❚❑❚❑❚❑❚❑❚❑❚ 56:171 Operations Research ❚❑❚❑❚❑❚❑❚❑❚
❚❑❚❑❚❑❚❑❚❑❚ Midterm Examination ❚❑❚❑❚❑❚❑❚❑❚
❚❑❚❑❚❑❚❑❚❑❚ October 25, 1991 ❚❑❚❑❚❑❚❑❚❑❚

• ***Write your name on the first page, and initial the other pages.***
• ***Answer both questions of Part One, and 3 (out of 5) problems from Part Two.***

		Possible	Score
Part One:	1. True/False	15	______
	2. Sensitivity analysis (LINDO)	25	______
Part Two:	3. Simplex method	15	______
	4. Revised simplex method	15	______
	5. LP duality	15	______
	6. Transportation problem	15	______
	7. Project scheduling	15	______
	total possible:	85	______

❚❑❚❑❚❑❚❑❚❑❚ PART ONE ❚❑❚❑❚❑❚❑❚❑❚

(1.) ***True/False:*** Indicate by "+" or "o" whether each statement is "true" or "false", respectively:

____ a. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate.

____ b. "Crashing" a critical path problem is a technique used to find a good initial feasible soltuion.

____ c. In the two-phase simplex method, an artificial variable is defined for each constraint row lacking a slack variable (assuming the right-hand-side of the LP tableau is nonnegative).

____ d. If the primal LP feasible region is nonempty and unbounded, then the dual LP is infeasible.

____ e. In PERT, the total completion time of the project is assumed to have a BETA distribution.

____ f. The Revised Simplex Method, for most LP problems, requires fewer pivots than the ordinary simplex method.

____ g. All tasks on the critical path of a project schedule have their latest start time equal to their earliest start time.

____ h. When maximizing in the simplex method, the value of the objective function increases at every iteration unless a degenerate tableau is encountered.

____ i. The critical path in a project network is the shortest path from a specified source node (beginning of project) to a specified destination node (end of project).

____ j. The assignment problem is a special case of a transportation problem.

____ k. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible.

____ l. A basic solution of an LP is always feasible, but not all feasible solutions are basic.

____ m. In Phase One of the 2-Phase method, one should never pivot in the column of an artificial variable.

____ n. In a transportation problem if the total supply exceeds total demand, a "dummy" destination should be defined.

____ o. If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must also be positive.

(2.) ***Sensitivity Analysis in LP.*** Recall the Sequoia Clinic Nurse Staffing Problem discussed in class:

• Required # nurses on duty (minimum):

MON	TUES	WED	THUR	FRI	SAT	SUN
17	14	12	15	22	10	15

• Work schedules for full-time nurses must have two consecutive days off per week. Pay is $120/day, except for Saturdays ($150) and Sundays ($180)

• The clinic may also hire part-time nurses who will work Fri-Sun-Mon schedules, for $240/weekend.

Decision Variables

MON = # full-time nurses who start 5-day shift on Mondays,
TU = # full-time nurses who start 5-day shift on Tuesdays
WED = etc.

P = # part-time nurses

LINDO output

```
:LOOK ALL

 MIN     600 MON + 630 TU + 690 WED + 690 TH + 690 FR + 690 SA + 660 SU + 240 P
 SUBJECT TO
        2)   MON + TH + FR + SA + SU + P >=   17
        3)   MON + TU + FR + SA + SU     >=   14
        4)   MON + TU + WED + SA + SU    >=   12
        5)   MON + TU + WED + TH + SU    >=   15
        6)   MON + TU + WED + TH + FR + P >=   22
        7)   TU + WED + TH + FR + SA     >=   10
        8)   WED + TH + FR + SA + SU + P >=   15
 END

: GO
         OBJECTIVE FUNCTION VALUE

  1)         12180.0000

    VARIABLE          VALUE            REDUCED COST
      MON            2.000000             0.000000
      TU             9.000000             0.000000
      WED            0.000000           180.000000
      TH             1.000000             0.000000
      FR             0.000000            60.000000
      SA             0.000000            60.000000
      SU             3.000000             0.000000
      P             11.000000             0.000000

     ROW     SLACK OR SURPLUS           DUAL PRICES
      2)        0.000000             -180.000000
      3)        0.000000             -180.000000
      4)        2.000000                0.000000
      5)        0.000000             -240.000000
      6)        1.000000                0.000000
      7)        0.000000             -210.000000
      8)        0.000000              -60.000000

   RANGES IN WHICH THE BASIS IS UNCHANGED

                           OBJ COEFFICIENT RANGES

    VARIABLE       CURRENT            ALLOWABLE              ALLOWABLE
                     COEF             INCREASE               DECREASE
      MON          600.000000          60.000000             180.000000
      TU           630.000000          30.000000             180.000000
      WED          690.000000          INFINITY              180.000000
      TH           690.000000         180.000000              60.000000
```

