190.000000
C2	0.000000	190.000000	210.000000
C3	0.000000	210.000000	202.500000
C4	0.000000	0.000000	17.500000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	2.000000	1.000000	1.000000
3	0.000000	1.000000	1.000000
4	0.000000	INFINITY	1.500000
5	0.000000	INFINITY	0.500000
6	-600.000000	200.000000	200.000000
7	-800.000000	200.000000	200.000000
8	-500.000000	100.000000	300.000000
9	-400.000000	0.000000	0.000000
10	2.000000	0.000000	INFINITY
11	0.000000	200.000000	200.000000
12	0.000000	200.000000	200.000000
13	0.000000	100.000000	300.000000
14	0.000000	0.000000	0.000000
15	0.000000	0.000000	0.000000

THE TABLEAU

ROW	(BASIS)	R1	R2	R3	R4	I1	I2
1	ART	0.000	0.000	0.000	3500.000	190.000	210.000
2	R1	1.000	0.000	0.000	0.000	-0.005	0.000
3	R2	0.000	1.000	0.000	0.000	0.005	-0.005
4	S3	0.000	0.000	0.000	0.000	0.005	-0.010
5	R3	0.000	0.000	1.000	0.000	0.000	0.005
6	B1	0.000	0.000	0.000	0.000	-0.005	0.000
7	B2	0.000	0.000	0.000	0.000	0.010	-0.005
8	S4	0.000	0.000	0.000	1.000	0.000	0.005
9	ART	0.000	0.000	0.000	-200.000	0.000	0.000
10	SLK 10	0.000	0.000	0.000	-1.000	0.000	0.000
11	C1	0.000	0.000	0.000	0.000	-1.000	0.000
12	C2	0.000	0.000	0.000	0.000	1.000	-1.000
13	C3	0.000	0.000	0.000	0.000	0.000	1.000
14	C4	0.000	0.000	0.000	-200.000	0.000	0.000

15 ART	0.000	0.000	0.000	0.000	0.000	0.000	

ROW	I3	I4	B1	B2	B3	B4	S1
1	202.500	200.000	0.000	0.000	2000.000	2000.000	2000.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.005	0.000	0.000	0.000	-1.000	0.000	0.000
5	-0.005	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	1.000	0.000	0.000	0.000	-1.000
7	0.000	0.000	0.000	1.000	0.000	0.000	0.000
8	-0.005	0.000	0.000	0.000	0.000	-1.000	0.000
9	-1.000	1.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	-1.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000

ROW	S2	S3	S4	D1	D2	D3	D4
1	2000.000	0.000	0.000	310.000	290.000	297.500	300.000
2	0.000	0.000	0.000	0.005	0.000	0.000	0.000
3	0.000	0.000	0.000	-0.005	0.005	0.000	0.000
4	0.000	1.000	0.000	-0.005	0.010	-0.005	0.000
5	0.000	0.000	0.000	0.000	-0.005	0.005	0.000
6	0.000	0.000	0.000	0.005	0.000	0.000	0.000
7	-1.000	0.000	0.000	-0.010	0.005	0.000	0.000
8	0.000	0.000	1.000	0.000	-0.005	0.005	0.000
9	0.000	0.000	0.000	0.000	0.000	1.000	-1.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	1.000	0.000	0.000	0.000
12	0.000	0.000	0.000	-1.000	1.000	0.000	0.000
13	0.000	0.000	0.000	0.000	-1.000	1.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	1.000

ROW	C1	C2	C3	C4	SLK 10	SLK 11	SLK 12
1	0.000	0.000	0.000	0.000	0.000	2.500	12.500
2	0.000	0.000	0.000	0.000	0.000	-0.005	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	-0.005
4	0.000	0.000	0.000	0.000	0.000	0.000	-0.005
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	-0.005	0.000
7	0.000	0.000	0.000	0.000	0.000	0.005	-0.005
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	1.000	0.000	0.000
11	1.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	1.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	1.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	1.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000

ROW	SLK 13	SLK 14	
1	2.500	0.000	-9750.000
2	0.000	0.000	3.000
3	0.000	0.000	4.000
4	0.005	0.000	1.500
5	-0.005	0.000	2.500
6	0.000	0.000	1.000
7	0.000	0.000	1.000
8	-0.005	0.000	0.500
9	0.000	1.000	0.000
10	0.000	0.000	0.000
11	0.000	0.000	600.000
12	0.000	0.000	800.000
13	0.000	0.000	500.000
14	0.000	1.000	400.000
15	0.000	0.000	0.000

56:171 Operations Research
Homework #5 -- Fall 2000

1. Linear Programming sensitivity. A paper recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are:

Input type	Cost $/ton	Pulp content
Box board	5	15%
Tissue paper	6	20%
Newsprint	8	30%
Book paper	10	40%

Two methods, de-inking and asphalt dispersion, can be used to process the four inputs into pulp. It costs $20 to de-ink a ton of any input. The process of de-inking removes 10% of the input's pulp. It costs $15 to apply asphalt dispersion to a ton of material. The asphalt dispersion process removes 20% of the input's pulp. At most 3000 tons of input can be run through the asphalt dispersion process or the de-inking process. Grade 1 paper can only be produced with newsprint or book paper pulp; grade 2 paper, only with book paper, tissue paper, or box board pulp; and grade 3 paper, only with newsprint, tissue paper, or box board pulp. To meet its current demands, the company needs 500 tons of pulp for grade 1 paper, 500 tons of pulp for grade 2 paper, and 600 tons of pulp for grade 3 paper. The LP model below was formulated to minimize the cost of meeting the demands for pulp.

Hint:: *you may wish to define the variables*

BOX = tons of purchased boxboard
TISS = tons of purchased tissue
NEWS = tons of purchased newsprint
BOOK = tons of purchased book paper
BOX1 = tons of boxboard sent through de-inking
TISS1 = tons of tissue sent through de-inking
NEWS1 = tons of newsprint sent through de-inking
BOOK1 = tons of book paper sent through de-inking
BOX2 = tons of boxboard sent through asphalt dispersion
TISS2 = tons of tissue sent through asphalt dispersion
NEWS2 = tons of newsprint sent through asphalt dispersion
BOOK2 = tons of book paper sent through asphalt dispersion
PBOX = tons of pulp recovered from boxboard
PTISS = tons of pulp recovered from tissue
PNEWS= tons of pulp recovered from newsprint
PBOOK = tons of pulp recovered from book paper
PBOX1 = tons of boxboard pulp used for grade 1 paper,
PBOX2 = tons of boxboard pulp used for grade 2 paper, etc.
...
PBOOK3 = tons of book paper pulp used for grade 3 paper.

The LP model using these variables is:

```
MIN  5 BOX +6 TISS +8 NEWS +10 BOOK +20 BOX1 +20 TISS1 +20 NEWS1
          +20 BOOK1 +15 BOX2 +15 TISS2 +15 NEWS2 +15 BOOK2
 SUBJECT TO
```

```
       2) - BOX + BOX1 + BOX2 <=   0
       3) - TISS + TISS1 + TISS2 <=   0
       4) - NEWS + NEWS1 + NEWS2 <=   0
       5) - BOOK + BOOK1 + BOOK2 <=   0
       6)   0.135 BOX1 + 0.12 BOX2 - PBOX =    0
       7)   0.18 TISS1 + 0.16 TISS2 - PTISS =    0
       8)   0.27 NEWS1 + 0.24 NEWS2 - PNEWS =    0
       9)   0.36 BOOK1 + 0.32 BOOK2 - PBOOK =    0
      10) - PBOX + PBOX2 + PBOX3 <=   0
      11) - PTISS + PTISS2 + PTISS3 <=   0
      12) - PNEWS + PNEWS1 + PNEWS3 <=   0
      13) - PBOOK + PBOOK1 + PBOOK2 <=   0
      14)   PNEWS1 + PBOOK1 >=   500
      15)   PBOX2 + PTISS2 + PBOOK2 >=   500
      16)   PBOX3 + PTISS3 + PNEWS3 >=   600
      17)   BOX1 + TISS1 + NEWS1 + BOOK1 <=   3000
      18)   BOX2 + TISS2 + NEWS2 + BOOK2 <=   3000
END
```

- Rows 2-5 state that only the supply of each input material which is purchased can be processed, either by de-inking or asphalt dispersion.
- Row 6 states that the recovered pulp in the boxboard is 90% of that in the boxboard which is processed by de-inking, i.e., (0.90)(0.15)BOX1, since boxboard is 15% pulp, plus 80% of that in the boxboard which is processed by asphalt dispersion, i.e., (0.80)(0.15)BOX2.
- Rows 7-9 are similar to row 6, but for different input materials.
- Rows 10-13 state that no more than the pulp which is recovered from each input may be used in making paper (grades 1, 2, &/or 3). Note that some variables are omitted, e.g., PBOX1, since boxboard cannot be used in Grade 1 paper.
- Rows 14-16 state that the demand for each grade of paper must be satisfied.
- Rows 17-18 state that each process (de-inking & asphalt dispersion) has a maximum throughput of 3000 tons. (The problem statement is unclear here, and might be interpreted as stating the total throughput of both processes cannot exceed 3000 tons, in which case rows 17&18 would be replaced by

```
17)   BOX1 + TISS1 + NEWS1 + BOOK1
    + BOX2 + TISS2 + NEWS2 + BOOK2 <=   3000
```

The solution found by LINDO is as follows:

```
 LP OPTIMUM FOUND AT STEP     25
       OBJECTIVE FUNCTION VALUE

       1)      140000.0

 VARIABLE        VALUE          REDUCED COST
      BOX         0.000000          0.000000
     TISS         0.000000          6.000000
     NEWS      2500.000000          0.000000
     BOOK      2833.333252          0.000000
     BOX1         0.000000         11.124999
    TISS1         0.000000          1.499999
    NEWS1         0.000000          0.249999
    BOOK1      2333.333252          0.000000
     BOX2         0.000000          9.333334
    TISS2         0.000000          0.222223
    NEWS2      2500.000000          0.000000
    BOOK2       500.000000          0.000000
     PBOX         0.000000          0.000000
    PTISS         0.000000          0.000000
```

PNEWS	600.000000	0.000000
PBOOK	1000.000000	0.000000
PBOX2	0.000000	19.444445
PBOX3	0.000000	0.000000
PTISS2	0.000000	19.444445
PTISS3	0.000000	0.000000
PNEWS1	0.000000	19.444445
PNEWS3	600.000000	0.000000
PBOOK1	500.000000	0.000000
PBOOK2	500.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	5.000000
3)	0.000000	0.000000
4)	0.000000	8.000000
5)	0.000000	10.000000
6)	0.000000	-102.777779
7)	0.000000	-102.777779
8)	0.000000	-102.777779
9)	0.000000	-83.333336
10)	0.000000	102.777779
11)	0.000000	102.777779
12)	0.000000	102.777779
13)	0.000000	83.333336
14)	0.000000	-83.333336
15)	0.000000	-83.333336
16)	0.000000	-102.777779
17)	666.666687	0.000000
18)	0.000000	1.666667

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
BOX	5.000000	INFINITY	5.000000
TISS	6.000000	INFINITY	6.000000
NEWS	8.000000	0.333334	4.666667
BOOK	10.000000	6.000000	1.999989
BOX1	20.000000	INFINITY	11.124999
TISS1	20.000000	INFINITY	1.499999
NEWS1	20.000000	INFINITY	0.249999
BOOK1	20.000000	0.249999	0.750001
BOX2	15.000000	INFINITY	9.333333
TISS2	15.000000	INFINITY	0.222222
NEWS2	15.000000	0.222221	4.666667
BOOK2	15.000000	0.666667	0.222221
PBOX	0.000000	INFINITY	77.777779
PTISS	0.000000	INFINITY	1.388890
PNEWS	0.000000	1.388890	19.444443
PBOOK	0.000000	19.444443	83.333336
PBOX2	0.000000	INFINITY	19.444443
PBOX3	0.000000	19.444443	77.777779
PTISS2	0.000000	INFINITY	19.444443
PTISS3	0.000000	19.444443	1.388890
PNEWS1	0.000000	INFINITY	19.444443
PNEWS3	0.000000	1.388890	19.444443
PBOOK1	0.000000	19.444443	83.333336
PBOOK2	0.000000	19.444443	83.333336

RIGHTHAND SIDE RANGES

ROW	CURRENT	ALLOWABLE	ALLOWABLE

	RHS	INCREASE	DECREASE
2	0.000000	0.000000	INFINITY
3	0.000000	INFINITY	0.000000
4	0.000000	2500.000000	INFINITY
5	0.000000	2833.333252	INFINITY
6	0.000000	0.000000	600.000000
7	0.000000	0.000000	600.000000
8	0.000000	120.000008	600.000000
9	0.000000	240.000015	840.000000
10	0.000000	600.000000	0.000000
11	0.000000	600.000000	0.000000
12	0.000000	600.000000	120.000008
13	0.000000	840.000000	240.000015
14	500.000000	240.000015	500.000000
15	500.000000	240.000015	500.000000
16	600.000000	120.000008	600.000000
17	3000.000000	INFINITY	666.666687
18	3000.000000	2625.000000	500.000000

THE TABLEAU

ROW	(BASIS)	BOX	TISS	NEWS	BOOK	BOX1	TISS1
1	ART	0.000	6.000	0.000	0.000	11.125	1.500
2	BOOK	0.000	0.000	0.000	1.000	-0.062	-0.083
3	SLK 3	0.000	-1.000	0.000	0.000	0.000	1.000
4	SLK 17	0.000	0.000	0.000	0.000	0.500	0.333
5	BOOK1	0.000	0.000	0.000	0.000	0.500	0.667
6	PBOX	0.000	0.000	0.000	0.000	-0.135	0.000
7	PTISS	0.000	0.000	0.000	0.000	0.000	-0.180
8	PNEWS	0.000	0.000	0.000	0.000	0.135	0.180
9	PBOOK	0.000	0.000	0.000	0.000	0.000	0.000
10	PBOX3	0.000	0.000	0.000	0.000	-0.135	0.000
11	PTISS3	0.000	0.000	0.000	0.000	0.000	-0.180
12	PNEWS3	0.000	0.000	0.000	0.000	0.135	0.180
13	PBOOK2	0.000	0.000	0.000	0.000	0.000	0.000
14	PBOOK1	0.000	0.000	0.000	0.000	0.000	0.000
15	NEWS2	0.000	0.000	0.000	0.000	0.562	0.750
16	NEWS	0.000	0.000	1.000	0.000	0.562	0.750
17	BOX	1.000	0.000	0.000	0.000	-1.000	0.000
18	BOOK2	0.000	0.000	0.000	0.000	-0.562	-0.750

ROW	NEWS1	BOOK1	BOX2	TISS2	NEWS2	BOOK2	PBOX
1	0.250	0.000	9.333	0.222	0.000	0.000	0.000
2	-0.125	0.000	0.056	0.037	0.000	0.000	0.000
3	0.000	0.000	0.000	1.000	0.000	0.000	0.000
4	0.000	0.000	0.444	0.296	0.000	0.000	0.000
5	1.000	1.000	-0.444	-0.296	0.000	0.000	0.000
6	0.000	0.000	-0.120	0.000	0.000	0.000	1.000
7	0.000	0.000	0.000	-0.160	0.000	0.000	0.000
8	0.000	0.000	0.120	0.160	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	-0.120	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	-0.160	0.000	0.000	0.000
12	0.000	0.000	0.120	0.160	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	1.125	0.000	0.500	0.667	1.000	0.000	0.000
16	0.125	0.000	0.500	0.667	0.000	0.000	0.000
17	0.000	0.000	-1.000	0.000	0.000	0.000	0.000
18	-1.125	0.000	0.500	0.333	0.000	1.000	0.000

ROW	PTISS	PNEWS	PBOOK	PBOX2	PBOX3	PTISS2	PTISS3
1	0.000	0.000	0.000	19.444	0.000	19.444	0.000
2	0.000	0.000	0.000	3.241	0.000	3.241	0.000

3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.926	0.000	0.926	0.000
5	0.000	0.000	0.000	-0.926	0.000	-0.926	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	1.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	1.000	0.000	-1.000	0.000	-1.000	0.000
9	0.000	0.000	1.000	1.000	0.000	1.000	0.000
10	0.000	0.000	0.000	1.000	1.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	1.000	1.000
12	0.000	0.000	0.000	-1.000	0.000	-1.000	0.000
13	0.000	0.000	0.000	1.000	0.000	1.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	-4.167	0.000	-4.167	0.000
16	0.000	0.000	0.000	-4.167	0.000	-4.167	0.000
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	4.167	0.000	4.167	0.000

ROW	PNEWS1	PNEWS3	PBOOK1	PBOOK2	SLK 2	SLK 3	SLK 4
1	19.444	0.000	0.000	0.000	5.000	0.000	8.000
2	3.241	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	1.000	0.000
4	0.926	0.000	0.000	0.000	0.000	0.000	0.000
5	-0.926	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	-1.000	0.000	0.000	0.000	0.000	0.000	0.000
9	1.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	1.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	1.000	0.000	0.000	0.000
14	1.000	0.000	1.000	0.000	0.000	0.000	0.000
15	-4.167	0.000	0.000	0.000	0.000	0.000	0.000
16	-4.167	0.000	0.000	0.000	0.000	0.000	-1.000
17	0.000	0.000	0.000	0.000	-1.000	0.000	0.000
18	4.167	0.000	0.000	0.000	0.000	0.000	0.000

ROW	SLK 5	SLK 10	SLK 11	SLK 12	SLK 13	SLK 14	SLK 15
1	10.000	102.778	102.778	102.778	83.333	83.333	83.333
2	-1.000	0.463	0.463	0.463	-2.778	-2.778	-2.778
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	3.704	3.704	3.704	2.778	2.778	2.778
5	0.000	-3.704	-3.704	-3.704	-2.778	-2.778	-2.778
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	-1.000	-1.000	-1.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	-1.000	-1.000	-1.000
10	0.000	1.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	1.000	0.000	0.000	0.000	0.000
12	0.000	-1.000	-1.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.000	0.000	-1.000
14	0.000	0.000	0.000	0.000	0.000	-1.000	0.000
15	0.000	-4.167	-4.167	-4.167	0.000	0.000	0.000
16	0.000	-4.167	-4.167	-4.167	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	4.167	4.167	4.167	0.000	0.000	0.000

ROW	SLK 16	SLK 17	SLK 18	RHS
1	0.10E+03	0.00E+00	1.7	-0.14E+06
2	0.463	0.000	0.111	2833.333
3	0.000	0.000	0.000	0.000
4	3.704	1.000	0.889	666.667
5	-3.704	0.000	-0.889	2333.333
6	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000

```
 8    -1.000     0.000     0.000   600.000
 9     0.000     0.000     0.000  1000.000
10     0.000     0.000     0.000     0.000
11     0.000     0.000     0.000     0.000
12    -1.000     0.000     0.000   600.000
13     0.000     0.000     0.000   500.000
14     0.000     0.000     0.000   500.000
15    -4.167     0.000     0.000  2500.000
16    -4.167     0.000     0.000  2500.000
17     0.000     0.000     0.000     0.000
18     4.167     0.000     1.000   500.000
```

a. *Complete the following statement:* the optimal solution is to purchase only newsprint and book paper, process ______ tons of the book paper and _____ tons of the newsprint by asphalt dispersion, and the remaining book paper by de-inking. This yields ______ tons of pulp from the newsprint and ______ tons of pulp from the book paper. One-half of the pulp from book paper is used in each of grades 1 & 2 paper, and the newsprint is used in grade 3 paper. This plan will use ______% of the de-inking capacity and ______% of the asphalt dispersion capacity. (Note that BOX is a basic variable, but has a value of zero, categorizing this solution as ______________.

b. How much must tissue drop in price in order that it would enter the solution? ________

c. If tissue were to enter the solution (e.g., because of the drop in price you determined in (b)), how much would be purchased? ________________ *(Hint: use the minimum ratio test!)*

d. How much would the cost decrease if 10 additional tons of pulp for grade 1 paper were required? ________________

e. If ten additional tons of pulp for grade 1 paper were required, how would the quantities of raw materials (boxboard, newsprint and book paper) change?_______________ *(Hint: if the surplus variable for the row stating the requirement were to increase, what effect would it have on BOX, NEWS, and BOOK?)*

2. ***Transportation problem***. A bank has two sites at which checks are processed. Site 1 can process 9,000 checks per day and site 2 can process 7000 checks per day. The bank processes three types of checks: vendor checks, salary checks, and personal checks. Each day 5000 checks of each type must be processed. The processing cost per check depends on the site at which the check is processed:

Type of check	Site #1	Site #2
Vendor checks	5¢	3¢
Salary checks	4¢	4¢
Personal checks	2¢	5¢

(a.) Formulate a balanced transportation problem to minimize the daily cost of processing checks. (That is, provide the transportation tableau for the problem.) What is the number of basic variables in any basic solution to this problem?

(b.) Use the Northwest-Corner to find a basic feasible solution to the problem. Compute the total cost for this basic feasible solution.
(c.) Starting with the Northwest-Corner solution, perform the simplex algorithm to find the optimal solution to this problem. At each iteration, state the values of the dual variables and the reduced costs of each nonbasic "shipment".

56:171 Operations Research
Homework #6 -- Fall 2000

1. Data Envelopment Analysis. The following data are available for each of seven university departments which are to be evaluated by the university administration:

- Number of staff persons
- Academic staff salaries (in thousands of British pounds)
- Support staff salaries (in thousands of British pounds)
- Number of undergraduates
- Number of graduate students
- Number of research papers

Dept	#Staff	Acad-sal	Supp-sal	#UG	#Grad	Papers
1	12	400	20	60	35	17
2	19	750	70	139	41	40
3	42	1500	70	225	68	75
4	15	600	100	90	12	17
5	45	2000	250	253	145	130
6	19	730	50	132	45	45
7	41	2350	600	305	159	97

It was decided to use DEA to compute the relative "efficiencies" of the departments. The results were less than helpful-- all but one department was rated as 100% efficient!

i	Efficiency
1	1
2	1
3	1
4	0.8197
5	1
6	1
7	1

A look at the "prices" assigned by each DMU (department) to each input and output help to explain this result.

i	#UG	#Grad	Papers	#Staff	Acad-salary	Supp-salary
1	0.00613375	0.0179087	0.000304169	0.0791606	0	0.00250366
2	0.0052393	0	0.00679343	0.0472869	0.000135398	0
3	0	0.00257257	0.0110009	0	0.000303082	0.0077911
4	0.00910818	0	0	0.0641504	0.0000629054	0
5	0	0.00280219	0.00456679	0	0.0003988	0.000809599
6	0.00376481	0.0109921	0.000186695	0.0485876	0	0.00153671
7	0.0012067	0.00333687	0.00104531	0.00731881	0.000297842	0

Note, for example, that department #2 places zero value on both the number of graduate students and support staff salaries-- which might be explained by the fact that their support staff salaries (an input) were relatively high and the number of graduate students (an output) were relatively low, compared to the other departments.
This illustrates a limitation of DEA when the number of inputs and outputs is relatively large compared to the number of DMUs being evaluated-- most DMUs are able to find some combination of input & output in which they "shine" and are thereby able to assign appropriate prices in order to earn a 100% efficiency rating.

The analysis which follows used a *single* input-- only the total number of staff-- and used all three of the previous outputs.

a. Write the LP which is solved in order to compute the efficiency of department #5, and solve it with LINDO. What are the values assigned to each of the three outputs? (Enter the efficiency and values assigned to outputs in the tables below.)

The results of the DEA, i.e., the seven LP solutions, are now:

Dept	Efficiency
1	0.7521
2	0.9834
3	0.7383
4	0.8066
5	______
6	0.9692
7	1

Prices:

Dept	#UG	#Grad	Papers
1	0	0.0214885	0
2	0.00707506	0	0
3	0.0015207	0	0.00528225
4	0.00896175	0	0
5	__________	________	_________
6	0.00336154	0	0.0116766
7	0.00155779	0	0.00541109

Weighted Output Values (%)

i	#UG	#Grad	papers
1	0.0	100.0	0.0
2	100.0	0.0	0.0
3	46.3	0.0	53.7
4	100.0	0.0	0.0
5	35.9	0.0	64.1
6	45.8	0.0	54.2
7	47.5	0.0	52.5

For example, department 6 placed no value on graduate students and assigned values to undergraduate students and research papers so that they accounted for approximately 45% and 55%, respectively.

b. Which department(s) seem to specialize in graduate education, i.e., give the number of graduate students a high priority? ___________________

c. Which department(s) seem to specialize in undergraduate education, i.e., give the number of undergraduate students a high priority? ________________

2. ***Assignment Problem.*** An accounting firm has three new clients, each of which is to be assigned a project leader. Based upon the different backgrounds and experiences of the available leaders the various assignments differ in expected completion times, which are (in days):

Project leader	Client A	Client B	Client C
Jackson	10	16	32
Ellis	14	22	40
Smith	22	24	34

Use the Hungarian algorithm to find the optimal assignment.

3. Assignment Problem. A Manufacturer of small electrical devices has purchased an old warehouse and converted it into a primary production facility. The physical dimensions of the existing building left the architect with little leeway for designing locations for the company's five assembly lines and five inspection and storage areas, but these have now been constructed and now exist in fixed areas within the building.

As items are taken off the assembly lines, they are temporarily stored in bins at the end of each line. At 30-minute intervals, the bins are physically transported to one of the five inspection areas. Because different volumes of product are manufactured at each assembly line and different distances must be traversed from each assembly line to each inspection station, different times are required. The company must designate a separate inspection area for each assembly line.

An IE has performed a study showing the times needed to transport finished products from each assembly line to each inspection area in minutes:

	A	B	C	D	E
1	10	4	6	10	12
2	11	7	7	9	14
3	13	8	12	14	16
4	14	16	13	17	17
5	19	11	17	20	19

a. Under the current arrangement, which has been operational since they moved into the building, work on assembly lines 1, 2, 3, 4, and 5 is transported to inspection areas A, B, C, D, and E, respectively. Given that the average worker costs $12 per hour, what is the annual labor cost for this arrangement, assuming two 8-hour shifts per day, 250 days per year? ________________

b. Use the Hungarian algorithm to find an optimal assignment of assembly lines to inspection areas.

Assembly Line	Inspection Area
1	
2	
3	
4	
5	

c. What is the annual savings which management could expect if this assignment were made? ________________

56:171 Operations Research
Homework #7 -- Due Monday, 23 October 2000

1. The campus bookstore must decide how many textbooks to order for a freshman economics course to be offered next semester. The bookstore believes that either seven, eight, nine, or ten sections of the course will be offered, each section consisting of 40 students. The publisher is offering bookstores a discount if they place their orders early. If the bookstore orders too few texts and runs out, the publisher will air express additional books at the bookstore's expense. If it orders too many texts, the store can return unsold texts to the publisher for a partial credit. The bookstore is considering ordering either 280, 320, 360, or 400 textbooks in order to get the discount. Taking into account the discounts, air express expenses, and credits for returned texts, the bookstore manager estimates the following resulting profits:

# books ordered	7 sections	8 sections	9 sections	10 sections
280	$2800	$2720	$2640	$2480
320	$2600	$3200	$3040	$2880
360	$2400	$3000	$3600	$3440
400	$2200	$2800	$3400	$4000

(a.) What is the decision if the manager uses the maximax criterion?

(b.) What is the decision if the manager uses the maximin criterion?

(c.) What is the decision if the manager uses the minimax regret criterion?

Suppose now that, based upon conversations held with the chairperson of the economics department, the bookstore manager believes the following probabilities hold:

P{7 sections offered} = 10%
P{8 sections offered} = 30%
P{9 sections offered} = 40%
P{10 sections offered} = 20%

(d.) Using the expected value criterion, determine how many books the manager should purchase in order to maximize the store's expected profit.

(e.) Based upon the probabilities given, determine the expected value of perfect information and interpret its meaning.

2. T. Bone Puckett, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants located in South Carolina. Three alternative decisions are being considered:

- ❑ Expand the plant and produce light-weight, durable materials for possible sales to the military, a market with little foreign competition.
- ❑ Maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition.
- ❑ Sell the plant now.

If one the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends upon foreign market conditions, including the status of a trade embargo bill in Congress. The following payoff table describes this decision situation.

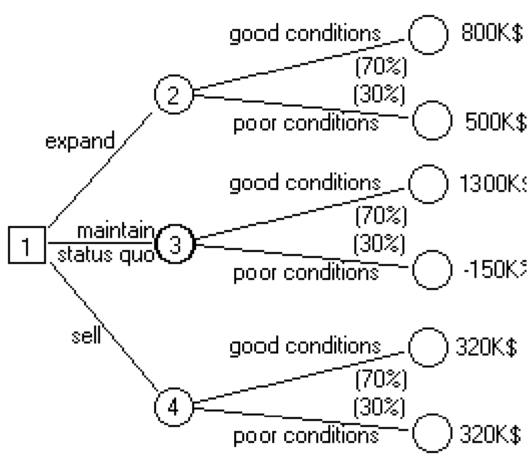
Decision	Good foreign competitive conditions	Poor foreign competitive conditions
Expand	$800,000	$500,000
Maintain status quo	$1,300,000	- $150,000
Sell now	$320,000	$320,000

(a.) Determine the best decision using the following decision criteria:
- Maximax
- Maximin
- Minimax regret

(b.) Assume it is now possible to estimate a probability of 70% that good foreign competitive conditions will exist and a probability of 30% that poor conditions will exist. Determine the best decision using expected value and expected opportunity loss.

(c.) Compute the expected value of perfect information.

(d.) Fold back the decision tree below:



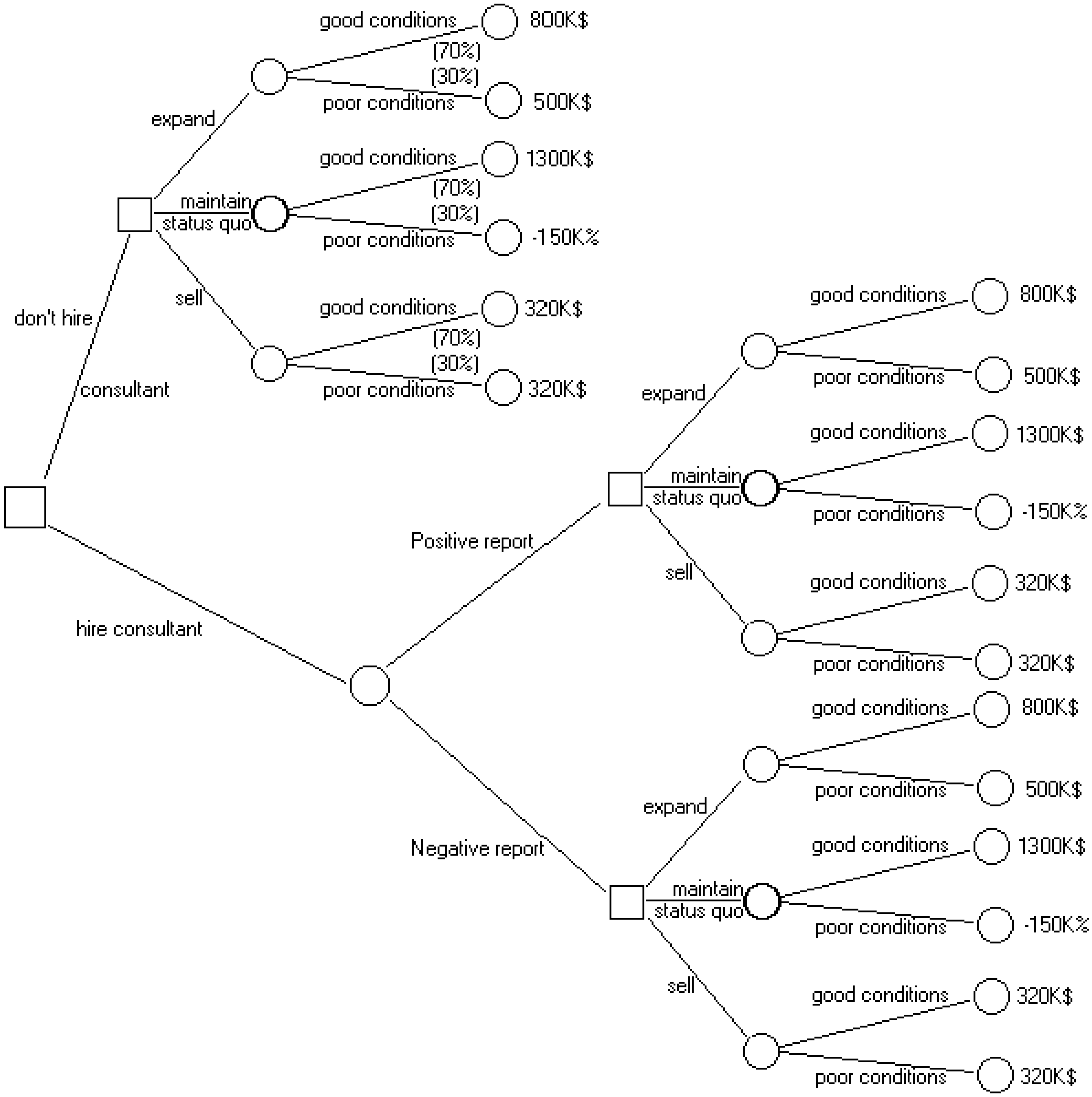
Puckett has hired a consulting firm to provide a report on future political and market situations. The report will be positive (P) or negative (N), indicating either a good (g) or poor (p) future foreign competitive situation. The conditional probability of each report outcome given each state of nature is

P{P|g} = 70%
P{N|g} = 30%
P{P|p} = 20%
P{N|p} = 80%

(e.) Determine the posterior probabilities using Bayes' rule:

P{g| P} = _______ %
P{p| P} = _______ %
P{g| N} = _______ %
P{p| N} = _______ %

(f.) Perform a decision tree analysis using the posterior probabilities that you have just computed.



56:171 Operations Research
Homework #8 -- Due Wednesday, 25 October 2000

It is June 1, and popular recording star Chocolate Cube is planning to add a separate recording studio to his palatial complex in rural Connecticut. The blueprints have been completed, and the following table lists the time estimates of the activities in the construction project. *(Based upon exercise in Applied Mgmt Science, by Lawrence & Pasternack.)*

	Activity	Immediate Predecessors	Optimistic time (days)	Most likely time (days)	Pessimistic time (days)	Expected time (days)
A	Order materials	none	1	2	9	
B	Clear land	none	2.5	4.5	9.5	
C	Obtain permits	none	2	5	14	
D	Hire subcontractors	C	4	6.5	18	
E	Unload/store materials	A	2	4	18	
F	Primary structure	B,D,E	22	30	50	
G	Install electrical work	F	15	20	37	
H	Install plumbing	F	4.5	10	21.5	
I	Finish/paint	G,H	12	15	24	
J	Complete electrical studio	H	14	14.5	48	
K	Clean-up	I,J	5	5	5	

1. Compute the expected duration of each activity, based upon the three time estimates.

2. Draw the AON (Activity-on-Node) network for the project.

3. Draw the AOA (Activity-on-Arrow) network for the project and label the nodes so that i<j if there is an arrow from node i to node j.

4. For each node (event), compute the ET (early time) and LT (late time), based upon the expected durations.

5. For each activity, compute the ES (early start), EF (early finish), LS (late start), LF (late finish), and TS (total slack).

	Activity	ES	EF	LS	LF	TS
A	Order materials					
B	Clear land					
C	Obtain permits					
D	Hire subcontractors					
E	Unload/store materials					
F	Primary structure					
G	Install electrical work					
H	Install plumbing					
I	Finish/paint					
J	Complete electrical studio					
K	Clean-up					

6. Which activities are on the critical path?

7. What is the expected date of completion of this project (assuming a 7-day work week, including July 4 and Labor Day)?

8. Chocolate Cube has committed himself to a recording session beginning September 8 (99 days from now). What is the probability that he will be able to begin recording in his own personal studio on that date?

9. If his studio is not ready in 99 days, Chocolate Cube will be forced to lease his record company's studio, which will cost $120,000. For $3,5000 extra, Eagle Electric, the company hired for the electrical installation (activity G) will work double time; each of the time estimates for this activity will therefore be reduced by 50%. Using an expected cost approach, determine if the $3,500 should be spent.

56:171 Operations Research
Homework #9 - Due Wednesday, November 1, 2000

1. **Integer Programming Formulation** *(#5, page 547, of O.R. text by W. Winston)* The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS). The cost of signing each pitcher and the number of victories each pitcher will add to the Cubs are shown below.

Pitcher	Cost of signing ($million)	Right- or Left-handed?	Victories added to Cubs
RS	$6	Right	6
BS	$4	Right	5
DE	$3	Right	3
ST	$2	Left	3
TS	$2	Right	2

Subject to the following restrictions, the Cubs want to sign the pitchers who will add the most victories to the team.

- At most $12 can be spent.
- If DE and ST are signed, then BS cannot be signed.
- At most two right-handed pitchers can be signed.
- The cubs cannot sign both BS and RS.

Formulate an integer LP to help the Cubs determine whom they should sign. Solve the problem, using LINDO (or equivalent) software.

2. **Integer Programming Formulation.** *(#4, p. 547, O.R. text, W. Winston)* A court decision has stated that the enrollment of each high school in Metropolis must be at least 20% black. The numbers of black and white high school students in each of the city's five school districts are shown in the table below.

District	Whitestudents	Black students
1	80	30
2	70	5
3	90	10
4	50	40
5	60	30

The distance (in miles) that a student in each district must travel to each high school is:

District	HS #1	HS #2
1	1	2
2	0.5	1.7
3	0.8	0.8
4	1.3	0.4
5	1.5	0.6

School board policy requires that all the students in a given district attend the same school. Assuming that each school must have an enrollment of at least 150 students, formulate an integer LP that will minimize the total distance that Metropolis students must travel to high school. Find the solution, using LINDO (or equivalent) software.

First Passage Probabilities

n	$f_{80}^{(n)}$
1	0
2	0
3	0.032
4	0.0459
5	0.0295
6	0.023933
7	0.028956
8	0.02948
9	0.026245
10	0.024937
11	0.025001
12	0.02431
13	0.023174
14	0.022383
15	0.02178
16	0.021073
17	0.020333
18	0.019668
19	0.019042
20	0.018418
sum:	0.45613

Mean First Passage array:

\to From	0	1	2	3	4	5	6	7	8
0	32.256	11.02	7.825	6.4131	6.6646	5.2769	4.383	10.239	22.708
1	32.256	11.02	7.825	6.4131	6.6646	5.2769	4.383	10.239	22.708
2	32.256	11.02	7.825	6.4131	6.6646	5.2769	4.383	10.239	22.708
3	25.842	7.0106	6.7	6.4511	7.8411	6.4534	5.5595	11.415	23.885
4	31.546	8.4243	4.9883	5.7033	6.959	6.7994	5.9055	11.761	24.231
5	30.205	9.5462	5.7966	4.3629	6.4345	6.3366	6.5609	12.417	24.886
6	30.986	9.5981	6.8052	5.1436	4.9139	5.8946	6.1143	13.049	25.519
7	31.927	10.474	7.0795	6.0844	5.6498	4.51	5.6535	11.581	26.094
8	32.256	11.02	7.825	6.4131	6.6646	5.2769	4.383	10.239	22.708

Results of simulation:

run	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1)	8	5	3	1	8	5	2	6	3	2	6	5	3	2	6	5	2	7	5	3	1
2)	8	7	5	4	3	1	5	3	2	8	5	3	1	6	3	2	8	8	7	6	3
3)	8	7	5	4	3	1	6	6	4	1	6	4	2	6	5	3	1	8	6	3	2
4)	8	5	4	2	7	5	3	0	7	4	3	2	6	4	2	6	5	3	1	8	5
5)	8	6	3	1	8	7	6	3	1	8	7	5	3	2	6	3	3	1	6	5	3
6)	8	5	3	0	8	8	6	3	1	7	4	1	7	4	2	5	3	2	7	5	2
7)	8	7	5	4	1	8	5	2	7	4	3	1	6	5	5	5	3	3	1	7	5
8)	8	6	4	2	5	2	6	3	2	8	7	5	4	3	1	6	4	2	6	4	3
9)	8	8	5	4	2	8	6	4	3	3	1	5	3	1	6	3	0	6	5	2	7
10)	8	7	7	7	6	5	3	3	3	3	1	7	5	4	3	1	5	3	1	7	7

P^2 = square of transition probability matrix:

From\to	0	1	2	3	4	5	6	7	8
0	0	0	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225
1	0	0	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225
2	0	0	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225
3	0.03	0.06	0.0375	0.0225	0	0.17	0.34	0.2125	0.1275
4	0.05	0.13	0.1225	0.075	0.0225	0.12	0.24	0.15	0.09
5	0.08	0.21	0.23	0.1825	0.075	0.0625	0.08	0.05	0.03
6	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225	0	0
7	0	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225	0
8	0	0	0.04	0.16	0.26	0.26	0.1825	0.075	0.0225

Sum of first 20 powers of P:

from\to	0	1	2	3	4	5	6	7	8
0	0.569	1.678	2.411	2.999	2.847	3.251	3.457	1.841	0.947
1	0.569	1.678	2.411	2.999	2.847	3.251	3.457	1.841	0.947
2	0.569	1.678	2.411	2.999	2.847	3.251	3.457	1.841	0.947
3	0.768	2.042	2.555	2.993	2.678	3.066	3.265	1.739	0.895
4	0.591	1.914	2.774	3.109	2.805	3.011	3.208	1.709	0.880
5	0.632	1.812	2.670	3.316	2.880	3.084	3.101	1.653	0.851
6	0.608	1.807	2.541	3.195	3.099	3.154	3.174	1.598	0.823
7	0.579	1.728	2.506	3.050	2.993	3.372	3.249	1.725	0.798
8	0.569	1.678	2.411	2.999	2.847	3.251	3.457	1.841	0.947

Steady state distribution:

i	name	P{i}
1	SOH.0	0.0310024
2	SOH.1	0.0907444
3	SOH.2	0.127795
4	SOH.3	0.155012
5	SOH.4	0.143698
6	SOH.5	0.157814
7	SOH.6	0.163551
8	SOH.7	0.0863466
9	SOH.8	0.0440368

c. Write the transition probability matrix.

from\to	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									
5									
6									
7									
8									

For the following questions, consult the computations below.

d. Over a long period of time, what is the percent of the weeks in which you would expect there to be a stockout (zero inventory)?

e. What will be the average end-of-week inventory level?

f. How often (i.e. once every how many weeks?) will the inventory be full at the end of the week?

g. How often will the inventory be restocked?

h. What is the expected number of weeks, starting with a full inventory, until a stockout occurs?

i. Starting with a full inventory, what is the expected number of stockouts during the first 20 weeks? What is the expected number of times that the inventory is restocked?

j. This inventory system was simulated ten times for 20 weeks, starting in state 8.

- In each simulated history, what is the number of stockouts during the first 20 weeks?
- In each simulated history, how many times did restocking occur?
- Compute the average number of stockouts and restocking of the inventory during the ten 20-week intervals which were simulated. How do these values compare with the answers you found in (i)?

56:171 Operations Research
Homework #10 – Due Wednesday, 8 November 2000

1. Markov Chains. *(Based upon Exercise 4, §19.5, page 982 of text by W. Winston)*
At the beginning of each year, my car is in good, fair, or broken-down condition. A good car will be good at the beginning of next year with probability 85%, fair with probability 10%, or broken-down with probability 5%. A fair car will be fair at the beginning of the next year with probability 75%, or broken-down with probability 25%. It costs $9000 to purchase a good car; a fair car can be traded in for $2500; and a broken-down car has no trade-in value and must immediately be replaced by a good car. It costs $1000 per year to operate a good car and $1500 to operate a fair car. Should I replace my car as soon as it becomes a fair car, or should I drive my car until it breaks down? Assume that the cost of operating a car during a year depends on the type of car on hand at the *beginning* of the year (after a new car, if any, arrives).

Define a Markov chain model with three states (Good, Fair, & Broken-down). Assume, as implied by the problem statement, that break-down occurs only at the *end* of a year, and then (at the beginning of the next year) the broken-down car "must immediately be replaced". For each of the two replacement policies mentioned, answer the following:

a. Draw a diagram of the Markov chain and write down the transition probability matrix.

b. Write down the equations which could be solved to obtain the steadystate probabilities.

c. Solve the equations, either manually or using appropriate computer software.

d. Compute the average cost per year for the replacement policy.

e. What is the expected time between break-downs?

f. What replacement policy do you recommend?

Note: assume that ***state 1= Good, state 2= Fair, and state 3= Broken-down***.

2. Consider a *reorder-point/order-up-to* type of inventory control system, sometimes referred to as *(s,S)*. Suppose that the inventory is counted at the end of the week (Saturday evening), and if s=2 or fewer items remain, enough is ordered to bring the level up to S=8 before the business reopens on Monday morning. The probability distribution of demand is:

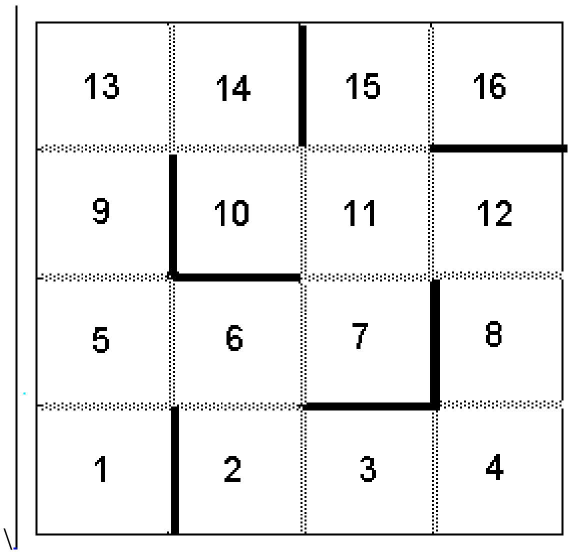
P{D=0}= 0.15 P{D=1}= 0.25 P{D=2}=0.4 P{D=3}= 0.2

a. What are the states in the Markov Chain model of this system? (That is, how many states are there, and what does each state signify?)

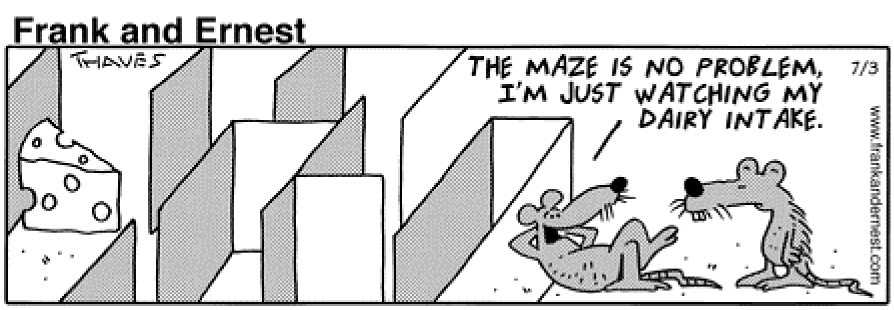
b. Draw the diagram for this Markov Chain.

56:171 Operations Research
Homework #11 – Due Wednesday, 15 November 2000

We wish to model the passage of a rat through a maze. Consider a maze in the form of a 4x4 array of boxes, such as the one below on the left:



The solid lines represent walls, the shaded lines represent doors. We will assume that a rat is placed into box #1. While in any box, the rat is assumed to be equally likely to choose each of the doors leaving the box (including the one by which he entered the box). For example, when in box #2 above, the probability of going next to boxes 3 and 6 are each $^1/_2$, regardless of the door by which he entered the box. *This assumption implies that no learning takes place if the rat tries the maze several times!*



Transition probabilities:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1)	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2)	0	0	0.5	0	0	0.5	0	0	0	0	0	0	0	0	0	0
3)	0	0.5	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0
4)	0	0	0.5	0	0	0	0	0.5	0	0	0	0	0	0	0	0
5)	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0	0	0	0	0
6)	0	0.333	0	0	0.333	0	0.333	0	0	0	0	0	0	0	0	0
7)	0	0	0	0	0	0.5	0	0	0	0	0.5	0	0	0	0	0
8)	0	0	0	0.5	0	0	0	0	0	0	0	0.5	0	0	0	0
9)	0	0	0	0	0.5	0	0	0	0	0	0	0	0.5	0	0	0
10)	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0
11)	0	0	0	0	0	0	0.25	0	0	0.25	0	0.25	0	0	0.25	0
12)	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0	0	0	0
13)	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0.5	0	0
14)	0	0	0	0	0	0	0	0	0	0.5	0	0	0.5	0	0	0
15)	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0.5

16)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

a. On the diagram representing the Markov chain, write the transition probabilities on each transition in each direction.

b. Write one of the equations (other than $\sum_i p_i = 1$) that determines the steady-state distribution of the rat's location.

c. In steady state, which box will be visited most frequently by the rat?

d. Suppose that in box #16 a reward (e.g. food) is placed. What is the expected number of moves of the rat required to reach this reward from box #1?

e. Count the minimum number of moves (M) required to reach the reward from box #1. What is the probability that the rat reaches the reward in **exactly** this number of moves?

f. What is the probability that the rat reaches the reward with no more than 4 unnecessary moves?

g. If the food were placed in box #7, into which box should we place the rat, if we want the largest expected number of moves to find the food?

Mean first passage times:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1)	34	31	43.6	49.1	1	11.3	23.1	47.6	19.6	33	23.3	39	31.1	35.6	54.3	87.3
2)	59.7	17	18.3	29.5	26.7	8.67	20.5	33.6	39.6	36	20.7	30.7	45.5	44.3	51.7	84.7
3)	65.2	11.2	17	15.7	32.2	15.3	23.7	24.4	44	37	20.5	26	48.7	46.4	51.5	84.5
4)	68.7	20.4	13.7	17	35.7	20	25	13.2	46.4	36	18.4	19.3	50	46.5	49.4	82.4
5)	33	30	42.6	48.1	11.3	10.3	22.1	46.6	18.6	32	22.3	38	30.1	34.6	53.3	86.3
6)	52.1	20.8	34.5	41.2	19.1	11.3	15.2	40.8	33.2	33	18.8	33.3	40.2	40.1	49.8	82.8
7)	60.7	29.4	39.7	43	27.7	12	17	39.2	38.4	28	10.4	28.3	42	38.5	41.4	74.4
8)	70.3	27.6	25.5	16.3	37.3	22.7	24.3	17	46.8	33	14.3	10.7	49.3	44.7	45.3	78.3
9)	43.9	35.2	46.7	51.1	10.9	16.7	25.1	48.4	17	27	21.9	38.7	16.1	25.1	52.9	85.9
10)	64.5	38.8	46.9	47.9	31.5	23.7	21.9	41.8	34.2	17	8.47	28.7	29.9	18.5	39.5	72.5
11)	67.3	36	42.9	42.8	34.3	22	16.8	35.6	41.6	21	8.5	21.3	41.8	34.9	31	64
12)	69.8	32.8	35.2	30.5	36.8	23.3	21.5	18.8	45.2	28	8.13	17	46.5	40.8	39.1	72.1
13)	52.7	38.4	48.7	52	19.7	21	26	48.2	13.4	20	19.4	37.3	17	13.5	50.4	83.4
14)	59.6	39.6	48.8	50.9	26.6	23.3	24.9	46	24.8	11	14.9	34	15.9	17	45.9	78.9
15)	70.3	39	45.9	45.8	37.3	25	19.8	38.6	44.6	24	3	24.3	44.8	37.9	17	33
16)	71.3	40	46.9	46.8	38.3	26	20.8	39.6	45.6	25	4	25.3	45.8	38.9	1	34

First-passage probabilities:

n	$f_{1,16}^{(n)}$	$p_{1,16}^{(n)}$
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0.00694	0.00694
7	0	0
8	0.0126	0.0161
9	0	0
10	0.0165	0.025
11	0	0
12	0.0189	0.0326
13	0	0
14	0.0203	0.0388
15	0	0
16	0.021	0.0437
17	0	0
18	0.0213	0.0474
19	0	0
20	0.0213	0.0503
21	0	0
22	0.0211	0.0524
23	0	0
24	0.0208	0.0541
25	0	0
26	0.0204	0.0553
27	0	0
28	0.02	0.0562
29	0	0
30	0.0196	0.0568
sum=	0.241	0.536

Steadystate distribution:

i	π_i
1	0.0294
2	0.0588
3	0.0588
4	0.0588
5	0.0882
6	0.0882
7	0.0588
8	0.0588
9	0.0588
10	0.0588
11	0.118
12	0.0588
13	0.0588
14	0.0588
15	0.0588
16	0.0294

2. Coldspot manufactures refrigerators. The company has issued a warranty on all refrigerators that requires free replacement of any refrigerator that fails before it is three years old.

- 4% of all new refrigerators fail during their first year of operation.
- 6% of all 1-year-old refrigerators fail during their second year of operation.
- 7% of all 2-year-old refrigerators fail during their third year of operation.
- 8% of all 3-year-old refrigerators fail during their fourth year of operation.

Replacement refrigerators are not covered by the warranty.

Define a discrete-time Markov chain, with states

(0.) new refrigerators
(1.) 1-year-old refrigerators
(2.) 2-year-old refrigerators
(3.) refrigerators that have passed their third anniversary
(4.) replacement refrigerators

Define the stages to be years, with the refrigerator observed on the anniversaries of its purchase.

a. Draw the transition diagram and write the transition probability matrix.

b. Which states are transient, and which are absorbing?

c. Identify the matrices Q (probabilities of transitions between transient states) and R (probabilities of transitions from transient states to absorbing states).

d. Calculate the matrix E (expected number of visits) and A (absorption probabilities).

e. What fraction of the refrigerators will Coldspot expect to replace?

f. Suppose that it costs $500 to replace a refrigerator, and that the company sells 10,000 units per year. What is the expected annual replacement cost?

g. They are considering extending the warranty period to four years. Assuming that this would have no effect on sales, what would be the increased replacement costs?

3. Birth-death model of queue. A parking lot consists of four spaces. Cars making use of these spaces arrive according to a Poisson process at the rate of six cars per hour. Parking time is exponentially distributed with mean of 30 minutes. Cars who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves, but may get impatient and leave before a parking space opens up. Assume that the time that a driver is willing to wait has exponential distribution with an average of 10 minutes. The temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere.

a. Model this system as a birth-death process, with states 0, 1, … 6.

b. Find the steadystate probability distribution of the number of cars in the system.

c. What is the fraction of the time that there is at least one empty space?

d. What is the average number of cars in the lot? … the average number of cars waiting?

e. What is the average arrival rate (keeping in mind that the arrival rate is zero when n=6)?

f. According to Little's Law, what is the average time that a car waits for a parking space?

56:171 Operations Research
Homework #12 – Due Wednesday, 29 November 2000

1. Birth/Death Process. A small service station has one gasoline pump. Cars wanting gasoline arrive according to a Poisson process at a mean rate of 15/hour. However, if the pump is already being used, these potential customers may balk (drive on to another service station). The probability that an arriving customer will balk is n/3 for n=1,2,3, where n = number of cars in the station (including the one using the pump.) The time required by a customer to fill a tank and pay the cashier has exponential distribution with a mean of 4 minutes.

(a.) Construct the diagram showing the birth & death rates.
(b.) Compute the steady state probability distribution of the number of cars in the station.
(c.) Compute the average number of cars *waiting* in the station.
(d.) Compute the average arrival rate $\underline{\lambda}$.
(e.) How many customers are expected per day if the station is open 12 hours?
(f.) What is the average time that a customer waits for use of the pump?

2. Deterministic Dynamic Programming Model: Power Plant Capacity Planning (see class notes):

This DP model schedules the construction of powerplants over a six-year period, given
R[t] = cumulative number of plants required at the end of year t (t=1,2,…6)
C[t] = cost per plant (in $millions) during year t

Year t	C_t	R_t
1	5.4	1
2	5.5	2
3	5.6	4
4	5.7	6
5	5.8	7
6	5.9	8

Eight plants are to be constructed during the six-year period, with a restriction that no more than three may be built during each one-year period. Any year in which a plant is built, a cost of $1.2 million is incurred (independent of number of plants built). A discount factor β = 85% is used to account for the time value of money, i.e., $1 spent a year from today is equivalent to $0.85 spent today.

In addition to a difference in the cost data, the computer output below differs from that in the notes in that the stages are numbered in *increasing* order, i.e., t=1 is the first year and t=6 is the final year.

Complete the computations for stage 1 (values in boxes):

(a) _______ (b)_________ (c)________ (d)________

(e) What is the present value of the minimum total cost? _________

(f) What is the optimal construction schedule? (X_t = # plants to be constructed in year t.)

Year t	X_t
1	
2	
3	
4	
5	
6	

Note: 999.999 in the output below represents +¥ to prevent an infeasible choice of state & decision combination.

```
               ---Stage 6---
         s \ x:   0        1
____________________________________

         7 | 999.9999   7.1000
         8 |   0.0000 999.9999

               ---Stage 5---
         s \ x:   0        1        2
____________________________________________

         6 | 999.9999  13.0350  12.8000
         7 |   6.0350   7.0000 999.9999
         8 |   0.0000 999.9999 999.9999

               ---Stage 4---
         s \ x:   0        1        2        3
_____________________________________________________

         4 | 999.9999 999.9999  23.4800  23.4297
         5 | 999.9999  17.7800  17.7298  18.3000
         6 |  10.8800  12.0298  12.6000 999.9999
         7 |   5.1297   6.9000 999.9999 999.9999
         8 |   0.0000 999.9999 999.9999 999.9999

               ---Stage 3---
         s \ x:   0        1        2        3
_____________________________________________________

         2 | 999.9999 999.9999  32.3153  33.0703
         3 | 999.9999  26.7153  27.4703  27.2480
         4 |  19.9153  21.8703  21.6480  22.3603
         5 |  15.0703  16.0480  16.7603  18.0000
         6 |   9.2480  11.1603  12.4000 999.9999
         7 |   4.3603   6.8000 999.9999 999.9999
         8 |   0.0000 999.9999 999.9999 999.9999

               ---Stage 2---
         s \ x:   0        1        2        3
_____________________________________________________

         1 | 999.9999  34.1680  34.9080  34.6280
         2 |  27.4680  29.4080  29.1280  30.5097
         3 |  22.7080  23.6280  25.0097  25.5608
         4 |  16.9280  19.5097  20.0608  21.4062
         5 |  12.8097  14.5608  15.9062  17.7000
         6 |   7.8608  10.4062  12.2000 999.9999
         7 |   3.7062   6.7000 999.9999 999.9999
         8 |   0.0000 999.9999 999.9999 999.9999
```

---Stage 1---

s \ x:	0	1	2	3
0	999.9999	(a)________	35.3478	36.7018

~~~~~~

Stage 6:

State	Optimal Values	Optimal Decisions	Resulting State
7	7.1000	1	8
8	0.0000	0	8

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
6	12.8000	2	8
7	6.0350	0	7
8	0.0000	0	8

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
4	23.4297	3	7
5	17.7298	2	7
6	10.8800	0	6
7	5.1297	0	7
8	0.0000	0	8

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
2	32.3153	2	4
3	26.7153	1	4
4	19.9153	0	4
5	15.0703	0	5
6	9.2480	0	6
7	4.3603	0	7
8	0.0000	0	8

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
1	34.1680	1	2
2	27.4680	0	2
3	22.7080	0	3
4	16.9280	0	4
5	12.8097	0	5
6	7.8608	0	6
7	3.7062	0	7
8	0.0000	0	8

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
0	(b)________	(c)____	(d)____
~~~~~~

56:171 Operations Research
Homework #13 – Due Wednesday, 6 December 2000

1. ***Production Planning*** We wish to plan production of an expensive, low-demand item for the next nine months (January through September).

- the cost of production is $5 for setup, plus $5 per unit produced, up to a maximum of 4 units.
- the storage cost for inventory is $2 per unit, based upon the level at the beginning of the month.
- a maximum of 3 units may be kept in inventory at the end of each month; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each month:

demand d	0	1	2
P{D=d}	0.2	0.5	0.3

- there is a penalty of $10 per unit for any demand which cannot be immediately satisfied. A maximum of 2 units may be backordered.
- the initial inventory (i.e., the inventory at the end of December) is 1.
- a salvage value of $4 per unit is received for any inventory remaining at the end of the last month (September)

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 9= January, stage 8= February, etc.** (i.e., n = # months remaining in planning period.) We define

S_n = inventory position at stage n, i.e., stock on hand if positive, # backordered if negative.

$f_n(S_n)$ = minimum total cost for the last n months if at the beginning of stage n the inventory position is S_n.

(a.) What is the optimal production quantity for January? ________

(b.) What is the total expected cost for the nine months, if there is one unit of stock on hand initially?

____

(c.) If, during January, the demand is 1 unit, how many units should be produced in February? _______

(d.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit in February when the inventory is 1 at the end of January._____________ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_7(0)$, i.e., the minimum total cost of the last 7 months (March through September) if there is no stock on hand (& no backorders) at the beginning of March. ____________
- the corresponding optimal decision $X_7^*(0)$ ____________

---Stage 1---

s \ x:	0	1	2	3	4
-2	55.00	62.00	55.50	45.20	41.40
-1	42.00	40.50	30.20	26.40	27.40
0	20.50	15.20	11.40	12.40	14.20
1	7.20	8.40	9.40	11.20	15.00
2	0.40	6.40	8.20	12.00	17.00
3	-1.60	5.20	9.00	14.00	19.00

etc.

---Stage 7 (March)---

s \ x:	0	1	2	3	4
-2	113.36	120.34	114.78	106.80	103.68
-1	100.34	99.78	91.80	88.68	88.57
0	79.78	76.80	73.68	73.57	75.18
1	68.80	70.68	70.57	72.18	76.25
2	62.68	67.57	69.18	73.25	78.25
3	59.57	66.18	70.25	75.25	80.25

---Stage 8 (February)---

s \ x:	0	1	2	3	4
-2	123.68	130.66	125.11	117.12	114.01
-1	110.66	110.11	102.12	99.01	98.90
0	90.11	87.12	84.01	83.90	85.51
1	79.12	_____	80.90	82.51	86.57
2	73.01	77.90	79.51	83.57	88.57
3	69.90	76.51	80.57	85.57	90.57

---Stage 9 (January)---

s \ x:	0	1	2	3	4
1	89.44	91.33	91.22	92.83	96.90

The values of f_n and X_n are:

Stage 9 (*January*)

State	Optimal Values	Optimal Decision
1 Stock1	89.44	0 Idle

Stage (*February*)

State	Optimal Values	Optimal Decision
-2 Back2	114.01	4 Prod 4
-1 Back1	98.90	4 Prod 4
0 Empty	83.90	3 Prod 3
1 Stock1	79.12	0 Idle
2 Stock2	73.01	0 Idle
3 Stock3	69.90	0 Idle

Stage 7 (*March*)

State	Optimal Values	Optimal Decision
-2 Back2	103.68	4 Prod 4
-1 Back1	88.57	4 Prod 4
0 Empty	_____	_________
1 Stock1	68.80	0 Idle
2 Stock2	62.68	0 Idle
3 Stock3	59.57	0 Idle

Stage 6 (*April*)

State	Optimal Values	Optimal Decision
-2 Back2	93.36	4 Prod 4
-1 Back1	78.25	4 Prod 4
0 Empty	63.25	3 Prod 3
1 Stock1	58.47	0 Idle
2 Stock2	52.36	0 Idle
3 Stock3	49.25	0 Idle

Stage 5 (*May*)

State	Optimal Values	Optimal Decision
-2 Back2	83.04	4 Prod 4
-1 Back1	67.93	4 Prod 4
0 Empty	52.93	3 Prod 3
1 Stock1	48.15	0 Idle
2 Stock2	42.04	0 Idle
3 Stock3	38.93	0 Idle

Stage 4 (*June*)

State	Optimal Values	Optimal Decision
-2 Back2	72.73	4 Prod 4
-1 Back1	57.60	4 Prod 4
0 Empty	42.60	3 Prod 3
1 Stock1	37.83	0 Idle
2 Stock2	31.73	0 Idle
3 Stock3	28.60	0 Idle

Stage 3 (*July*)

State	Optimal Values	Optimal Decision
-2 Back2	62.36	4 Prod 4
-1 Back1	47.28	4 Prod 4
0 Empty	32.28	3 Prod 3
1 Stock1	27.54	0 Idle
2 Stock2	21.36	0 Idle
3 Stock3	18.28	0 Idle

Stage 2 (*August*)

State	Optimal Values	Optimal Decision
-2 Back2	52.10	4 Prod 4
-1 Back1	37.04	4 Prod 4
0 Empty	22.04	3 Prod 3
1 Stock1	17.06	0 Idle
2 Stock2	11.10	0 Idle
3 Stock3	8.04	0 Idle

Stage 1 (*September*)

State	Optimal Values	Optimal Decision
-2 Back2	41.40	4 Prod 4
-1 Back1	26.40	3 Prod 3
0 Empty	11.40	2 Prod 2
1 Stock1	7.20	0 Idle
2 Stock2	0.40	0 Idle
3 Stock3	-1.60	0 Idle

2. *Stochastic Machine Replacement*-- A component of a machine has an active life, measured in weeks, that is a random variable T, where

t	P{T=t}
1	0.1

2	0.2
3	0.4
4	0.3

Note that the component never survives more than four weeks! Suppose that we start with a fresh component, and wish to plan the replacement strategy for the next eight weeks, after which the machine will be retired. At the beginning of each week, the component is inspected and determined to be either operational or broken down. (That is, the component is not continuously monitored, and so the broken-down condition is only discovered at the beginning of the week.) At the beginning of the week, after determining the condition of the component, we may decide to replace it with a fresh component or, if it is still operational, to continue with the current component. The machine earns $500 in revenues each week that it is operational with no breakdowns. Assume that no revenues are earned in a week if the component fails. A replacement for the component costs $300. We began last week with a fresh component.

We define the stages to be the weeks, with the number of the stage being the number of weeks *remaining* in the eight-week period. S_n = state = age (in weeks) of the component at the beginning of stage n, where S_n = 4 means either that it is 4 weeks old or has broken down. The decisions are X_n= 0 (keep) or 1 (replace). The optimal value $f_n(S_n)$is the maximum expected value (revenues minus replacement costs) for the last n weeks, if the current state of the component is S_n.

(a.) What is the failure probability for a 2-week-old component? ________

(b.) What is the failure probability for a 3-week-old component? ________

(c.) What is the expected total value for the seventh and eighth weeks if, in the seventh week (i.e., n=2), the part is 3 weeks old and is kept? $________

= $________ (for the current week) + $_________ (for the remaining weeks).

(d.) In the third week (i.e., n=6 weeks to go), if the component is replaced the total expected value will be $___________

= $________ (for the current week) + $_________ (for the remaining weeks).

(e.) What is the maximum total expected value for the eight week period, if the component is one week old at the beginning of the period? $___________

(f.) If the part survives the first week, should it be replaced? ______

(g.) In general, the optimal replacement rule seems to be "replace when ___ weeks old or when it has failed", except in the final week, when the rule is

____

```
=======================================================================
                ---Stage 1---                              ---Stage 4---
        s \ x:   0        1                        s \ x:   0        1
---------------------------                   ---------------------------
        1 |   450.00   150.00                      1 |  1407.05  1177.35
        2 |   388.89   150.00                      2 |  1329.39  1177.35
        3 |   214.29   150.00                      3 |  1154.79  1177.35
        4 |     0.00   150.00                      4 |   940.50  1177.35

                ---Stage 2---                              ---Stage 5---
        s \ x:   0        1                        s \ x:   0        1
---------------------------                   ---------------------------
        1 |   815.00   570.00                      1 |  1764.19  1534.08
        2 |   588.89   570.00                      2 |  1566.24  1534.08
        3 | (c)_____    570.00                     3 |  1391.64  1534.08
        4 |   150.00   570.00                      4 |  1177.35  1534.08

                ---Stage 3---                              ---Stage 6---
        s \ x:   0        1                        s \ x:   0        1
---------------------------                   ---------------------------
        1 |  1037.00   940.50                      1 |  2013.02  (d)_____
        2 |   958.89   940.50                      2 |  1922.97  (d)_____
        3 |   784.29   940.50                      3 |  1748.37  (d)_____
        4 |   570.00   940.50                      4 |  1534.08  (d)_____
```

---Stage 7---

s \ x:	0	1
1	2369.79	2150.84
2	2280.06	2150.84
3	2105.46	2150.84
4	1891.17	2150.84

---Stage 8---

s \ x:	0	1
1	2717.14	2497.89
4	2150.84	2497.89

The optimal values (f_n) & decisions (X_n):

Stage 8 (i.e., initial week)

State	Optimal Values	Optimal Decision
1)1 wk old	2717.1409	0 keep
4)failed	2497.89	1 replace

Stage 7

State	Optimal Values	Optimal Decision
1)1 wk old	2369.7895	0 keep
2)2 wk old	2280.0634	0 keep
3)3 wk old	2150.8382	1 replace
4) failed	2150.8382	1 replace

Stage 6

State	Optimal Values	Optimal Decision
1)1 wk old	2013.0230	0 keep
2)2 wk old	1922.9689	0 keep
3)3 wk old	1891.1745	1 replace
4) failed	1891.1745	1 replace

Stage 5

State	Optimal Values	Optimal Decision
1)1 wk old	1764.1850	0 keep
2)2 wk old	1566.2389	0 keep
3)3 wk old	1534.0800	1 replace
4) failed	1534.0800	1 replace

Stage 4

State	Optimal Values	Optimal Decision
1)1 wk old	1407.0500	0 keep
2)2 wk old	1329.3889	0 keep
3)3 wk old	1177.3500	1 replace
4) failed	1177.3500	1 replace

Stage 3

State	Optimal Values	Optimal Decision
1)1 wk old	1037.0000	0 keep
2)2 wk old	958.8889	0 keep
3)3 wk old	940.5000	1 replace
4) failed	940.5000	1 replace

Stage 2

State	Optimal Values	Optimal Decision
1)1 wk old	815.0000	0 keep
2)2 wk old	588.8889	0 keep
3)3 wk old	570.0000	1 replace
4) failed	570.0000	1 replace

Stage 1

State	Optimal Values	Optimal Decision
1)1 wk old	450.0000	0 keep
2)2 wk old	388.8889	0 keep
3)3 wk old	214.2857	0 keep
4) failed	150.0000	1 replace