56:171 Operations Research
Midterm Exam Solutions
Fall 1994

	Possible	Score
A. True/False & Multiple Choice	30	_____
B. Sensitivity analysis (LINDO)	20	_____
C.1. Transportation	15	_____
C.2. Decision Tree	15	_____
C.3. Simplex LP Method	15	_____
total possible:	80	_____

Part A

True/False:

<u>True</u> 1. A "pivot" in a nonbasic column of an LP simplex tableau will make it a basic column.

<u>True</u> 2. If you increase the right-hand-side of a "greater-than-or-equal" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

<u>False</u> 3. The "reduced cost" in LP provides an estimate of the change in the objective value when a right-hand-side of a constraint changes.

<u>False</u> 4. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

<u>True</u> 5. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

<u>True</u> 6. In the LP formulation of the project scheduling problem, the constraints include $Y_B - Y_A \quad d_A$ if activity A must precede activity B, where d_A = duration of activity A.

<u>False</u> 7. In CPM, the "forward pass" is used to determine the latest time (LT) for each event (node).

<u>True</u> 8. The "minimum ratio test" is used to determine the pivot row in the simplex method.

<u>True</u> 9. The A-O-N project network does not require any "dummy" activities, except for the "begin" and "end" activities.

<u>True</u> 10. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

<u>False</u> 11. In a transportation problem, if the current dual variables U_2=3 and V_4=1, and C_{24}=5, then the current basic solution *cannot* be optimal.

<u>True</u> 12. In a transportation problem, if the current dual variables U_2=3 and V_4=1, and C_{24}=5, then X_{24} *cannot* be basic.

<u>True</u> 13. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.

<u>True</u> 14. A "dummy" activity in an A-O-A project network always has duration zero, but can be on the critical path.

<u>False</u> 15. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.

<u>False</u> 16. When maximizing in the simplex method, the value of the objective function will not improve at the next pivot if the current tableau is degenerate.

<u>False</u> 17. When minimizing in the simplex method, you must select the column which has the smallest (i.e., the most negative) reduced cost as the next pivot column.

<u>True</u> 18. If the "float" ("slack") of an activity of a project is positive, then the activity cannot be "critical" in the schedule.

<u>False</u> 19. If a primal minimization LP problem has a cost which is unbounded below, then the dual maximization problem has an objective which is unbounded above.

False 20. If $X_{ij}=0$ in the transportation problem, then dual variables U and V *must* satisfy $C_{ij}=U_i+V_j$.

Multiple Choice: Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =None of the above).

__e_ 21. If, in the optimal *primal* solution of an LP problem (min cx st Ax≤b, x≥0), there is zero slack in constraint #1, then in the optimal dual solution,
(a) dual variable #1 must be zero (c) slack variable for dual constraint #1 must be zero
(b) dual variable #1 must be positive (d) dual constraint #1 must be slack (e) *NOTA*

__c_ 22. If, in the optimal *dual* solution of an LP problem (min cx st Ax≤b, x≥0), variable #2 is positive, then in the optimal primal solution,
(a) variable #2 must be zero (c) slack variable for constraint #2 must be zero
(b) variable #2 must be positive (d) constraint #2 must be slack (e)*NOTA*

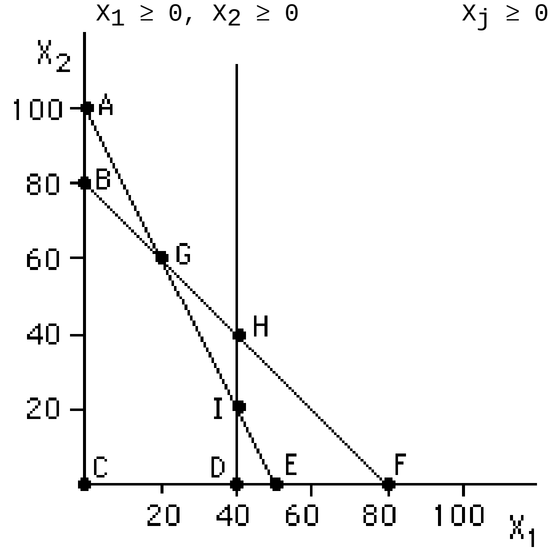
__b_ 23. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
(a) will be nonbasic (c) will have a worse objective value
(b) will be nonfeasible (d) will be degenerate (e) *NOTA*

__c_ 24. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau will
(a) be nonbasic (c) have a worse objective value
(b) be nonfeasible (d) be degenerate (e) *NOTA*

__d_ 25. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
(a) will be nonbasic (c) will have a worse objective value
(b) will be nonfeasible (d) will be degenerate (e) *NOTA*

The problems (26)-(30) below refer to the following LP: *(with inequalities converted to equations:)*

Maximize $3X_1 + 2X_2$	Maximize $3X_1 + 2X_2$
subject to $2X_1 + X_2 \le 100$	subject to $2X_1 + X_2 + X_3 = 100$
$X_1 + X_2 \le 80$	$X_1 + X_2 + X_4 = 80$
$X_1 \le 40$	$X_1 + X_5 = 40$
$X_1 \ge 0,\ X_2 \ge 0$	$X_j \ge 0,\ j=1,2,3,4,5$



__d_ 26 .The feasible region includes points
(a) I, D, & E (c) E, F, H, & I
(b) G, H, & I (d) B, D, G, I, & C (e) *NOTA*

__c_ 27. At point G, the basic variables <u>include</u> the variables
(a) X_2 & X_3 (c) X_1 & X_5
(b) X_3 & X_4 (d) X_1 & X_4 (e) *NOTA*

__e__ 28. Which point is degenerate in this problem?
(a) point B (c) point H
(b) point G (d) point I (e) *NOTA*

__f_ 29. <u>If</u> point G is optimal, then which dual variables must be zero, according to the Complementary Slackness Theorem?
(a) both Y_1 and Y_2 (d) Y_1 only
(b) both Y_1 and Y_3 (e) Y_2 only
(c) both Y_2 and Y_3 (f) Y_3 only

30. For each alternative pair in parentheses, check the appropriate choice to obtain the dual LP of the above primal problem (with the inequality constraints):

```
(___Max/ _X_Min)                100Y1 + 80Y2 + 40Y3
  subject to    2Y1 +  Y2  +  Y3  (___≤ / _X_≥ )   3
                 Y1 +   Y2        (___≤ / _X_≥ )   2
  Y1 (___≤ / _X_≥ ) 0, Y2 (___≤ / _X_≥ ) 0, Y3 (___≤ / _X_≥ )  0
```

Part B

LINDO analysis

Problem Statement: McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

i) Red Baron must contain no more than 75% of A.
ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of \$3.35 for Diablo and \$2.85 for Red Baron. A and B cost \$1.60 and \$2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

Define *D = quarts of Diablo to be produced*
R = quarts of Red Baron to be produced
AD= quarts of A used to make Diablo
AR = quarts of A used to make Red Baron
BD = quarts of B used to make Diablo
BR = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

```
MAX     3.35 D + 2.85 R - 1.6 AD - 1.6 AR - 2.05 BD - 2.05 BR
SUBJECT TO
       2) - D + AD + BD =    0
       3) - R + AR + BR =    0
       4)   AD + AR <=   40
       5)   BD + BR <=   30
       6) - 0.25 D + AD >=   0
       7) - 0.5 D + BD >=   0
       8) - 0.75 R + AR <=   0
END
         OBJECTIVE FUNCTION VALUE
 1)        99.0000000

VARIABLE          VALUE               REDUCED COST
     D           50.000000              0.000000
     R           20.000000              0.000000
     AD          25.000000              0.000000
```

```
       AR             15.000000
       BD             25.000000
       BR              5.000000

     ROW              SLACK OR SURPLUS
       2)                0.00                  -2.350000
       3)                                      -4.350000
       4)                                       0.750000
       5)                                       2.300001
       6)                                       0.000000
       7)                                      -1.999999
       8)                                       2.000000

RANGES IN WHICH THE BASIS IS UNCHANGED


     VARIABLE         CURRENT                                 ALLOWABLE
                         COEF          INCREASE
         D             3.350000                                0.500000
         R                             0.500000                0.375000
                      -1.600000        1.500001
         AR           -1.600000                                0.500000
         BD                            1.500001                1.000000
                      -2.050000        1.000000

                            RIGHTHAND SIDE RANGES
       ROW                              ALLOWABLE             ALLOWABLE
                           RHS            INCREASE
         2             0.000000                                10.000000
         3                               16.666668              3.333333
                      40.000000          50.000000
         5            30.000000                                16.666664
         6                               12.500000              INFINITY
                       0.000000           6.250000
         8             0.000000                                12.500000

THE TABLEAU:

       (BASIS) D            AD     AR           BR    SLK 4        SLK 6
   1          0.000  0.000        0.000  0.000        0.750  2.300
   2   AD            0.000  1.000        0.000  0.000        1.500  0.000
       R      0.000     00  0.000        0.000  0.000       -2.000  0.000
       AR     0.000         0.000  1.000        0.000  1.500         0.000
   5          0.000  0.000        0.000  0.000        0.500 -0.500
   6   SLK 6         0.000  0.000        0.000  0.000        0.750  1.000
       D      1.000         0.000  0.000        0.000 -1.000  3.000
   8   BD            0.000  0.000        1.000  0.000        1.500  0.000

       SLK 7      SLK 8
   1    2.000               99.000
   2               2.000    25.000
      -4.000    -4.000
   4  -3.000                15.000
   5              -2.000     5.000
       2.000     1.000
   7    4.000               50.000
   8               2.000    25.000
```

20 quarts__

99.00__

additional amount should the firm be willing to pay to have another quart of ingredient

2.30_____

the*total*

$__4.35

How many quarts should they be willing to buy at this price? __10 quarts (___increase/___decrease) the "slack variable" in row #5 by _one

5. Using one more quart of ingredient B (i.e., a total of 31 quarts) would result the following changes in the variables:

(_X 0.75_

• the quantity of A used in producing "Red Baron"? (___increase/ X_decrease) by _____

• the quantity of A used in producing " (_X 1.5_

Diablo" produced? (___increase/___decrease) by __3.0

• the total quantity of "Red Baron" produced? (___increase/ X_decrease) by _____

6. How much can the price of "Red Baron" increase before the composition of the current 0.50___

problem? $ _99.00 _X minimized / ___maximized.

❚❏❚❏❚❏❚❏❚❏ Part C ❏❚❏❚❏❚❏❚❏❚

(C.1) **Transportation Problem.** Consider the problem with the initial solution specified on the left below:

(Each cell: cost; allocation in brackets where shown.)

source \ destination	1	2	3	4	supply
1	4	7	[3] 9	[3] 2	6
2	[1] 3	[6] 5	[1] 7	3	8
3	[4] 2	4	5	4	4
demand	5	6	4	3	

Dual Variables:

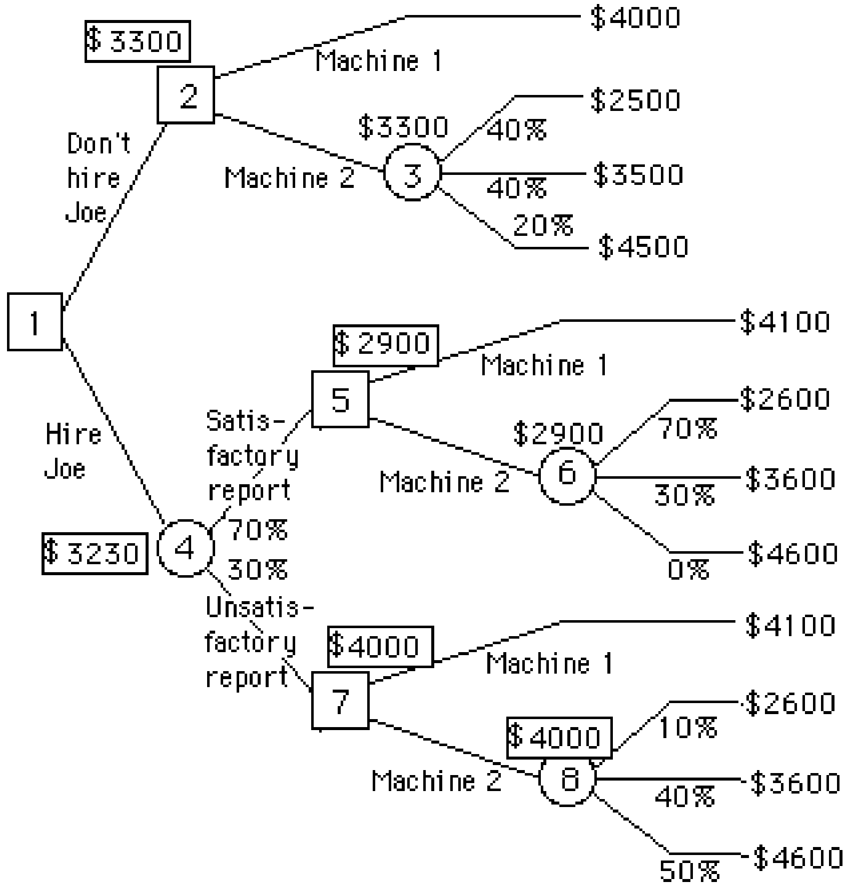
	v_1= +5	v_2=	v_3= +9	v_4=
u_1= 0	4	7	[3] 9	[3] 2
u_2= -2	[1] 3	[6] 5	[1] 7	3
u_3=	[4] 2	4	5	4

1. Is this a basic solution? (*circle:* **Yes** / No)
2. Is this a degenerate solution? (*circle:* Yes / **No**)
3. If the dual variable u_1 (for supply constraint #1) were assigned the value zero, the values of the other dual variables are: u_2= -2, u_3 = __-3_ , v_1= +5, v_2= _+7_, v_3= +9 , v_4= __+2_.
4. If X_{33} were to be increased, what would be the change in the cost function per unit increase in X_{33}? __**-1**_
5. If X_{33} were to enter the basis, what would be its new value? __1____
6. If X_{33} were to enter the basis, what variable(s) would leave the basis?
   Circle as many as apply: X_{13} X_{14} X_{21} X_{22} **X_{23}** X_{31}
7. If X_{33} were to enter the basis, would the new basic solution be degenerate? (*circle:* Yes / **No**)
8. Which other variable *instead* of X_{33}would result in a degenerate basic solution if it were to enter the basis? *Circle as many as apply:* X_{11} X_{12} X_{24} X_{32} X_{33} X_{34} **none**
9. What is the cost of the initial solution shown above? __81_

C.2. Decision Tree. The decision sciences department is trying to determine which of two copying machines to purchase. Both machines will satisfy the department's needs for the next **ten** years.

Machine 1 costs $3000 and has a maintenance agreement, which, for an annual fee of $100, covers all repairs. Machine 2 costs $2500, and its annual maintenance cost is a random variable. At present, the decision sciences department believes there is a 40% chance that the annual cost for machine 2 will be $0, a 40% chance it will be $100, and a 20% chance it will be $200.

For a fee of $100 the department can have Joe, a trained repairman, evaluate the quality of machine 2 before the purchase decision is made. If Joe believes that machine 2 is *satisfactory* , there is a 70% chance that its annual maintenance cost will be $0 and a 30% chance that it will be $100. If Joe believes that machine 2 is *unsatisfactory* , there is a 10% chance that the annual maintenance cost will be $0, a 40% chance it will be $100, and a 50% chance it will be $200. The department believes that there is a 70% probability that Joe will give a satisfactory report after evaluating machine 2. Based upon this data, the decision tree below is drawn:



Fill the blank boxes in the decision tree above and answer the following questions:

__d_ 1. If Joe's evaluation of machine 2 is "unsatisfactory" and they were to select machine 2, what would be their expected cost (including the cost of Joe's evaluation)?

a. $ 2600	c. $ 3300	e. $ 4500
b. $ 2900	d. $ 4000	f. *NOTA*

__e_ 2. This computation is referred to as "folding back" which node in the tree above?

a. Node # 4	c. Node #6	e. Node #8
b. Node # 5	d. Node #7	f. *NOTA*

__f_ 3. What is EVSI (i.e., the expected value of Joe 's evaluation)?

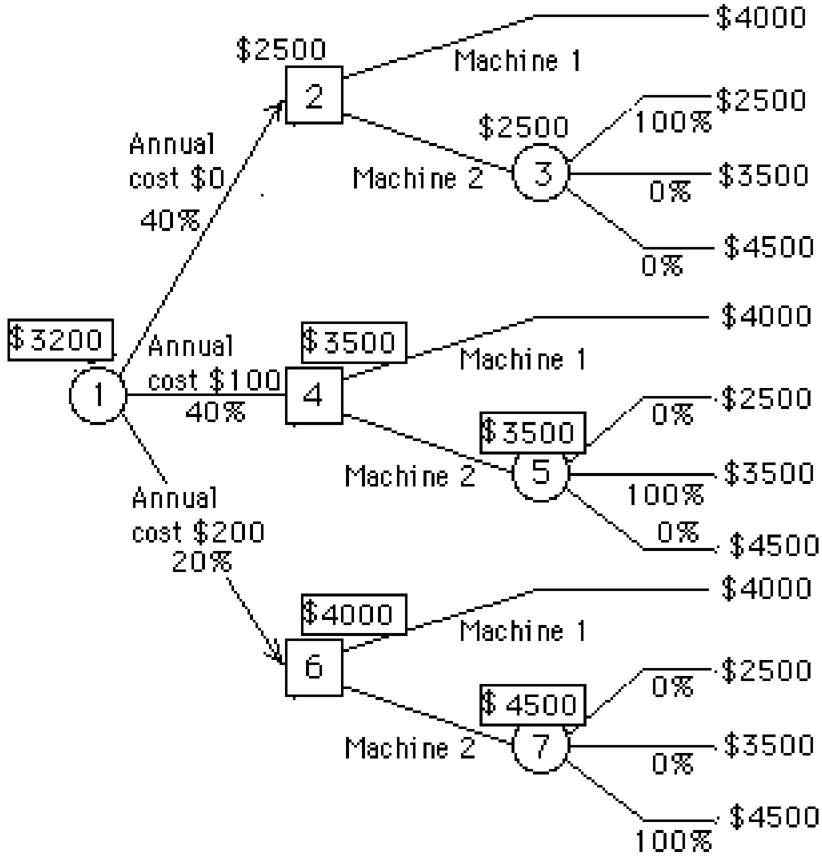
a. *negative* $ 30
b. *negative* $ 20
c. $ 40
d. $ 70
e. $ 120
f. *NOTA*

Question #4 was disregarded... the answer depends upon the report of the repairman!

____ 4. Which machine should the department buy?

a. Machine #1
b. Machine #2
c. a "toss-up"
d. *NOTA*

Suppose that the department were able to get "perfect information", i.e., a perfect prediction of the annual maintenance costs. Based upon this supposition, the department head drew the decision tree below:



Fill the blank boxes in the decision tree above and answer the following questions:

__c_ 5. What is the minimum expected cost to the department if they were able to obtain this perfect prediction?

a. $ 2600 c. $ 3200 e. $ 4500
b. $ 2900 d. $ 4000 f. *NOTA*

__b_ 6. What is EVPI (expected value of perfect information)?

a. $ 70 c. $ 150 e. $ 300
b. $ 100 d. $ 200 f. *NOTA*

C.3. Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, the tableau is:

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
1	2	0	0	0	0	-3	1	-2	-15
0	-4	2	1	0	0	0	1	1	4
0	1	-1	0	1	0	-1	0	-1	2
0	2	0	0	0	1	-3	3	2	6

1. What are the basic variables for this tableau? (*circle*: X_1 X_2 **X_3** **X_4** **X_5** X_6 X_7 X_8 RHS)

2. What are the current values of the variables?

X_1 = _0, X_2 = _0, X_3 = _4_, X_4 = _2_, X_5 = _6_, X_6 = _0, X_7 = _0, X_8 = _0

3. Increasing X_1 would (*circle*: **increase** / decrease) the objective function.
4. Increasing X_6 would (circle: increase / **decrease**) the objective function.

5. What is the substitution rate of X_1 for X_3? __-4___
That is, if X_1 is increased by 1 unit, X_3 (*circle*: **<u>increases</u>** / decreases) by a quantity _4__.

6. Suppose that X_6 and X_7 are slack variables in the first 2 constraints, and X_8 a surplus variable in the the last constraint. (That is, the first two constraints were originally constraints, and the third was originally a constraint, all converted to equations.) What are the values of the simplex multipliers for this tableau? $_1$ = __+3_, $_2$ = __-1_, $_3$ = __-2_

7. If the objective is to (*circle:* maximize / **<u>minimize</u>**) the objective z, the optimal solution is unbounded.

8. If the objective is <u>not</u> that which you specified in (7), perform a pivot to improve the objective function, and write the new tableau below. *(You need only complete the unshaded portion.)*

If the objective is the maximize, then either X_1 or X_7 could be selected to enter the basis.

If X_1 is entered into the basis:

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
			0	-2	0				-19
			1	4	0				
			0	1	0				
			0	-2	1				

If X_7 is entered into the basis:

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
			-1	0	0				-19
			1	0	0				
			0	1	0				
			-3	0	1				