FR	690.000000	INFINITY	60.000000
SA	690.000000	INFINITY	60.000000
SU	660.000000	180.000000	60.000000
P	240.000000	20.000000	180.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	17.000000	3.000000	0.500000
3	14.000000	1.000000	1.000000
4	12.000000	2.000000	INFINITY
5	15.000000	0.500000	1.000000
6	22.000000	1.000000	INFINITY
7	10.000000	3.000000	0.500000
8	15.000000	1.000000	3.000000

: TABLEAU

ROW	(BASIS)	MON	TU	WED	TH
1	ART	0.000	0.000	180.000	0.000
2	P	0.000	0.000	-1.000	0.000
3	MON	1.000	0.000	-1.000	0.000
4	SLK 4	0.000	0.000	-1.000	0.000
5	TH	0.000	0.000	1.000	1.000
6	SLK 6	0.000	0.000	-2.000	0.000
7	TU	0.000	1.000	0.000	0.000
8	SU	0.000	0.000	1.000	0.000

ROW	FR	SA	SU	P	SLK 2
1	60.000	60.000	0.000	0.000	180.000
2	3.000	3.000	0.000	1.000	-1.000
3	0.000	0.000	0.000	0.000	-1.000
4	1.000	0.000	0.000	0.000	0.000
5	-1.000	-1.000	0.000	0.000	0.000
6	3.000	4.000	0.000	0.000	-2.000
7	2.000	2.000	0.000	0.000	0.000
8	-1.000	-1.000	1.000	0.000	1.000

ROW	SLK 3	SLK 4	SLK 5	SLK 6	SLK 7
1	180.000	0.000	240.000	0.000	210.000
2	-1.000	0.000	2.000	0.000	-1.000
3	0.000	0.000	0.000	0.000	0.000
4	-1.000	1.000	0.000	0.000	0.000
5	1.000	0.000	-1.000	0.000	0.000
6	-1.000	0.000	2.000	1.000	-2.000
7	-1.000	0.000	1.000	0.000	-1.000
8	0.000	0.000	-1.000	0.000	1.000

ROW	SLK 8	RHS
1	60.	-0.12E 05
2	0.000	11.000
3	1.000	2.000
4	0.000	2.000
5	0.000	1.000
6	1.000	1.000
7	0.000	9.000
8	-1.000	3.000

:PARARHS
ROW:2
NEW RHS VAL=30

VAR OUT	VAR IN	PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
			17.0000	-180.000	12180.0
SU	FR	5	20.0000	-180.000	12720.0

SLK 4	SA	4	22.0000	-240.000	13200.0
SLK 6	SLK 8	6	24.5000	-240.000	13800.0
TU	SLK 4	7	24.5000	-240.000	13800.0
FR	SLK 6	5	26.5000	-240.000	14280.0
			30.0000	-240.000	15120.0

Consult the LINDO output to answer the following questions. *(If not enough information is available in the output, answer "no info".)*

a. On which days is the minimum required number of nurses exceeded? _______________

b. If the minimum requirement on Monday were to increase by 1,
what would be the effect on the objective function? ________________________
what would be the effect on the basic variables? __________________________

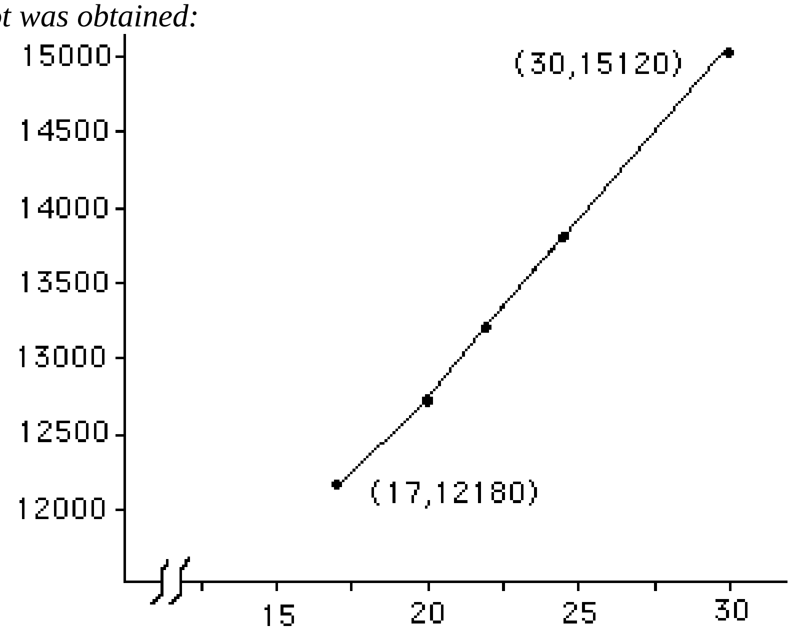
c. If the minimum requirement on Friday were to increase by 1, what would be the effect on the objective function? ________________________

d. If the salary of part-time workers were to increase by $10 per shift, would the solution be changed? _______________

e. If the salary of the persons working Monday through Friday were to be increased by $30 per week, what, if any, is the effect on the objective function?______________
what, if any, is the effect on the basic variables? ________________

f. In the optimal solution, no one is to work the shift beginning on Saturday. Suppose that for unspecified reasons, it is required that one person work this shift.
- How much will this increase the cost? __________
- How will this change the number of persons working the shift beginning on Sunday? _______________
- How will this change the number of part-time persons to be employed? ___________

Near the end of the LINDO output is the result of the command PARARHS. Using this, the following plot was obtained:



g. What quantity is represented by the horizontal axis? ________________________

h. What quantity is represented by the vertical axis? __________________________

i. There is information elsewhere in the output which allows you to extend this curve (either to the right or left.) Draw the extension on the plot, labeling the new endpoint of the curve.

❚❑❚❑❚❑❚❑❚❑❚ PART TWO ❚❑❚❑❚❑❚❑❚❑❚

(3.) ***Simplex Method.*** Classify each simplex tableau below, using the following classifications, and write the appropriate letter on the right of the tableau. If class B, D, or E, indicate, by circling,the additional information requested.

A. UNIQUE OPTIMUM.
B. OPTIMAL, but with ALTERNATE optimal solutions. *Indicate (by circling) a pivot element which would yield an alternate basic optimal solution.*
C. INFEASIBLE
D. FEASIBLE but NOT OPTIMAL. *Indicate (by circling) a pivot element which would yield an improved solution.*
E. FEASIBLE but UNBOUNDED. *Indicate a variable which, by increasing without limits, will improve the objective without limit.*

Take careful note of whether the LP is being **min**imized or **max**imized! Note also that (-z), rather than z, appears in the first column (i.e., corresponding to the approach used in my notes instead of that in the text by Winston).

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MIN	1	2	0	4	-2	-3	0	1	0	-10	
	0	0	0	2	-4	0	0	-1	1	3	_____
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MIN	1	0	0	4	2	3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	_____
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	_____
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	2	0	4	-2	-3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	-3	_____
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS

MAX	1	-2	0	-4	-2	-3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	_____
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

(4.) ***Revised Simplex Method.*** We wish to solve the LP problem

$$\begin{aligned} \text{Maximize } & z=10X_1 + 6X_2 + 4X_3 \\ \text{subject to: } & X_1 + X_2 + X_3 \le 100 \\ & 10X_1 + 4X_2 + 5X_3 \le 600 \\ & 2X_1 + 2X_2 + 6X_3 \le 300 \\ & X_j \ge 0,\ j=1,2,3 \end{aligned}$$

After several iterations, we obtain the tableau below (in which some values have been omitted):

-Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
				$-10/3$			
			$5/6$	$5/3$	$-1/6$	0	$200/3$
	1	0	$1/6$	$-2/3$	$1/6$	0	
				-2	0	1	100

a. What is the "substitution rate" of X_4 for X_1? _______

b. If X_4 increases by 1 unit, X_1 will (**in**crease/**de**crease) *(circle one)* by ________ units.

c. What are the values of the simplex multipliers (π) for this tableau: ____________

d. Using the results of (c), what is the relative profit of X_3? __________

e. Complete the missing portions of the tableau above.

f. Is the above tableau optimal? ________ If not, circle a pivot element which would improve the objective.

(5.) **LINEAR PROGRAMMING DUALITY**: Consider the following LP:

Maximize $2X_1 - 13X_2 - 3X_3 + -2X_4 - 5X_5$

subject to $X_1 - X_2 \quad - \; 4X_4 - X_5 \quad = 5$

$X_1 \quad - 7X_4 \quad -2X_5 \quad -1$

$5X_2 + X_3 + X_4 \quad +2\,X_5 \quad 5$

$3X_2 + X_3 - X_4 + X_5 \quad = 2$

X_j 0 for all j=1, 2, 3; X_4 0; X_5 unrestricted in sign

a. Write a dual of this LP problem.

b. If X=(6,0,1,0,1) is optimal in the primal problem, then which **dual** variables (including slack or surplus variables) must be **zero** in the dual optimal solution, according to the complementary slackness conditions for this primal-dual pair of problems?

(6.) Transportation Problem: Consider the transportation problem with the tableau below:

destinations

sources	1	2	3	supply
1	10 (cost 9)	2 (cost 5)	(cost 4)	12
2	(cost 7)	(cost 6)	(cost 12)	8
3	(cost 10)	(cost 9)	(cost 3)	10
demand	10	5	15	

a. If the ordinary simplex tableau were to be written for this problem, how many rows (including the objective) will it have? _____
How many columns (including the right-hand-side and objective value -z) will it have? ________

b. Why does this problem not require a "dummy" destination? ______________________________

c. How many basic variables will this problem have? __________

d. An initial basic feasible solution is found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau above.

e. If U_1 (the dual variable for the first source) is equal to 10, what is the value of V_2 (the dual variable for the second destination)? ________

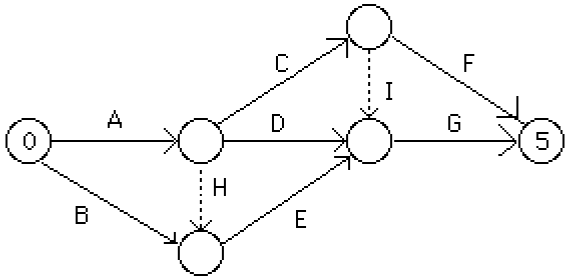
f. What is the reduced cost of the variable X_{31}? ___________ (Explain your computation.)

g. Will increasing X_{31} improve the objective function? ___________

h. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{31} enters?_________

i. What will be the value of X_{31} if it is entered into the solution as in (h)? ___________

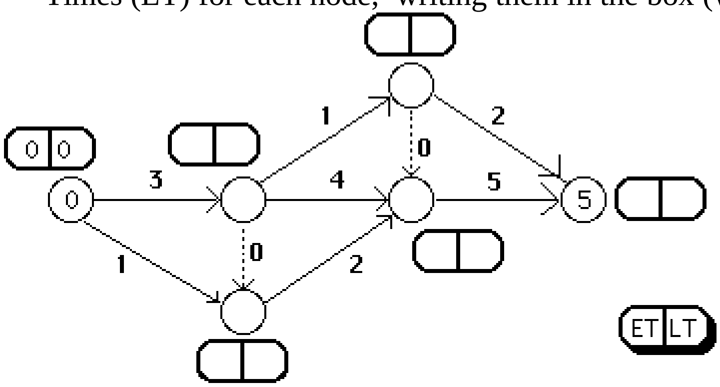
(7.) ***Project Scheduling.*** Consider the project with the A-O-A (activity-on-arrow) network given below.



a. How many activities (i.e., tasks), not including "dummies", are required to complete this project? ________

b. Complete the labeling of the nodes on the network above.

c. The activity durations are given below on the arrows. Compute the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

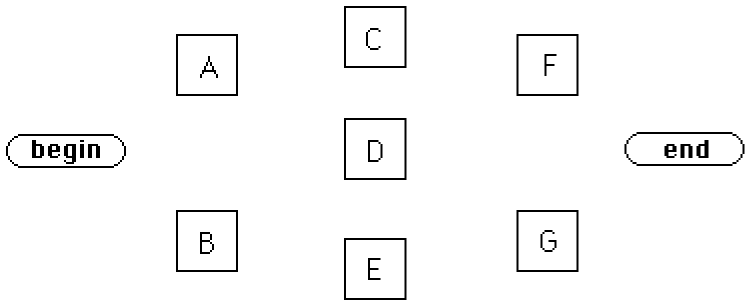


d. Find the slack ("total float") for activity C. _________

e. Which activities are critical? _________

f. What is the earliest completion time for the project? ___________

g. Complete the A-O-N (activity-on-node) network below for this same project.



h. Suppose that the arrow labelled "I" is deleted. Indicate the resulting A-O-N network below: