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Midterm Exam Solutions

✣ ✣ ✣ ✣ ✣ ✣ ✣ Fall 2001 ✣ ✣ ✣ ✣ ✣ ✣ ✣

True/False: Indicate by "+" or "o" whether each statement is "true" or "false", respectively:

__o_ 1. If a primal LP constraint is slack at the optimal solution, then the optimal value of the dual variable for that same constraint must be positive.

__+_ 2. In reference to LP, the terms "dual variable" and "simplex multiplier" are synonymous.

__+_ 3. If you make a mistake in choosing the pivot row in the simplex method, the next basic solution will have one or more negative basic variables.

__+_ 4. An assignment problem is a special type of linear programming problem.

__+_ 5. Every basic feasible solution of an assignment problem is degenerate.

__o_ 6. A degenerate solution of an LP is one which has more nonbasic than basic variables.

__+_ 7. If a basic feasible solution of a transportation problem is not degenerate, the next iteration must result in an improvement of the objective.

__o_ 8. The two-phase simplex method solves for the primal variables in phase one, and then solves for the dual variables in phase two.

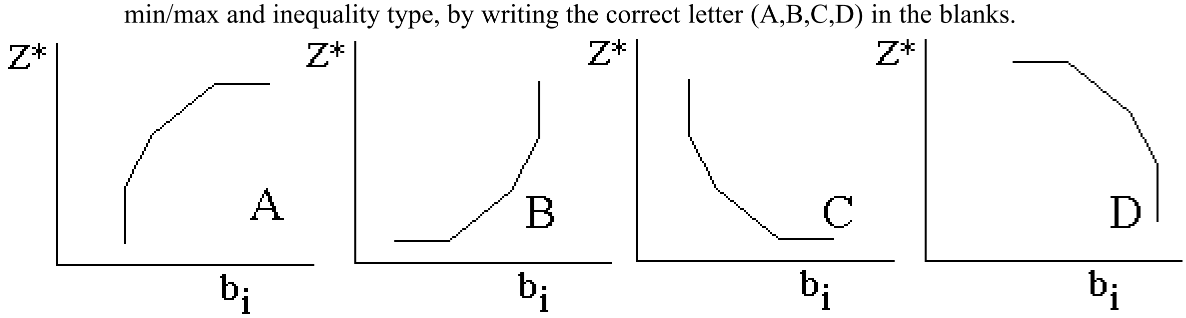
__o_ 9. In a "balanced" transportation problem, the number of sources equals the number of destinations

__+_ 10. The minimum expected regret is never less than the expected value of perfect information.

__o_ 11. A dual variable for an equality constraint is always zero.

__o_ 12. In a maximization LP problem, if the right-hand-side of a "greater-than-or-equal" constraint is increased, the objective function will either remain the same or increase.

Match the four hypothetical graphs of optimal value vs right-hand-side to the appropriate combination of min/max and inequality type, by writing the correct letter (A,B,C,D) in the blanks.



_B_ 13. Min cx st $Ax \geq b$

_C__ 14. Min cx st $Ax \leq b$

_D_ 15. Max cx st $Ax \geq b$

_A_ 16. Max cx st $Ax \leq b$

__e_ 17. If, in the optimal *primal* solution of an LP problem (min cx st $Ax \geq b$, $x \geq 0$), there is zero slack in constraint #1, then in the optimal dual solution,

a. dual variable #1 must be zero
b. dual variable #1 must be positive
c. slack variable for dual constraint #1 must be zero
d. dual constraint #1 must be slack
e. *NOTA*

__c_ 18. If, in the optimal solution of the *dual* of an LP problem (min cx subject to: $Ax \geq b$, $x \geq 0$), dual variable #2 is positive, then in the optimal *primal* solution,

a. variable #2 must be zero
b. variable #2 must be positive
c. slack variable for constraint #2 must be zero
d. constraint #2 must be slack
e. *NOTA*

__d_ 19. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau

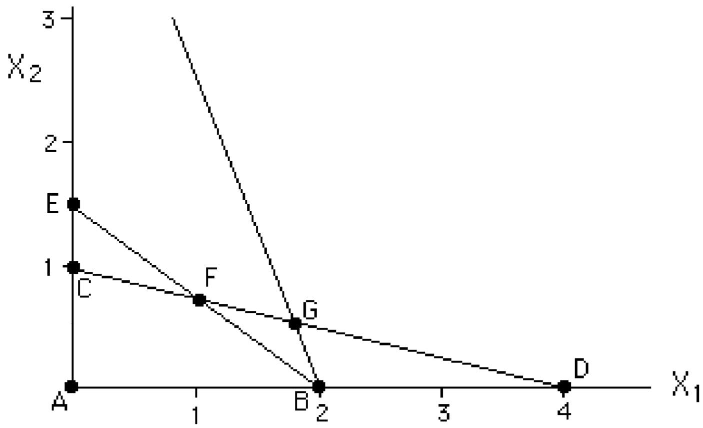
a. will be nonbasic
a. will be nonfeasible
c. will have a worse objective value
d. will be degenerate
e. *NOTA*

__b_ 20. Bayes' Rule is used to compute

a. the joint probability of a "state of nature" and the outcome of an experiment
b. the conditional probability of a "state of nature" given the outcome of an experiment
c. the conditional probability of the outcome of an experiment given a "state of nature"
d. NOTA

The problems below refer to the following LP:

	(with inequalities converted to equations:)
Minimize $8X_1 + 4X_2$	Minimize $8X_1 + 4X_2$
subject to $3X_1 + 4X_2 \leq 6$	subject to $3X_1 + 4X_2 + X_3 = 6$
$X_1 + 4X_2 \geq 4$	$X_1 + 4X_2 - X_4 = 4$
$5X_1 + 2X_2 \leq 10$	$5X_1 + 2X_2 + X_5 = 10$
$X_1 \geq 0, X_2 \geq 0$	$X_j \geq 0$, j=1,2, 3,4,5



_c__ 21 .The feasible region includes points
- a. B, F, & G
- b. A, B, C, & F
- c. C, E, & F
- d. E, F, &G
- e. B, D, & G
- f. *NOTA*

_e__ 22. At point F, the basic variables include the variables
- a. X_2 & X_3
- b. X_3 & X_4
- c. X_4 & X_5
- d. X_1 & X_4
- e. X_2 & X_5
- f. *NOTA*

_c__ 23. The dual of the original LP (before introducing slack & surplus variables) has the following constraints (not including nonnegativity or nonpositivity constraints):
- a. 2 constraints of type (≥)
- b. one each of type ≤ & ≥
- c. 2 constraints of type (≤)
- d. one each of type ≥ & =
- e. *NOTA*

_b__ 24. The dual of the LP has the following types of variables:
- a. two non-negative variables and one non-positive variable
- b. one non-negative and two non-positive variables
- c. two non-negative variables and one unrestricted in sign
- d. three non-negative variables
- e. three non-positive variables
- f. NOTA

_f__ 25. If point F is optimal, then which dual variables must be zero, according to the *Complementary Slackness Theorem* ?
- a. Y_1 and Y_2
- b. Y_1 and Y_3
- c. Y_2 and Y_3
- d. Y_1 only
- e. Y_2 only
- f. Y_3 only

_d__ 26. When using the Hungarian method to solve assignment problems, if the number of lines drawn to cover the zeroes in the reduced matrix is equal to the number of rows,
- a. this indicates that no solution exists.
- b. a dummy row or column must be introduced
- c. a mistake occurred; one should review previous steps.
- d. this means an optimal solution has been reached.

The following is a transportation tableau, with an initial set of shipments indicated:

	1	2	3	4	supply
1	4 [5]	3 [1]	[3]	[5]	7
2	[6]	1 [2]	4 [3]	[4]	5
3	[3]	[3]	2 [4]	1 [4]	3
demand	4	4	6	1	

27. Is the solution above a basic feasible solution? _yes_

Complete the computation of a set of dual variables for the above transportation tableau, starting by assigning the dual variable for source #1 equal to zero:

28. Dual variables for the supply constraints: $U_1 = 0$, $U_2 =$ _+1_ , $U_3 = +2$

29. Dual variables for demand constraints: $V_1 = +5$, $V_2 = +1$, $V_3 = +2$, $V_4 =$ _+2_

30. Compute the reduced costs for X_{21} _0_ = 6 – (1+5)

31. & X_{31} _–4_ =3–(2+5)

_b_ 32. Which of these two variables should enter the basis?

a. X_{21} b. X_{31} c. both d. neither

_c_ 33. Which basic variable should leave the basis? ______

a. X_{11} b. X_{12} c. X_{22} d. X_{23} e. X_{33} f. *NOTA*

_b_ 34. If $X_{ij}>0$ in the transportation problem, then dual variables U and V *must* satisfy

a. $C_{ij} > U_i + V_j$ c. $C_{ij} < U_i + V_j$ e. $C_{ij} = U_i - V_j$

b. $C_{ij} = U_i + V_j$ d. $C_{ij} + U_i + V_j = 0$ f. *NOTA*

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Sensitivity Analysis in LP. Consult the LINDO output below to answer the questions:

_a_ 35. The number of optimal solutions of this LP is

a. exactly one c. infinite e. NOTA

b. exactly two d. none

Suppose that a failure of one of the Wheeling presses in June shuts down production for ten hours.

_a_ 36. The resulting increase in cost is (in dollars)

a. between 0 and 100 *(10×0.333)* c. between 200 and 300 e. between 400 and 500

b. between 100 and 200 d. between 300 and 400 f. greater than 500

_c_ 37. Taking into account this failure is equivalent to

a. increasing the variable WN1 by 10 c. increasing the variable SLK2 by 10 e. *NOTA*

b. decreasing the variable WN1 by 10 d. decreasing the variable SLK2 by 10

_a_ 38. If a pivot were performed to enter SLK2 into the basis, the variable leaving the basis would be

a. SLK3 *by min ratio test* c. WN1 e. RN1 g. *NOTA*

b. SLK7 d. WG1 f. RG1

_c_ 39. If a pivot were performed to enter SLK2 into the basis, the resulting value of SLK2 would be

a. between 0 and 100 c. between 200 and 300 *(278.667/1.067)* e. between 400 and 500

b. between 100 and 200 d. between 300 and 400 f. greater than 500

40-41. The effect on variable WN1 of an increase of 10 in the variable IG2 would be to

(increase / [decrease]) by _8_

_b_ 42. Suppose that the monthly storage cost of a nylon tire at the end of June were to increase from 10 cents to 15 cents. Then…

a. neither the basis nor the values of the variables will change

b. both the basis and the values of the variables will change *since the basis B changes*

c. the basis will not change, but the values of the variables will change

d. insufficient information is available to answer this question

e. NOTA

_c_ 43. Suppose that the demand for fiberglass tires in June were to double. Then…

a. neither the basis nor the values of the variables will change

b. both the basis and the values of the variables will change

c. the basis will not change, but the values of the variables will change *since* $x_B = (A_B)^{-1} b$ *&b changes*

d. insufficient information is available to answer this question

e. NOTA

An automobile tire manufacturer has the ability to produce both nylon and fiberglass tires. During the next 3 months, they have agreed to deliver the following quantities:

Date	Nylon	Fiberglass
June 30	4000	1000
July 31	8000	5000
August 31	3000	5000

The company has two presses, referred to as the Wheeling and Regal machines, and appropriate molds which can be used to produce these tires, with the following production hours available in the upcoming months:

Month	Wheeling	Regal
June	700	1500
July	300	400
August	1000	300

The production rates for each machine-and-tire combination, in terms of hours per tire, are as follows:

Tire	Wheeling	Regal
Nylon	0.15	0.16
Fiberglass	0.12	0.14

Production costs per tire, including labor and materials, are

Tire	Wheeling	Regal
Nylon	0.75	0.80
Fiberglass	0.60	0.70

The inventory carrying charge is $0.10 per tire per month.

Decision variables:

- WNt = number of nylon tires to be produced on the Wheeling machine during month t, t=1,2,3
- RNt = number of nylon tires to be produced on the Regal machine during month t, t=1,2,3
- WGt = number of fiberglass tires to be produced on the Wheeling machine during month t, t=1,2,3
- RGt = number of fiberglass tires to be produced on the Regal machine during month t, t=1,2,3
- INt = number of nylon tires put into inventory at the end of month t, t=1,2
- IGt = number of fiberglass tires put into inventory at the end of month t, t=1,2

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MIN      0.75 WN1 + 0.8 RN1 + 0.6 WG1 + 0.7 RG1 + 0.1 IN1 + 0.1 IG1
      + 0.75 WN2 + 0.8 RN2 + 0.6 WG2 + 0.7 RG2 + 0.1 IN2 + 0.1 IG2
      + 0.75 WN3 + 0.8 RN3 + 0.6 WG3 + 0.7 RG3
  SUBJECT TO
         2)   0.15 WN1 + 0.12 WG1 <=   700
         3)   0.16 RN1 + 0.14 RG1 <=   1500
         4)   0.15 WN2 + 0.12 WG2 <=   300
         5)   0.16 RN2 + 0.14 RG2 <=   400
         6)   0.15 WN3 + 0.12 WG3 <=   1000
         7)   0.16 RN3 + 0.14 RG3 <=   300
         8)   WN1 + RN1 - IN1 =    4000
         9)   WG1 + RG1 - IG1 =    1000
        10)   IN1 + WN2 + RN2 - IN2 =    8000
        11)   IG1 + WG2 + RG2 - IG2 =    5000
        12)   IN2 + WN3 + RN3 =    3000
        13)   IG2 + WG3 + RG3 =    5000
  END
        OBJECTIVE FUNCTION VALUE
        1)      19173.33

  VARIABLE        VALUE          REDUCED COST
       WN1      1866.666626          0.000000
       RN1      7633.333496          0.000000
       WG1      3500.000000          0.000000
       RG1         0.000000          0.060000
       IN1      5500.000000          0.000000
       IG1      2500.000000          0.000000
       WN2         0.000000          0.025000
       RN2      2500.000000          0.000000
       WG2      2500.000000          0.000000
       RG2         0.000000          0.047500
       IN2         0.000000          0.200000
       IG2         0.000000          0.200000
       WN3      2666.666748          0.000000
       RN3       333.333344          0.000000
       WG3      5000.000000          0.000000
       RG3         0.000000          0.060000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.333333
        3)       278.666656          0.000000
        4)         0.000000          1.166667
```

5)	0.000000	0.625000
6)	0.000000	0.333333
7)	246.666672	0.000000
8)	0.000000	-0.800000
9)	0.000000	-0.640000
10)	0.000000	-0.900000
11)	0.000000	-0.740000
12)	0.000000	-0.800000
13)	0.000000	-0.640000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
WN1	0.750000	0.025000	0.059375
RN1	0.800000	0.075000	0.050000
WG1	0.600000	0.047500	0.020000
RG1	0.700000	INFINITY	0.060000
IN1	0.100000	0.025000	0.054286
IG1	0.100000	0.047500	0.020000
WN2	0.750000	INFINITY	0.025000
RN2	0.800000	0.054286	INFINITY
WG2	0.600000	0.020000	INFINITY
RG2	0.700000	INFINITY	0.047500
IN2	0.100000	INFINITY	0.200000
IG2	0.100000	INFINITY	0.200000
WN3	0.750000	0.050000	0.075000
RN3	0.800000	0.075000	0.050000
WG3	0.600000	0.060000	INFINITY
RG3	0.700000	INFINITY	0.060000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	700.000000	1145.000000	261.249969
3	1500.000000	INFINITY	278.666656
4	300.000000	300.000000	261.249969
5	400.000000	880.000000	278.666656
6	1000.000000	50.000004	231.250000
7	300.000000	INFINITY	246.666672
8	4000.000000	1741.666626	7633.333496
9	1000.000000	2177.083252	3500.000000
10	8000.000000	1741.666626	5500.000000
11	5000.000000	2177.083252	2500.000000
12	3000.000000	1541.666748	333.333344
13	5000.000000	1927.083252	416.666687

THE TABLEAU

ROW	(BASIS)	WN1	RN1	WG1	RG1	IN1	IG1
1	ART	0.000	0.000	0.000	0.060	0.000	0.000
2	WN1	1.000	0.000	0.000	-0.800	0.000	0.000
3	SLK 3	0.000	0.000	0.000	0.012	0.000	0.000
4	WG2	0.000	0.000	0.000	0.000	0.000	0.000
5	RN2	0.000	0.000	0.000	0.000	0.000	0.000
6	WN3	0.000	0.000	0.000	0.000	0.000	0.000
7	SLK 7	0.000	0.000	0.000	0.000	0.000	0.000
8	RN1	0.000	1.000	0.000	0.800	0.000	0.000
9	WG1	0.000	0.000	1.000	1.000	0.000	0.000
10	IN1	0.000	0.000	0.000	0.000	1.000	0.000
11	IG1	0.000	0.000	0.000	0.000	0.000	1.000
12	RN3	0.000	0.000	0.000	0.000	0.000	0.000
13	WG3	0.000	0.000	0.000	0.000	0.000	0.000

ROW	WN2	RN2	WG2	RG2	IN2	IG2	WN3
1	0.025	0.000	0.000	0.047	0.200	0.200	0.000
2	1.000	0.000	0.000	-0.800	0.000	0.800	0.000
3	0.000	0.000	0.000	0.012	0.160	0.128	0.000
4	1.250	0.000	1.000	0.000	0.000	0.000	0.000
5	0.000	1.000	0.000	0.875	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	-0.800	1.000
7	0.000	0.000	0.000	0.000	-0.160	-0.128	0.000
8	0.000	0.000	0.000	-0.075	-1.000	-0.800	0.000
9	-1.250	0.000	0.000	1.000	0.000	-1.000	0.000
10	1.000	0.000	0.000	-0.875	-1.000	0.000	0.000
11	-1.250	0.000	0.000	1.000	0.000	-1.000	0.000
12	0.000	0.000	0.000	0.000	1.000	0.800	0.000
13	0.000	0.000	0.000	0.000	0.000	1.000	0.000

ROW	RN3	WG3	RG3	SLK 2	SLK 3	SLK 4	SLK 5
1	0.000	0.000	0.060	0.333	0.000	1.167	0.625
2	0.000	0.000	0.000	6.667	0.000	6.667	0.000
3	0.000	0.000	0.000	1.067	1.000	1.067	1.000
4	0.000	0.000	0.000	0.000	0.000	8.333	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	6.250
6	0.000	0.000	-0.800	0.000	0.000	0.000	0.000
7	0.000	0.000	0.012	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	-6.667	0.000	-6.667	-6.250
9	0.000	0.000	0.000	0.000	0.000	-8.333	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	-6.250
11	0.000	0.000	0.000	0.000	0.000	-8.333	0.000
12	1.000	0.000	0.800	0.000	0.000	0.000	0.000
13	0.000	1.000	1.000	0.000	0.000	0.000	0.000

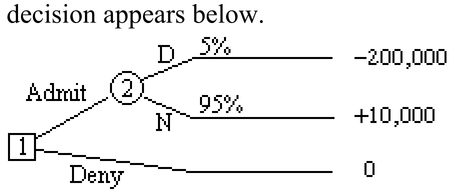
ROW	SLK 6	SLK 7	
1	0.333	0.000	-0.19E+05
2	0.000	0.000	1866.667
3	0.000	0.000	278.667
4	0.000	0.000	2500.000
5	0.000	0.000	2500.000
6	6.667	0.000	2666.667
7	1.067	1.000	246.667
8	0.000	0.000	7633.333
9	0.000	0.000	3500.000
10	0.000	0.000	5500.000
11	0.000	0.000	2500.000
12	-6.667	0.000	333.333
13	0.000	0.000	5000.000

Decision Analysis
The government is attempting to determine whether immigrants should be tested for a certain contagious disease. Let's assume that the decision will be made strictly on a financial basis.

Assume that each immigrant who is allowed into the country and has the disease (event **D**) costs the U.S. $200,000, and each immigrant who enters and does not have the disease (event **N**) will contribute $10,000 to the national economy. Assume that 5% of all potential immigrants have this disease.

The government's goal is to maximize (per potential immigrant) expected benefits minus expected costs.

A tree representing the government's decision appears below.



_b_ 44. The optimal decision is

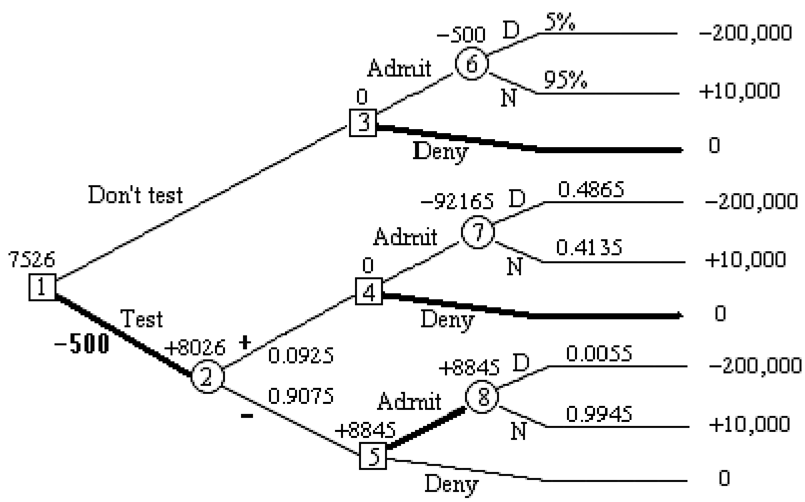
a. admit all immigrants
b. deny admission to all immigrants
c. the government is indifferent
d. *NOTA*

Suppose that there is a medical test which may be administered before determining whether a potential immigrant should be admitted. The cost of this test is $500 per person. The test result is either **positive** (event +) indicating presence of the disease or **negative** (event −) indicating absence of the disease, but the test is somewhat unreliable: 10% of all people with the disease test negative, and 5% of the persons without the disease test positive.

45-46. Complete the following blanks

P{D} *(prior probability)*	0.05	P{N} *(prior probability)*	0.95
P{+|D}	0.90	P{−|D}	0.10
P{+|N}	0.05	P{−|N}	0.95
P{+}	0.0925	P{−}	0.9075
P{D|+}	0.4865	P{D|−}	0.0055
P{N|+}	0.5135	P{N|−}	0.9945

The decision tree below includes the decision as to whether or not administer the medical test. *Note that the $500 cost of the test has not been incorporated in the "payoffs" at the far right.*



47-49. "Fold back" nodes 2 through 8, and write the missing values of the nodes below:

Node	Value	Node	Value	Node	Value
8	+8845	5	+8845	2	+8026
7	−92165	4	0	1	+7526
6	−500	3	0		

_e_ 50. The expected value of the test result (in $) is *(Choose nearest value)*:

a. ≤ 0
b. 500
c. 1000
d. 5,000
e. 7,500 *($8026)*
f. 10,000
g. 20,000
h. ≥ 20,000

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