56:171 Operations Research
Midterm Exam
October 22, 1992

• ***Write your name on the first page, and initial the other pages.***
• ***Answer both questions of Part One, and 3 (out of 4) problems from Part Two.***

		Possible	Score
Part One:	1. True/False & Multiple Choice	15	_____
	2. Sensitivity analysis (LINDO)	25	_____
Part Two:	3. Simplex method	15	_____
	4. LP duality	15	_____
	5. Transportation problem	15	_____
	6. Project scheduling	15	_____
	total possible:	85	_____

PART ONE

(1.) ***True/False:*** Indicate by "+" ="true" or "o" ="false" :

____ a. A "dummy" activity in CPM has duration zero and cannot be on the critical path.

____ b. In PERT, the total completion time of the project is assumed to be a random variable with a normal distribution.

____ c. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

____ d. During any iteration of the simplex method, if x_j is the variable entering the basis, its value after the pivot is the value of the minimum ratio.

____ e. The revised simplex method usually requires fewer iterations than the ordinary simplex method.

____ f. In a transportation problem, if the total supply exceeds total demand, a "dummy" destination should be defined.

____ g. All tasks on the critical path of a project schedule have their latest start time equal to their earliest start time.

____ h. When maximizing in the simplex method, the value of the objective function increases at every iteration unless a degenerate tableau is encountered.

____ i. A basic solution of an LP is always feasible, but not all feasible solutions are basic.

____ j. The assignment problem is a special case of a transportation problem.

Multiple Choice: Write the appropriate letter (a, b, c, or d) in the blank:

____ k. If, in the optimal *primal* solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), constraint #1 is slack, then in the optimal dual solution,

- a. variable #1 must be zero
- b. variable #1 must be positive
- c. slack variable for constraint #1 must be zero
- d. constraint #1 must be slack
- e. None of the above

____ l. If, in the optimal *dual* solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), variable #2 is positive, then in the optimal primal solution,

- a. variable #2 must be zero
- b. variable #2 must be positive
- c. slack variable for constraint #2 must be zero
- d. constraint #2 must be slack
- e. None of the above

____ m. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau

- a. will be nonbasic
- b. will be nonfeasible
- c. will have a worse objective value
- d. will be degenerate
- e. None of the above

____ n. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau

- a. will be nonbasic
- b. will be nonfeasible
- c. will have a worse objective value
- d. will be degenerate

e. None of the above

____ o. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau

a. will be nonbasic
b. will be nonfeasible
c. will have a worse objective value
d. will be degenerate
e. None of the above

(2.) ***Sensitivity Analysis in LP.*** (Tire Manufacturing Problem)
An automobile tire company has the ability to produce both nylon (N) and fiberglass (G) tires. During the next three months they have agreed to deliver tires as follows:

Date	Nylon	Fiberglass
June 30	4000	1000
July 31	8000	5000
August 31	3000	5000
Total	15000	11000

The company has two presses, a Wheeling machine and a Regal machine, and appropriate molds that can be used to produce these tires, with the following production hours available in the upcoming months:

Month	Wheeling machine	Regal machine
June	700	1500
July	300	400
August	1000	300

The production rates for each machine-and-tire combination, in terms of *hours per tire*, are as follows:

Tire	Wheeling machine	Regal machine
Nylon	0.15	0.16
Fiberglass	0.12	0.14

The variable costs of producing tires are $5.00 per operating hour, regardless of which machine is being used or which tire is being produced. There is also an inventory-carrying charge of $0.10 per tire per month. The objective is to minimize the cost of meeting the delivery schedule.
Definition of variables: Variables representing production quantities are named as follows:

MTi = # of tires of type T produced on machine M in month i, where M=W (Wheeling) or R (Regal), T = N (Nylon) or G (Fiberglass), and i=1 (June), 2 (July), or 3 (August)

Variables representing inventory are named as follows:

ITi = # of tires of type T (N or G) stored at the end of month i (1, 2, or 3)

Consult the attached LINDO output to answer the following questions. If there is not sufficient information in the LINDO output, answer "NSI".

a. If the number of Nylon tires which the company has agreed to deliver on July 31 were to increase by 1000, the cost would (*circle:* increase / decrease) by $ __________.

b. If the number of Nylon tires which the company has agreed to deliver on August 31 were to increase by 1000, the cost would (*circle:* increase / decrease) by $ __________.

c. If the number of hours available on the Wheeling machine in July were to decrease by 10, fhe cost will increase by $__________, and the following adjustments should be made in the production schedule:

