Name/Initial _______________________________

▼▲▼▲▼▲▼ 56:171 Operations Research ▼▲▼▲▼▲▼
▲▼▲▼▲▼▲ Midterm Examination ▲▼▲▼▲▼
▼▲▼▲▼▲▼ October 21, 1998 ▼▲▼▲▼▲▼

• ***Write your name on the first page, and initial the other pages.***
• ***Answer both Parts A and B, and 4 (out of 5) problems from Part C.***

		Possible	Score
Part A:	True/False	15	_____
Part B:	Sensitivity analysis (LINDO)	25	_____
Part C:	1. Simplex method	15	_____
	2. LP duality	15	_____
	3. Transportation problem	15	_____
	4. Project scheduling	15	_____
	5. Decision analysis	15	_____
	total possible:	100	_____

▼▲▼▲▼▲▼ PART A ▼▲▼▲▼▲▼

1. ***True/False:*** Indicate by "**+**" or "**o**" whether each statement is "true" or "false", respectively:

____ a. If there is a tie in the "minimum-ratio test" of the simplex method, there will be no improvement in the objective in this iteration.

____ b. If the primal LP feasible region is nonempty and unbounded, then the dual LP is infeasible.

____ c. Data Envelopment Analysis (DEA) is an application of linear programming.

____ d. In PERT, the total completion time of the project is assumed to have a normal distribution.

____ e. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.

____ f. The transportation problem is a special case of an assignment problem.

____ g. The critical path in a project network is the longest path from a specified source node (beginning of project) to a specified destination node (end of project).

____ h. There is at most one critical path in a project network.

____ i. The latest times of the events in a project schedule must be computed before the earliest times of those events.

____ j. If the optimal value of a slack variable of a primal LP constraint is zero, then the optimal value of the dual variable for that same constraint must be positive.

____ k. If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must be zero

____ l. For any LP, the "DUAL PRICE" reported by LINDO is the same as the "DUAL VARIABLE".

____ m. The values in a "regret" table of a decision problem are always nonnegative.

____ n. Bayes' rule gives the value of a joint probability of a "state of nature" and the outcome of an experiment.

____ o. If they are both feasible, the optimal objective value of an LP problem is the same as the optimal objective value of the dual of its dual problem.

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis: Consider the Gasoline Blending Problem (which is found in the lecture notes): A refinery buys four "raw" gasolines and blends them to produce three types of fuel:

Raw Gas type	Octane Rating	Available (barrels/day)	Price ($/barrel)
1	68	4000	31.02
2	86	5050	33.15
3	91	7100	36.35
4	99	4300	38.75

Fuel blend	Minimum	Selling price	Demand pattern
1	95	45.15	≤ 10,000
2	90	42.95	any amt. can be sold
3	85	40.99	≥ 15,000

Raw gasolines not used in blending can be sold at

- \$38.95/barrel if octane rating ≥ 90
- \$36.85/barrel if octane rating < 90

Define variables:

- X_{ij} = barrels/day of raw gasoline of type i used in making fuel type j (i=1,2,3,4; j=1,2,3)
- Y_i = barrels/day of raw gasonline of type i sold "as is" on the market (i=1,2,3,4)

LINDO output:

```
MAX 14.13 X11 + 12 X21 + 8.8 X31 + 6.4 X41 + 11.93 X12 + 9.8 X22
      + 6.6 X32 + 4.2 X42 + 9.97 X13 + 7.84 X23 + 4.64 X33 + 2.24 X43
      + 5.83 Y1 + 3.7 Y2 + 2.6 Y3 + 0.2 Y4
  SUBJECT TO
        2) - 27 X11 - 9 X21 - 4 X31 + 4 X41 >=   0
        3) - 22 X12 - 4 X22 + X32 + 9 X42 >=   0
        4)  ??? X13 + X23 + 6 X33 + 14 X43 >=   0
        5)   X11 + X12 + X13 + Y1 <=   4000
        6)   X21 + X22 + X23 + Y2 <=   5050
        7)   X31 + X32 + X33 + Y3 <=   7100
        8)   X41 + X42 + X43 + Y4 <=   4300
        9)   X11 + X21 + X31 + X41 <=   10000
       10)   X13 + X23 + X33 + X43 >=   15000
  END

       OBJECTIVE FUNCTION VALUE
       1)      140216.5

  VARIABLE        VALUE          REDUCED COST
       X11         0.000000         0.000000
       X21         0.000000         0.000000
       X31      2453.703613         0.000000
       X41      2453.703613         0.000000
       X12         0.000000         0.000000
       X22         0.000000         0.542424
       X32         0.000000         0.693098
       X42         0.000000         0.934175
       X13      3457.407471         0.000000
       X23      5050.000000         0.000000
       X33      4646.296387         0.000000
       X43      1846.296265         0.000000
        Y1       542.592590         0.000000
        Y2         0.000000         5.533333
        Y3         0.000000         4.970370
        Y4         0.000000         7.429630

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        -0.307407
        3)         0.000000        -0.277273
        4)         0.000000        -0.307407
        5)         0.000000         5.830000
        6)         0.000000         9.233334
        7)         0.000000         7.570370
        8)         0.000000         7.629630
        9)      5092.592773         0.000000
       10)         0.000000        -1.085926
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X11	14.130000	0.000000	INFINITY
X21	12.000000	0.000000	INFINITY
X31	8.800000	INFINITY	0.000000
X41	6.400000	0.000000	1.627273
X12	11.930000	2.283539	2.983334
X22	9.800000	0.542424	INFINITY
X32	6.600000	0.693098	INFINITY
X42	4.200000	0.934175	INFINITY
X13	9.970000	1.627273	0.000000
X23	7.840000	INFINITY	0.000000
X33	4.640000	0.000000	1.207331
X43	2.240000	1.627273	0.000000
Y1	5.830000	6.100000	2.932000
Y2	3.700000	5.533334	INFINITY
Y3	2.600000	4.970370	INFINITY
Y4	0.200000	7.429630	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	17096.773438	14650.000000
3	0.000000	0.000000	11937.037109
4	0.000000	93350.000000	14650.000000
5	4000.000000	INFINITY	542.592590
6	5050.000000	5538.888672	1627.777710
7	7100.000000	4334.782227	3662.500000
8	4300.000000	3662.500000	4274.193359
9	10000.000000	INFINITY	5092.592773
10	15000.000000	1465.000000	5864.706055

THE TABLEAU

ROW	(BASIS)	X11	X21	X31	X41	X12	X22
1	ART	0.000	0.000	0.000	0.000	0.000	0.542
2	X31	3.875	1.625	1.000	0.000	0.000	0.333
3	X12	0.000	0.000	0.000	0.000	1.000	0.182
4	X13	1.000	0.000	0.000	0.000	0.000	-0.333
5	X33	-3.875	-1.625	0.000	0.000	0.000	-0.333
6	X23	0.000	1.000	0.000	0.000	0.000	1.000
7	X41	-2.875	-0.625	0.000	1.000	0.000	0.333
8	X43	2.875	0.625	0.000	0.000	0.000	-0.333
9	SLK 9	0.000	0.000	0.000	0.000	0.000	-0.667
10	Y1	0.000	0.000	0.000	0.000	0.000	0.152

ROW	X32	X42	X13	X23	X33	X43	Y1
1	0.693	0.934	0.000	0.000	0.000	0.000	0.000
2	0.426	0.574	0.000	0.000	0.000	0.000	0.000
3	-0.045	-0.409	0.000	0.000	0.000	0.000	0.000
4	-0.148	0.148	1.000	0.000	0.000	0.000	0.000
5	0.574	-0.574	0.000	0.000	1.000	0.000	0.000
6	0.000	0.000	0.000	1.000	0.000	0.000	0.000
7	0.426	0.574	0.000	0.000	0.000	0.000	0.000
8	-0.426	0.426	0.000	0.000	0.000	1.000	0.000
9	-0.852	-1.148	0.000	0.000	0.000	0.000	0.000
10	0.194	0.261	0.000	0.000	0.000	0.000	1.000

ROW	Y2	Y3	Y4	SLK 2	SLK 3	SLK 4	SLK 5
1	5.533	4.970	7.430	0.307	0.277	0.307	5.830
2	0.333	0.426	0.574	0.144	0.000	0.019	0.000
3	0.000	0.000	0.000	0.000	0.045	0.000	0.000
4	-0.333	-0.148	0.148	0.037	0.000	0.037	0.000
5	-0.333	0.574	-0.574	-0.144	0.000	-0.019	0.000
6	1.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.333	0.426	0.574	-0.106	0.000	0.019	0.000
8	-0.333	-0.426	0.426	0.106	0.000	-0.019	0.000
9	-0.667	-0.852	-1.148	-0.037	0.000	-0.037	0.000
10	0.333	0.148	-0.148	-0.037	-0.045	-0.037	1.000

ROW	SLK 6	SLK 7	SLK 8	SLK 9	SLK 10	
1	9.2	7.6	7.6	0.00E+00	1.1	0.14E+06
2	0.333	0.426	0.574	0.000	0.315	2453.704
3	0.000	0.000	0.000	0.000	0.000	0.000
4	-0.333	-0.148	0.148	0.000	-0.370	3457.407
5	-0.333	0.574	-0.574	0.000	-0.315	4646.296
6	1.000	0.000	0.000	0.000	0.000	5050.000
7	0.333	0.426	0.574	0.000	0.315	2453.704
8	-0.333	-0.426	0.426	0.000	-0.315	1846.296
9	-0.667	-0.852	-1.148	1.000	-0.630	5092.593
10	0.333	0.148	-0.148	0.000	0.370	542.593

___ 1. How many thousands of barrels/day of blend #1 should be produced? *(choose nearest number!)*
a. none b. one c. two d. five
e. ten e. fifteen f. twenty g. fifty

___ 2. How many thousands of barrels/day of blend #2 should be produced? *(choose nearest number!)*
a. none b. one c. two d. five
e. ten e. fifteen f. twenty g. fifty

___ 3. How many raw gasolines should be sold on the market instead of (or in addition to) being used in blending?
a. none b. one c. two
d. three e. four

___ 4. What is the missing coefficient in row 4 of the LP model (which imposes the minimum octane requirement for blend #3)? *(Choose nearest number!)*
a. -30 b. -20 c. -10 d. zero
e. +10 f. +20 g. +30 h. +50

In the optimal solution, raw gasoline type #4 is not sold on the market, even though it can be sold for more than the price paid by the refinery.

___ 5. What increase in the selling price of raw gasoline #4 would be required in order to make its sale optimal? *(Choose nearest number!)*
a. $1 b. $2 c. $5 d. $8
e. $10 f. $15 g. $20 h. $50

___ 6. What would be the change in the quantity of raw gasoline #1 sold on the market, if 100 barrels of raw gasoline #4 were sold on the market? *(Choose nearest number!)*
a. decrease 15 b. decrease 10 c. decrease 5 d. no change
e. increase 5 f. increase 10 g. increase 15

7100 barrels/day of raw gasoline #3 is now available for $36.35/barrel.

___ 7. If 100 additional barrels would be available, by how much would the refinery be able to increase its profit? *(Choose nearest number!)*
a. 0 b. $1 c. $10 d. $100
e. $1000 f. $5000 g. $10000 h. $50000

___ 8. If 100 additional barrels of raw gasoline #3 were available, what would be the effect on the quantity of raw gasoline #4 used in blend #1? *(Choose nearest number!)*
a. decrease 100 b. decrease 50 c. decrease 10 d. no change
e. increase 10 f. increase 50 g. increase 100 h. increase 500

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▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

1. Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, in which the objective is to be **minimized**, the tableau is:

-z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
1	0	0	0	-2	0	6	-10
0	2	0	1	-4	0	1	4
0	0	1	0	1	0	-1	3
0	-2	0	0	2	1	3	1

1. What are the basic variables for this tableau? *(circle):*
   -Z X_1 X_2 X_3 X_4 X_5 X_6
2. The current value of the cost for this basic solution is (*circle:* +10 or -10)

_____ 3. The current value of X_1 for this basic solution is
   a. 0   b. 1   c. 3   d. 4   e. 10

_____ 4. The current value of X_2 for this basic solution is
   a. 0   b. 1   c. 3   d. 4   e. 10

_____ 5. Increasing X_4 would (*circle:* increase / decrease) the objective function.

_____ 6. What is the substitution rate of X_4 for X_5?
   a. 0   b. 1   c. -1   d. 2   e. -2

_____ 7. If X_4 were increased by **2** units, the value of X_5 will
   a. not change   b. increase by 2   c. decrease by 2
   d. increase by 4   e. decrease by 4   f. none of the above

_____ 8. If the original constraints were all of type "≤" where X_4, X_5, and X_6 are slack variables, the value of the first dual variable π_1 corresponding to the tableau given above is
   a. 0   b. 1   c. -1   d. 2   e. -2
   f. none of the above   g. cannot be determined

_____ 9. If the original constraints were of type "≥" and X_4, X_5, and X_6 are surplus variables, the value of the second dual variable π_2 corresponding to the tableau above is
   a. 0   b. 1   c. -1   d. 2   e. -2
   f. none of the above   g. cannot be determined

10. Perform a pivot to improve the objective function, and complete the blank entries in the tableau below:

-z	X_1	X_2	X_3	X_4	X_5	X_6	RHS

_____ 11. The improvement in the objective resulting from the pivot in (11) is
(choose the nearest value)
a. zero   b. 1   c. 2
d. 3   e. 4   f. ≥5

2. LP Duality: Consider the (primal) LP

Min w = $4X_1 + 2X_2 - X_3$

s.t. $X_1 + 2X_2 \leq 6$

$X_1 - X_2 + 2Y_3 = 8$

$X_1 \geq 0, X_2 \geq 0$ (X_3 unrestricted in sign)

a. The dual of this LP will have _____ variables.
b. The dual of this LP will have _____ constraints in addition to sign (e.g. nonnegativity) restrictions.
c. The first dual constraint will be of type *(circle):* ≤ = ≥
d. The right-hand-side of the second constraint will be *(circle):* positive negative zero
e. The third dual constraint will be of type *(circle):* ≤ = ≥

The point X=(0, 0, 4) is optimal in the above problem. If the dual variables are denoted by Y_i, which one or more of the below statements must therefore be true? *(Circle):*

i. $Y_1 > 0$ ii. $Y_1 = 0$ iii. $Y_1 < 0$
iv. $Y_2 > 0$ v. $Y_2 = 0$ vi. $Y_2 < 0$ vii. None of the above

3. Transportation Problem: Consider the transportation problem with the tableau below:

(Each cell shows the allocation, if any, followed by the cost in brackets.)

	D	E	F	G	supply
A	4 [5]	[1]	[4]	[2]	4
B	[4]	[2]	3 [2]	[3]	8
C	[6]	[3]	[1]	4 [2]	5
demand	6	3	4	4	

a. If the ordinary simplex tableau were to be written for this problem, how many rows (excluding the objective) will it have? _____
How many variables (excluding the objective value -z) will it have? ________

b. Is this transportation problem "balanced?" _______ (yes/no).

c. How many <u>basic</u> variables will this problem have? __________

d. An initial basic feasible solution is to be found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau above.

e. If U_1 (the dual variable for the first source) is equal to 0, what is the value of
U_2 (the dual variable for the second source)? ________
V_1 (the dual variable for the first destination)? _______
V_4 (the dual variable for the fourth destination)? ________

f. What is the reduced cost of the variable X_{14}? ___________________________ (Explain your computation.)

g. Will increasing X_{14} improve the objective function? _______ (yes/no).
h. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{14} enters?_________

i. What will be the value of X_{14} if it is entered into the solution as in (h)? ___________

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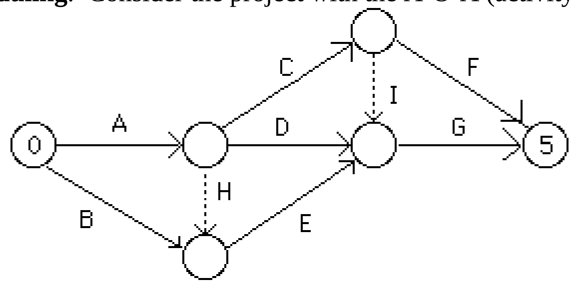
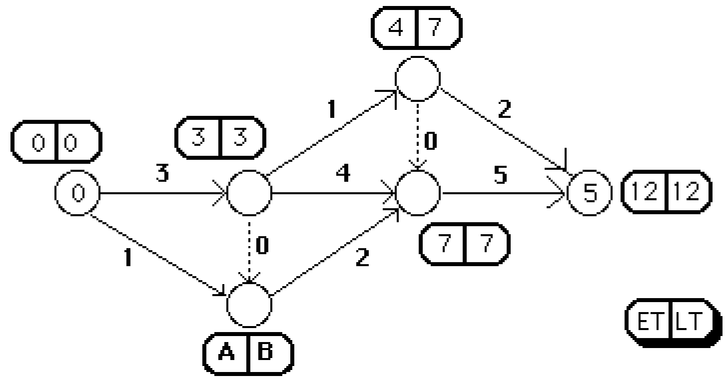
4. Project Scheduling. Consider the project with the A-O-A (activity-on-arrow) network:



1. Complete the labeling of the nodes on the network above.

___ 2. The number of activities (i.e., tasks), <u>not</u> including "dummies", which are required to complete this project is

a. six	c. eight	e. ten
b. seven	d. nine	f. *NOTA*



The activity durations are given above on the arrows. The Early Times (ET) and Late Times (LT) for each node are written in the box (with rounded corners) beside each node.

___ 3. The early time (ET) indicated by **A** in the network above is:

a. three	c. five	e. seven
b. four	d. six	f. *NOTA*

___ 4. The late time (LT) ndicated by **B** in the network above is:

a. three	c. five	e. seven
b. four	d. six	f. *NOTA*

___ 5. The slack ("total float") for activity C is

a. zero	c. two	e. four
b. one	d. three	f. *NOTA*

6. Which activities are critical? (*circle:* A B C D E F G H I J)

___ 7. The earliest completion time for the project is

a. four	c. seven	e. twelve
b. five	d. ten	f. *NOTA*

Suppose that the non-zero durations are *random*, with each value in the above network being the *expected* values and each *standard deviation* equal to 1.00. Then...

___ 8. The expected earliest completion time for the project is

a. four	c. seven	e. twelve
b. five	d. ten	f. *NOTA*

___ 9. The *variance* σ^2 of the earliest completion time for the project is

a. 1	c. 3	e. 5	g. 7
b. 2	d. 4	f. 6	h. *NOTA*

10. Add the arrows to complete the A-O-N (activity-on-node) network below for this same project.

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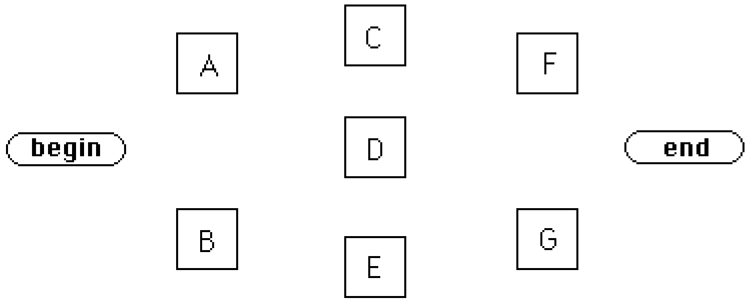


5. Decision Trees: General Custard Corporation is being sued by Sue Smith. Sue can settle out of court and win $60,000, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L).* If she wins, she will receive $200,000, and if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:

_____ 1. What is the decision which maximizes the expected value?

a. settle   b. go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL),* or he predicts a win *(event PW).* The consultant is correct 80% of the time.

_____ 2. The probability that the consultant will predict a win, i.e. P{PW} is *(choose nearest value)*

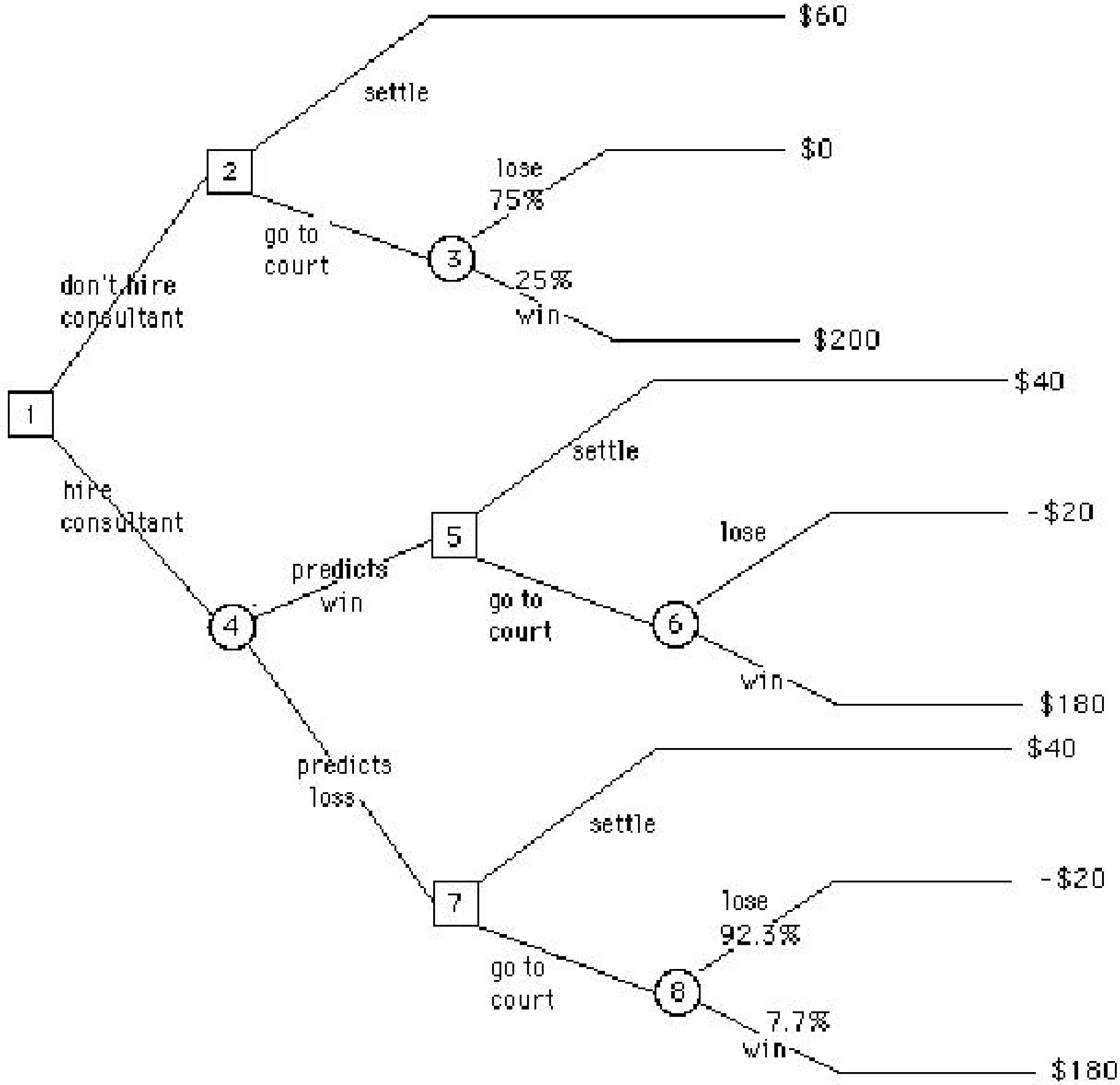
a. ≤25%   b. 30%   c. 35%
d. 40%   e. 45%   f. ≥ 50%

_____ 3. According to Bayes' theorem, the conditional probability that, *if* the consultant predicts a win, then in fact Sue *will* win, i.e. P{W | PW}, is *(choose nearest value)*

a. ≤40%   b. 45%   c. 50   d. 55%
e. 60%   f. 65%   f. ≥ 70%

_____ 4. The decision tree below includes Sue's decision as to whether or not to hire the consultant.

Note that the consultant's fee has already been deducted from the "payoffs" on the far right.



"Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	_____	5	94.286	2	______
7	40	4	59	1	______
6	94.286	3	50		

5. Should Sue hire the consultant? *Circle:* Yes No

____ 6. The expected value of the consultant's opinion is (in <u>thousands</u> of $) *(Choose nearest value)*:

a. ≤10 b. 15 c. 20 d. 25

e. 30 f. 35 g. 40 h. ≥45

_____ 7. What would be the expected value of "perfect information" which is *given* to Sue at no cost, i.e., a prediction which is 100% accurate, so that the portion of the tree containing nodes 4, 5, 6, 7, etc., would appear as below? *(Choose nearest value, in thousands of $)*

a. ≤10	b. 15	c. 20	d. 25
e. 30	f. 35	g. 40	h. ≥45