▼▲▼▲▼▲▼ 56:171 Operations Research ▼▲▼▲▼▲▼
▲▼▲▼▲▼▲ Midterm Examination Solutions ▲▼▲▼▲▼
▼▲▼▲▼▲▼ Fall 1997 ▼▲▼▲▼▲▼

• ***Write your name on the first page, and initial the other pages.***
• ***Answer both questions of Part One, and 4 (out of 5) problems from Part Two.***

		Possible	Score
Part One:	1. True/False	15	_____
	2. Sensitivity analysis (LINDO)	25	_____
Part Two:	3. Simplex method	15	_____
	4. LP duality	15	_____
	5. Transportation problem	15	_____
	6. Project scheduling	15	_____
	7. Decision analysis	15	_____
	total possible:	100	_____

▼▲▼▲▼▲▼ PART ONE ▼▲▼▲▼▲▼

(1.) ***True/False:*** Indicate by "+" or "o" whether each statement is "true" or "false", respectively:

_+_ a. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate.

_o_ b. "Crashing" a critical path problem is a technique used to find a good initial feasible solution.

_+_ c. In the two-phase simplex method, an artificial variable is defined for each constraint row lacking a slack variable (assuming the right-hand-side of the LP tableau is nonnegative).

_+_ d. If the primal LP feasible region is nonempty and unbounded, then the dual LP is infeasible.

_o_ e. In PERT, the total completion time of the project is assumed to have a BETA distribution.

_o_ f. The Revised Simplex Method, for most LP problems, requires fewer pivots than the ordinary simplex method.

_+_ g. All tasks on the critical path of a project schedule have their latest start time equal to their earliest start time.

_+_ h. When maximizing in the simplex method, the value of the objective function increases at every iteration unless a degenerate tableau is encountered.

_o_ i. The critical path in a project network is the shortest path from a specified source node (beginning of project) to a specified destination node (end of project).

_+_ j. The assignment problem is a special case of a transportation problem.

_o_ k. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible.

_o_ l. A basic solution of an LP is always feasible, but not all feasible solutions are basic.

_+_ m. In Phase One of the 2-Phase method, one should never pivot in the column of an artificial variable.

_+_ n. In a transportation problem if the total supply exceeds total demand, a "dummy" destination should be defined.

_o_ o. If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must also be positive.

(2.) ***Sensitivity Analysis in LP.***

Problem Statement: McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

i) Red Baron must contain no more than 75% of A.

ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of $3.35 for Diablo and $2.85 for Red Baron. A and B cost $1.60 and $2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

Define *D = quarts of Diablo to be produced*
R = quarts of Red Baron to be produced
AD= quarts of A used to make Diablo
AR = quarts of A used to make Red Baron
BD = quarts of B used to make Diablo
BR = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

```
MAX     3.35 D + 2.85 R - 1.6 AD - 1.6 AR - 2.05 BD - 2.05 BR
SUBJECT TO
       2) - D + AD + BD =    0
       3) - R + AR + BR =    0
       4)   AD + AR <=   40
       5)   BD + BR <=   30
       6) - 0.25 D + AD >=   0
       7) - 0.5 D + BD >=   0
       8) - 0.75 R + AR <=   0
END
        OBJECTIVE FUNCTION VALUE
 1)        99.0000000
```

VARIABLE	VALUE	REDUCED COST
D	50.000000	0.000000
R	20.000000	0.000000
AD	25.000000	0.000000
AR	15.000000	0.000000
BD	25.000000	0.000000
BR	5.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-2.350000
3)	0.000000	-4.350000
4)	0.000000	0.750000
5)	0.000000	2.300001
6)	12.500000	0.000000
7)	0.000000	-1.999999
8)	0.000000	2.000000

RANGES IN WHICH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
D	3.350000	0.750000	0.500000
R	2.850000	0.500000	0.375000
AD	-1.600000	1.500001	0.666666
AR	-1.600000	0.666666	0.500000

BD	-2.050000	1.500001	1.000000
BR	-2.050000	1.000000	1.500001

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	10.000000	10.000000
3	0.000000	16.666668	3.333333
4	40.000000	50.000000	10.000000
5	30.000000	10.000000	16.666664
6	0.000000	12.500000	INFINITY
7	0.000000	6.250000	5.000000
8	0.000000	2.500000	12.500000

THE TABLEAU:

ROW	(BASIS)	D	R	AD	AR	BD	BR	SLK 4	SLK 5	SLK 6
1	ART	0.000	0.000	0.000	0.000	0.000	0.000	0.750	2.300	0.000
2	AD	0.000	0.000	1.000	0.000	0.000	0.000	-0.500	1.500	0.000
3	R	0.000	1.000	0.000	0.000	0.000	0.000	2.000	-2.000	0.000
4	AR	0.000	0.000	0.000	1.000	0.000	0.000	1.500	-1.500	0.000
5	BR	0.000	0.000	0.000	0.000	0.000	1.000	0.500	-0.500	0.000
6	SLK 6	0.000	0.000	0.000	0.000	0.000	0.000	-0.250	0.750	1.000
7	D	1.000	0.000	0.000	0.000	0.000	0.000	-1.000	3.000	0.000
8	BD	0.000	0.000	0.000	0.000	1.000	0.000	-0.500	1.500	0.000

ROW	SLK 7	SLK 8	RHS
1	2.000	2.000	99.000
2	3.000	2.000	25.000
3	-4.000	-4.000	20.000
4	-3.000	-2.000	15.000
5	-1.000	-2.000	5.000
6	2.000	1.000	12.500
7	4.000	4.000	50.000
8	1.000	2.000	25.000

_a_1. If the selling price of DIABLO sauce were to increase from $3.35 /quart to $4.50/quart, the number of quarts of DIABLO to be produced would

a. increase c. remain the same e. *NOTA*
b. decrease d. insufficient info. given

Note: since the increase is $1.15 and the "ALLOWABLE INCREASE" in the objective coefficient of the variable D is $0.75, the basis will change. "Common sense" dictates that D will (in the absence of degeneracy) increase.

_a_2. The LP problem above has

a. exactly one optimal sol'n c. multiple solutions e. insufficient info. given
b. a degenerate solution d. no optimal solution f. *NOTA*

_c_3. If an additional 5 quarts of ingredient B were available, McNaughton's profits would be *(choose nearest value)* :

a. $90 c. $110 e. insufficient info. given
b. $100 d. $120 f. *NOTA*

Note: an additional 5 quarts is less than the "ALLOWABLE INCREASE" (which is 10). Since the dual variable for the right-hand-side of the constraint imposing the limit of 30 quarts is $2.30/quart, the profit will improve (increase) by $2.30/qt (5 qt) = $11.50 to $99+11.50 = $110.50.

_b_4. If the variable "SLK 4" were increased, this would be equivalent to

a. increasing A availability c. increasing B availability
b. decreasing A availability d. decreasing B availability e. *NOTA*

_b _5. If the variable "SLK 4" were decreased by 10 (i.e. from 0 to -10), the quantity of DIABLO produced would be *(choose nearest value)*

a. 30 quarts c. 50 quarts e. ≥70 quarts
b. 40 quarts d. 60 quarts f. *NOTA*

Note: the substitution rate of SLK4 for D is -1, and therefore the variable D will decrease at the rate of 1 quart per unit decrease of SLK4, i.e., from 50 to 40.

_d_6. If a pivot were to be performed to enter the variable SLK4 into the basis, then according to the "minimum ratio test", the value of SLK4 in the resulting basic solution would be *(choose nearest value)*

a. 20 c. 10 e. ≤0.5
b. 15 d. 0.10 f. *NOTA*

Note: minimum {2/20, 1.5/15, 0.5/5} = minimum{0.1, 0.1, 0.1} = 0.1. (The resulting solution will be degenerate, with all three of the variables R, AR, and BR decreasing to zero.)

_i_7. If the variable SLK4 were to enter the basis, then the variable leaving the basis is

a. A c. AD e. D g. SLK6 i. more than one answer is possible
b. B d. BD f. R h. any of the above j. *NOTA*

Note: there are three possible pivot rows (3, 4, 5), and so either R, AR, or BR could be selected to leave the basis.

_b_8. If the variable SLK4 were to enter the basis, then the next tableau

a. indicates multiple optimal sol'ns c. both of the above
b. is degenerate d. *NOTA*

_a_9. The dual of the LP above has an objective function which is to be

a. minimized c. both of the above
b. maximized d. *NOTA*

_c_10. The dual of the LP above has an optimal value which is *(choose nearest value)*

a. 0 c. 100 e. insufficient infomation given
b. 50 d. ≥150 f. *NOTA*

Note: the optimal dual objective = optimal primal objective = 99.

▼▲▼▲▼▲▼ PART TWO ▼▲▼▲▼▲▼

(3.) ***Simplex Method.*** Classify each simplex tableau below by writing "X" in the appropriate (one or more) columns, using the following classifications:

- Is the current solution feasible or not?
- Is the current solution degenerate or not?
- Is there an indication that the LP has an unbounded objective function?
- Is the current solution optimal?
- If the current solution is optimal, are there other optima?

In the tableaus which are <u>feasible</u> but <u>not</u> optimal, circle at least one valid pivot element to improve the objective. *Take careful note of whether the LP is being **min**imized or **max**imized! Note also that (-z), rather than z, appears in the first column (i.e., corresponding to the approach used in my notes instead of that in the text by Winston).*

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UnBounded?	Optimal?	MtiOt'
MIN	1	2	0	4	-3	-2	0	1	0	-10	(X)	(_)	(X)	(_)	(_
	0	0	0	2	-4	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS					
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	(X)	(_)	(_)	(_)	(_
	0	0	0	2	1	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS					
MIN	1	0	0	4	2	3	0	1	0	-10	(X)	(X)	(_)	(X)	(X
	0	0	0	2	1	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	0					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS					
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	(X)	(X)	(_)	(_)	(_
	0	0	0	2	1	0	0	-1	1	0					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS					
MAX	1	2	0	4	-2	-3	0	1	0	-10	(_)	(_)	(_)	(_)	(_
	0	0	0	2	1	0	0	-1	1	-3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	2					

4. **LINEAR PROGRAMMING DUALITY**: Consider the following LP:

Maximize $2X_1 - 13X_2 - 3X_3 - 2X_4 - 5X_5$

subject to

$X_1 - X_2 - 4X_4 - X_5 = 5$

$X_1 - 7X_4 - 2X_5 \geq -1$

$5X_2 + X_3 + X_4 + 2X_5 \leq 5$

$3X_2 + X_3 - X_4 + X_5 \geq 2$

$X_j \geq 0$ for all j=1, 2, 3; $X_4 \leq 0$; X_5 unrestricted in sign

At the primal point X=(6,0,1,0,1),
objective function = -4
left-hand-side of 1st constraint is 5
left-hand-side of 2nd constraint is 4
left-hand-side of 3rdconstraint is 3
left-hand-side of 4th constraint is 2

a. Is this solution feasible? _YES_

b. Is this solution basic? _NO_

Note: in addition to -z (objective), there should be four basic variables. There are five positive variables, namely X_1, X_3, and X_5, together with the surplus variable in the second constraint and the slack variable in the third constraint, and so the solution cannot be basic.

c. Is this solution degenerate? _NO_ *Note: since degeneracy is a property of basic solutions, the category doesn't apply.*

d. Complete the following properties of the dual problem of this LP:
Number of dual variables: _4__
Number of dual constraints (not including nonnegativity): __5__
Type of optimization: Minimize

e. Write out in full a dual problem of the LP above, denoting your dual variables by Y_1, Y_2, etc..

Minimize $5Y_1 - Y_2 + 5Y_3 + 2Y_4$

subject to

$Y_1 + Y_2 \geq 2$

$-Y_1 + 5Y_3 + 3Y_4 \geq -13$

$Y_3 + Y_4 \geq -3$

$-4Y_1 - 7Y_2 + Y_3 + 3Y_4 \leq -2$

$-Y_1 - 2Y_2 + 2Y_3 = -5$

Y_1 unrestricted in sign, $Y_2 \leq 0$, $Y_3 \geq 0$, $Y_4 \leq 0$

f. IF X=(6,0,1,0,1) is optimal in the primal problem, then which **dual** variables (including slack or surplus variables) must be **zero** in the dual optimal solution, according to the complementary slackness conditions for this primal-dual pair of problems? (circle. Ignore variables not defined i the dual problem, e.g., slack variables in nonexistent constraints.)

Y_1 Y_2 Y_3 Y_4 Y_5 Y_6 …..

Constraint #1 slack	Constraint #4 slack
Constraint #2 slack	Constraint #5 slack
Constraint #3 slack	Constraint #6 slack

…. etc.

According to the Complementary Slackness Theorem,

Note:
$X_1>0$ implies that dual constraint 1 must be tight, i.e., the slack must be zero
$X_3>0$ implies that dual constraint 3 must be tight, i.e., the slack must be zero
$X_5>0$ implies that dual constraint 5 must be tight, i.e., the slack must be zero
Primal constraint 2 slack implies that dual variable 2 must be zero
Primal constraint 3 slack implies that dual variable 3 must be zero

5. **Transportation Problem:** Consider the transportation problem with the tableau below:

destinations

sources	1	2	3	supply
1	5 [9]	[5]	[4]	5
2	[7]	[6]	[12]	18
3	[10]	[9]	[3]	7
demand	10	5	15	

a. If the ordinary simplex tableau were to be written for this problem, how many rows (including the objective) will it have? __7__
How many variables (excluding the objective value -z) will it have? _3x3 = 9__

b. Is this transportation problem "balanced?" __YES_

c. How many basic variables will this problem have? __m+n-1 = 3+3-1 = 5__

d. An initial basic feasible solution is found using the "Northwest Corner Method"; complete the computatio of this solution and write the values of the variables in the tableau above.
Solution: $X_{11} = 5$, $X_{21} = 5$, $X_{22}=5$, $X_{23}=8$, $X_{33}=7$.

e. If U_1 (the dual variable for the first source) is equal to 0, what is the value of
U_2 (the dual variable for the second source)? __-2___
V_1 (the dual variable for the first destination)? __+9___
V_2 (the dual variable for the second destination)? __+8____

Note: $U_1=0$ implies that (since X_{11} is basic) $V_1 = 9$. Then X_{21} basic implies that $U_2 = -2$. It then follows tha since X_{22} is basic, $V_2 =+8$.

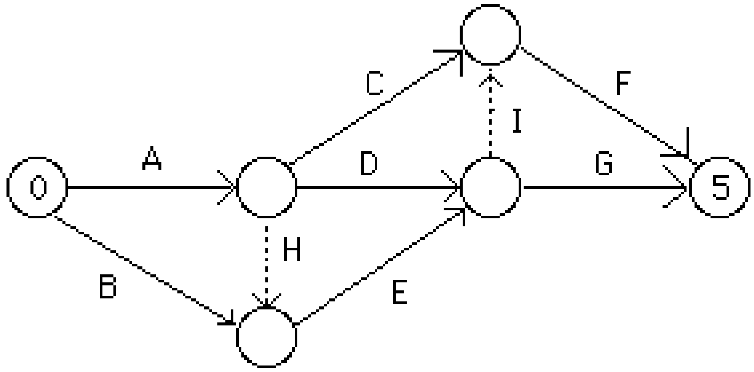
f. What is the reduced cost of the variable X_{12}? -3 = $C_{12} - (U_1 + V_2) = 5 - (0+8)$

g. Will increasing X_{12} improve the objective function? __YES__

h. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{12} enters?
either X_{11} or X_{22}

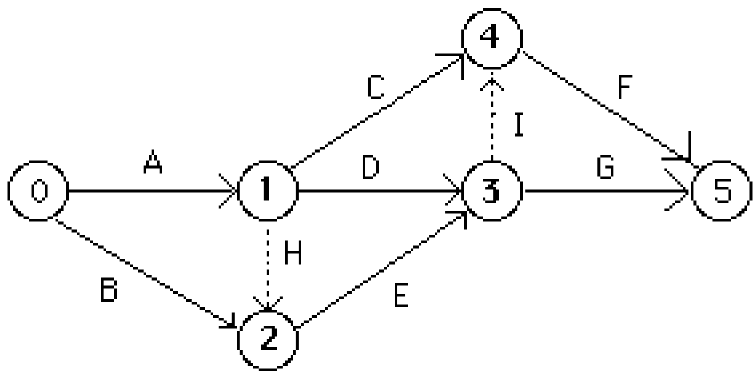
i. What will be the value of X_{12} if it is entered into the solution as in (h)? ___5___

6. ***Project Scheduling.*** Consider the project with the A-O-A (activity-on-arrow) network given below.

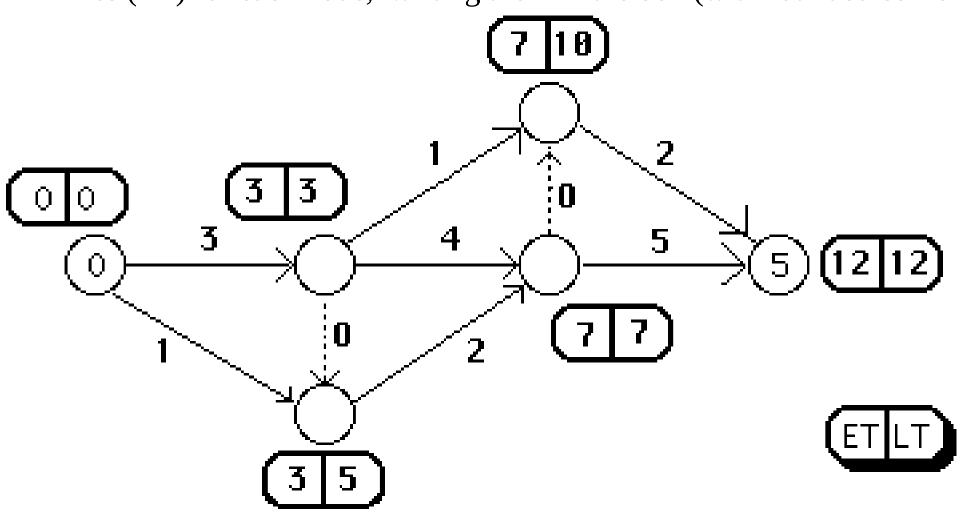


a. How many activities (i.e., tasks), <u>not</u> including "dummies", are required to complete this project? ___<u>seven</u>___

b. Complete the labeling of the nodes on the network above.



c. The activity durations (in days) are given below on the arrows. Compute the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

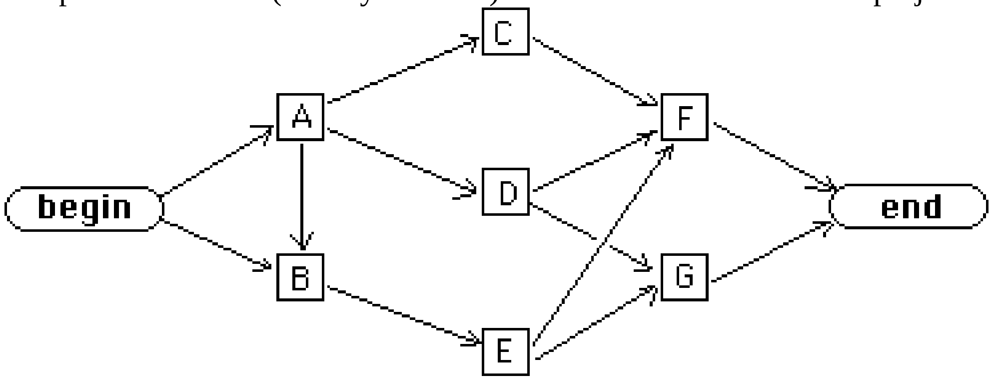


d. Find the slack ("total float") for activity B. __<u>4</u>__days
Note: Early start time for B is ES= 0, and latest finish time is LF= 5, which implies that the latest start time is LS= 5-1=4. Hence the slack is LS-ES = 4-0 = 4.

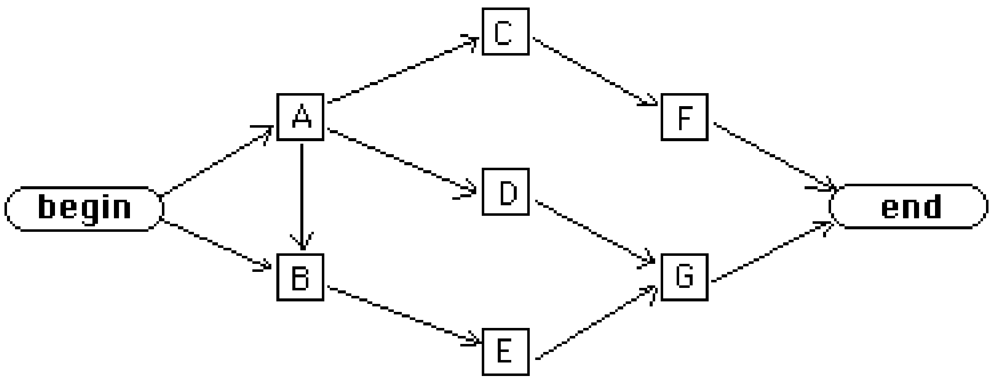
e. Which activities are critical? <u>A</u> B C <u>D</u> E F <u>G</u> H I

f. What is the earliest completion time for the project? ___<u>12</u>__days

g. Complete the A-O-N (activity-on-node) network below for this same project.



h. Suppose that the arrow labelled "I" in the original AOA network is deleted. Indicate the resulting A-O-N network below:



7. Decision Analysis. We have $1000 to invest in one of the following: Gold, Stock, or Money Market. The value of the $1000 investment a year from now depends upon the unknown state of the economy in the intervening year. The value of the investment one year from now is given by the table:

Investment	Weak	Moderate	Strong
Money market	$1100	$1100	$1100
Stock	$1000	$1100	$1200
Gold	$1600	$300	$1400

a. What is the optimal investment decision if your criterion is "maximin"? _Money market_
since Max{1100, 1000, 300} = 1100
What is the optimal investment decision if your criterion is "maximax"? __Gold
since Max{1100, 1200, 1600}= 1600

b. Complete the regret table:

Investment	Weak	Moderate	Strong
Money market	500	0	300
Stock	600	0	200
Gold	0	800	0

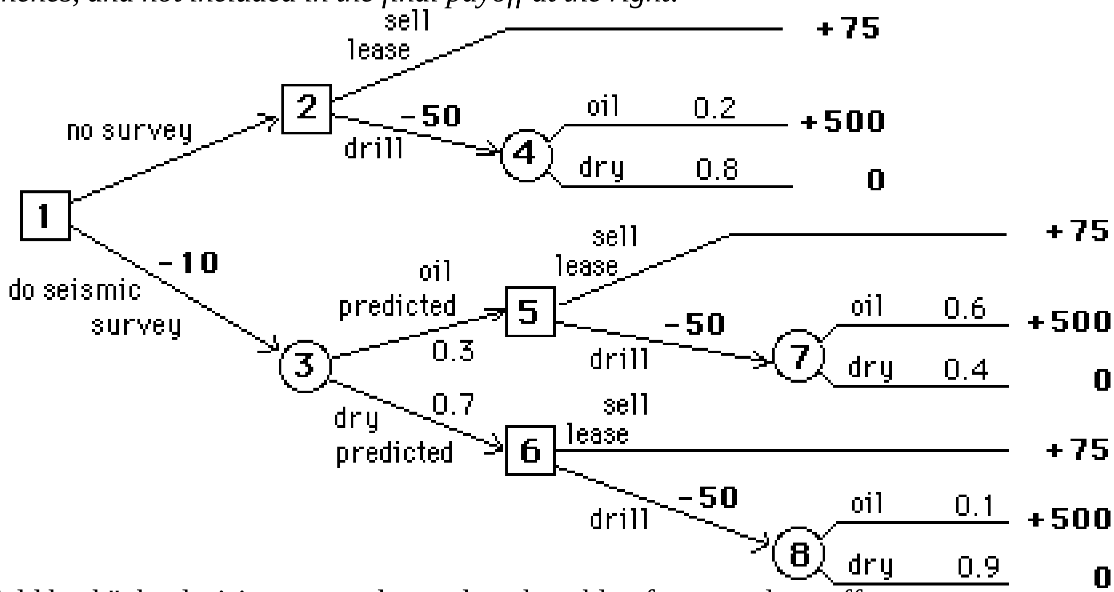
c. What is the optimal investment decision if your criterion is "minimax regret"? Money market
since Min{500, 600, 800} = 500

Suppose that you own a lease on the oil rights of a piece of land. You have the options of

- selling the lease for $75,000
- drilling for oil yourself, which costs you an investment of $50,000

If you choose to drill, the estimated probability of finding oil is 20%, in which case the "payoff" is $500,000.
If the oil well is "dry", there is no payoff, of course.
Before you make the above decision, you have the option of hiring a geologist to do a seismic survey for $10,000. The geologist will predict either that there is oil or that the well will be dry. If he predicts oil, he has been right 60% of the time. When he predicts a dry well, he is right 90% of the time.
The probability that the geologist will predict oil is 30%.

These values have been inserted in a decision tree shown below, with costs and payoffs expressed in thousands of dollars. *Note that the costs of the survey and of the drilling are indicated on the decision branches, and not included in the final payoff at the right!*



"Fold back" the decision tree and complete the table of expected payoffs:

node	E[payoff]	node	E[payoff]	node	E[payoff]
1	117.5	4	100	7	300
2	75	5	250	8	50
3	127.5	6	75		

d. Should you hire the geologist to perform the seismic survey? YES

e. What is the expected value of the geologist's survey? 52.5