Nylon tires produced in Wheeling machine in June: WN1 (*circle:* increase / decrease) by _______
F-Glass tires produced in Wheeling machine in June: WG1 (*circle:* increase / decrease) by _______
Nylon tires produced in Wheeling machine in July: WN2 (*circle:* increase / decrease) by _______
F-Glass tires produced in Wheeling machine in July: WG2 (*circle:* increase / decrease) by _______
Nylon tires in storage at end of July: IN2 (*circle:* increase / decrease) by _______
of idle hours on the Regal machine in August: (*circle:* increase / decrease) by _______

d. Suppose that the production plan is modified in order to produce 10 Nylon tires on the Wheeling machine in July. Then the cost will increase by $________ and the following adjustments should be made:

- \# Nylon tires produced in Wheeling machine in June: WN1 (*circle:* increase / decrease) by _______
- \# F-Glass tires produced in Wheeling machine in June: WG1 (*circle:* increase / decrease) by _______
- \# Nylon tires produced in Wheeling machine in July: WN2 (*circle:* increase / decrease) by _______
- \# F-Glass tires produced in Wheeling machine in July: WG2 (*circle:* increase / decrease) by _______
- \# Nylon tires stored at end of July: IN2 (*circle:* increase / decrease) by _______
- \# of idle hours on the Regal machine in August: (*circle:* increase / decrease) by _______

e. If the storage cost of Nylon tires at the end of June were to increase by 3¢ per tire (to 13¢), the production plan (*circle:* should / should not) be modified.

f. If the storage cost of Fiberglass tires at the end of June were to increase by 3¢ per tire (to 13¢), the production plan (*circle:* should / should not) be modified.

PART TWO

(3.) *Simplex Method.* Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, the tableau is:

-z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
1	0	0	0	-2	0	3	-15
0	0	-2	1	4	0	0	4
0	0	1	0	1	1	-1	0
0	1	0	0	-2	0	-3	3

(a.) What are the basic variables for this tableau? (circle:) -Z X_1 X_2 X_3 X_4 X_5 X_6

(b.) What are the current values of the variables?
Z = _____, X_1 = _____, X_2 = _____, X_3 = _____, X_4 = _____, X_5 = _____, X_6 = _____,

(c.) Increasing X_4 would (circle: increase / decrease) the objective function.

(d.) Increasing X_6 would (circle: increase / decrease) the objective function.

(e.) What is the substitution rate of X_4 for X_5? _______ That is, if X_4 is increased by 1 unit, X_5 (circle: increases / decreases) by a quantity _________.

(f.) Suppose that X_3 and X_4 are slack variables in the first 2 constraints, and X_5 a surplus variable in the the last constraint. (That is, the first two constraints were originally ≤ constraints, and the third was originally a ≥ constraint, all converted to equations.) What are the values of the simplex multipliers (dual variables) for this tableau? Π_1 = _____, Π_2 = _____, Π_3 = _____

(g.) If the objective is to (circle: maximize / minimize) z, the optimal solution is unbounded.

(h.) If the objective is <u>not</u> that which you specified in (g), perform a pivot to improve the objective function, and write the new tableau below:

$-Z$	X_1	X_2	X_3	X_4	X_5	X_6	rhs

vvvvvvvvvvvvvv

(4.) Linear Programming Duality: Consider the following LP:

Minimize $12X_1 + 8X_2$

subject to $3X_1 + 4X_2 \le 12$

$5X_1 + 2X_2 \le 10$

$X_1 + 2X_2 \ge 3$

$X_1 \ge 0,\ X_2 \ge 0$

(with inequalities replaced by equations:)

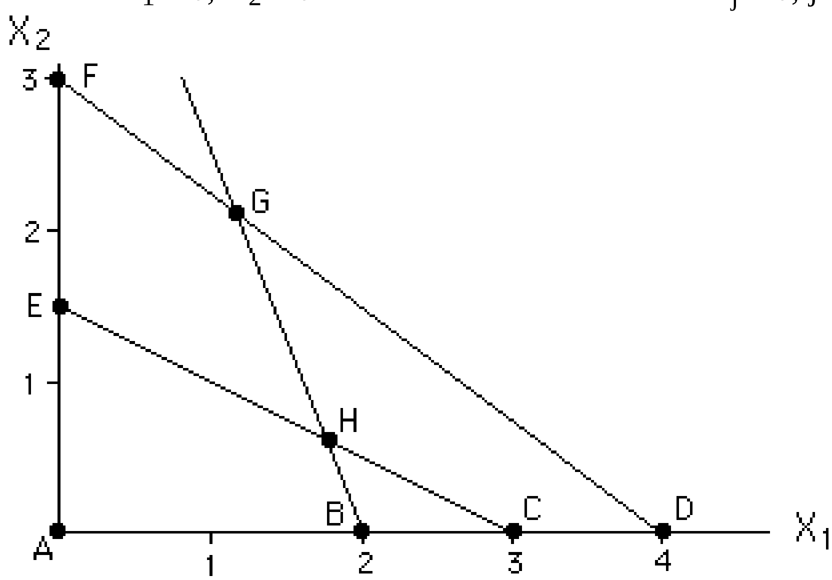
Minimize $12X_1 + 8X_2$

subject to $3X_1 + 4X_2 + X_3 = 12$

$5X_1 + 2X_2 + X_4 = 10$

$X_1 + 2X_2 - X_5 = 3$

$X_j \ge 0,\ j=1,2,3,4,5$



a. Which points above are feasible? (circle:) A B C D E F G H

b. At point E, which variables are basic? (circle:) X_1 X_2 X_3 X_4 X_5

c. At point G, which variables are basic? (circle:) X_1 X_2 X_3 X_4 X_5

d. Indicate (by shading) the feasible region of the LP.

e. Circle as appropriate to obtain the dual LP of the above problem (with inequality constraints):

(Max / Min) $12Y_1 + 10Y_2 + 3Y_3$

subject to $3Y_1 + 5Y_2 + Y_3$ $(\le / \ge)$ 12

$4Y_1 + 2Y_2 + 2Y_3$ $(\le / \ge)$ 8

Y_1 $(\le / \ge)$ 0, Y_2 $(\le / \ge)$ 0, Y_3 $(\le / \ge)$ 0

f. Circle as appropriate to obtain the dual LP with only equations:

(Max / Min) $12Y_1 + 10Y_2 + 3Y_3$

subject to $3Y_1 + 5Y_2 + Y_3 \;(+\;/\;-)\; Y_4 = 12$

$4Y_1 + 2Y_2 + 2Y_3 \;(+\;/\;-)\; Y_5 = 8$

Y_1 (≤ / ≥) 0, Y_2 (≤ / ≥) 0, Y_3 (≤ / ≥) 0, Y_4 (≤ / ≥) 0, Y_5 v 0

g. Which point is optimal in the primal problem? (circle:) A B C D E F G H

h. According to the Complementary Slackness Theorem, which variables must be zero at the optimum of the dual LP? (circle:) Y_1 Y_2 Y_3 Y_4 Y_5

i. The optimal dual solution is: Y_1 = ___, Y_2 = ___, Y_3 = ___, Y_4 = ___, Y_5 = ___,

vvvvvvvvvvvvv

(5.) Transportation Problem: Consider the transportation problem with the tableau below:

	D	E	F	G	supply
A	3	1	2	4	5
B	5	2	3	6	4
C	2	1	1	3	2
demand	3	3	2	3	

a. If the ordinary simplex tableau were to be written for this problem, it would have ___ rows, plus the objective row, and ____ columns (in addition to -z and the right-hand-side).

b. This problem will have ____ basic variables (plus -z).

c. Find an initial basic feasible solution using the "Northwest Corner Method" *(write the values of the variables in the tableau above.)*

d. What are the values of the dual variables for the solution in (c)? U_A = ____ , U_B = ____ , U_C = ____ , V_D = ____ , V_E = ____ , V_F = ____ , V_G = ____ .

e. What is the reduced cost of the variable X_{BD}? ___________ ... of the variable X_{AG}? ______

f. Will increasing X_{BD} improve the objective function? ___________

g. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{BD} enters?_________

h. What will be the value of X_{BD} if it is entered into the solution as in (g)? ___________

i. Which variable, if it were entered into the solution, would result in a degenerate solution?
(Circle none, one, or more: X_{AF} X_{CD} X_{CE})

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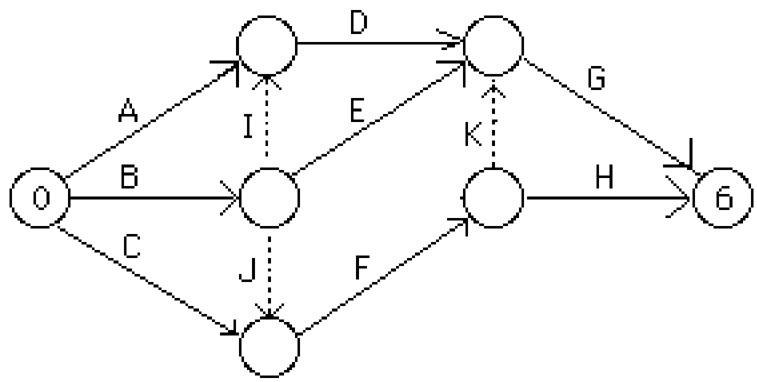
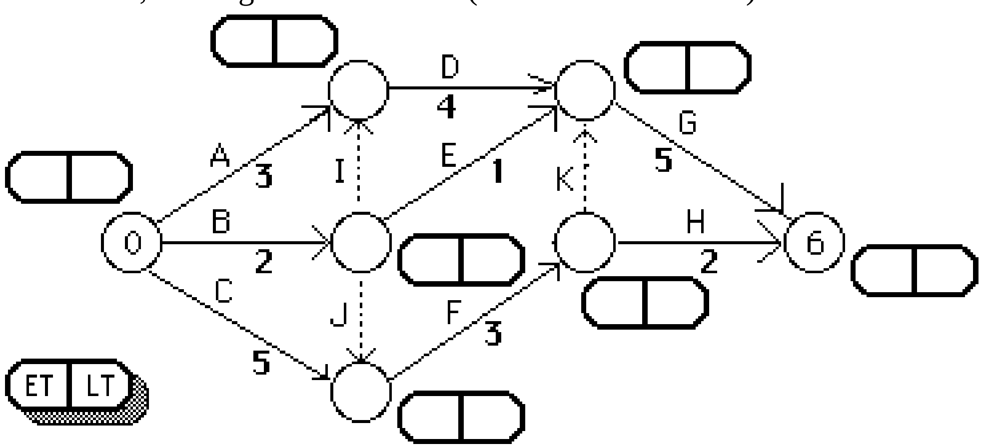
(6.) ***Project Scheduling.*** Consider the project with the A-O-A (activity-on-arrow) network given below.



a. How many activities (i.e., tasks), <u>not</u> including "dummies", are required to complete this project? ________

b. Complete the labeling of the nodes on the network above.

c. The activity durations are given below on the arrows. Compute the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.



d. Find the slack ("total float") for activity B. _________

e. Which activities are critical? (circle: A B C D E F G H I J K)

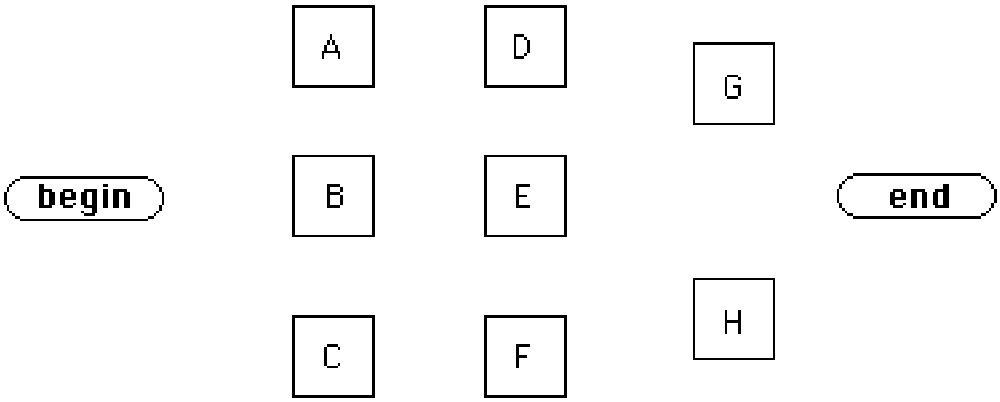
f. What is the earliest completion time for the project? ___________

g. Indicate by X which of the following constraint(s) would appear in the LP formulation of this problem:

____ $Y_F - Y_A \geq 3$ ____ $Y_F - Y_B \geq 3$ ____ $Y_F - Y_C \geq 3$

____ $Y_F - Y_A \geq 2$ ____ $Y_F - Y_B \geq 2$ ____ $Y_F - Y_C \geq 5$

h. Complete the A-O-N (activity-on-node) network below for this same project. (Add any "dummy" activities which are necessary.)



h. Suppose that the arrow labelled "K" is deleted. Indicate the resulting A-O-N network below: