◆ ◆ ◆ ◆ ◆ 56:171 Operations Research ◆ ◆ ◆ ◆ ◆
Midterm Exam - October 26, 1989
◆ ◆ ◆ ◆ ◆ Instructor: D.L. Bricker ◆ ◆ ◆ ◆ ◆

Answer **all** of Part One and two (of the four) problems of Part Two

| Problem: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | TOTAL |
|---|---|---|---|---|---|---|---|---|---|
| Possible: | 16 | 12 | 20 | 10 | 8 | 8 | 8 | 8 | 74 |
| Your Score | | | | | | | | | |

◆ ◆ ◆ ◆ ◆ PART ONE: ANSWER ALL FOUR PROBLEMS ◆ ◆ ◆ ◆ ◆

(1.) Indicate by "+" or "o" whether each statement is true or false, respectively.

_____ a. To be feasible, a set of values of the LP decision variables must satisfy at least one constraint.

_____ b. In a transportation problem with **m** sources and **n** destinations, the number of basic variables is **m+n+1.**

_____ c. An LP that does not have a unique optimal solution must be either infeasible or unbounded.

_____ d. For an LP to be unbounded, it is necessary (but not sufficient) that all decision variables should be able to increase without limit.

_____ e. It is possible to find an LP with only two alternative optimal solutions.

_____ f. The optimal basic solution to an LP with $m$ constraints (excluding non-negativity constraints) can have at most $m$ positive decision variables.

_____ g. The steepest descent algorithm requires the computation of partial derivatives.

_____ h. The dual variables computed during the solution of a transportation problem are always nonnegative.

_____ i. The "Northwest Corner Rule" ignores the cost of shipping goods to their destinations when making allocations to get an initial feasible solution to a transportation problem.

_____ j. A function which is not convex is called *concave*.

_____ k. If the primal LP has an equality constraint, the corresponding dual variable must be zero.

_____ l. A basic solution of an LP is always feasible.

_____ m. A feasible solution of an LP is always basic.

_____ n. The dual of an LP problem is always a MAXIMIZE problem with "≤" constraints.

_____ o. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.

_____ p. Any point which satisfies the Kuhn-Tucker conditions is optimal.

(2.) Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through

**F**, according to the descriptions below. Also answer the question accompanying each classification, if any.

(A) Nonoptimal, nondegenerate tableau with no indication of being unbounded. *Circle a pivot element which would improve the objective.*

(B) Nonoptimal, degenerate tableau with no indication of being unbounded. *Circle an appropriate pivot element. Would the objective improve with this pivot?*

(C) Unique optimum.

(D) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*

(E) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal.

**Warning:** Some of these classifications might be used for several tableaus, while others might not be used at all!

| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | -1 | 0 | 0 | -2 | 0 | -84 |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 |
| 0 | 4 | 1 | 2 | -5 | 0 | 0 | 2 | 1 | 8 |
| 0 | -6 | 0 | 3 | -2 | 1 | 0 | -4 | 3 | 15 |

_________

| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | 1 | 0 | 0 | 2 | 12 | -84 |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 |
| 0 | 4 | 1 | 2 | -5 | 0 | 0 | 2 | 1 | 8 |
| 0 | -6 | 0 | 3 | -2 | 1 | 0 | -4 | 3 | 15 |

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| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | -1 | 0 | 0 | -2 | 0 | -84 |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 |
| 0 | 4 | 1 | 2 | 5 | 0 | 0 | 2 | 1 | 8 |
| 0 | -6 | 0 | 3 | -2 | 1 | 0 | -4 | 3 | 15 |

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| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | 3 | 0 | 0 | 2 | 0 | -84 |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 |
| 0 | 4 | 1 | 2 | -5 | 0 | 0 | 2 | 1 | 8 |
| 0 | -6 | 0 | 3 | -2 | 1 | 0 | -4 | 3 | -15 |

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| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | -1 | 0 | 0 | -2 | 0 | -84 | |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 | ________ |
| 0 | 4 | 1 | 2 | -5 | 0 | 0 | 2 | 1 | 0 | |
| 0 | -6 | 0 | 3 | 2 | 1 | 0 | -4 | 3 | 8 | |

| -z | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | RHS | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 1 | 3 | 0 | 0 | 0 | 0 | -84 | |
| 0 | 0 | 0 | -4 | 0 | 0 | 1 | 3 | 0 | 13 | ________ |
| 0 | 4 | 1 | 2 | -5 | 0 | 0 | 2 | 1 | 8 | |
| 0 | -6 | 0 | 3 | -2 | 1 | 0 | -4 | 3 | 15 | |

(3.) Consider the following product-mix problem:

- Five products can be manufactured: *A, B, C, D, & E*
- Each product requires time on each of three machines:

| *Product* | MACHINE *1* | *2* | *3* | |
|---|---|---|---|---|
| *A* | 12 | 8 | 5 | |
| *B* | 7 | 9 | 10 | *machine time* |
| *C* | 8 | 4 | 7 | *requirement* |
| *D* | 10 | 0 | 3 | *(minutes/lb)* |
| *E* | 7 | 11 | 2 | |

- 128 hours per week are available on each machine
- Any amounts which are produced may be sold at prices of
  $5, $4, $5, $4, & $4 per pound, respectively
- Variable labor costs are $4 per hour for machines 1 & 2, and $3 per hour for machine 3
- Material costs for products A and C are $2 per pound, and for products B, D, and E: $1 per pound

The company wishes to know how much of each product should be manufactured per week, in order to maximize profits.

**The LINDO output for this problem is as follows:**

```
:LOOK ALL

 MAX     1.417 A + 1.43 B + 1.85 C + 2.183 D + 1.7 E
 SUBJECT TO
        2)   12 A + 7 B + 8 C + 10 D + 7 E <=   7680
        3)   8 A + 9 B + 4 C + 11 E             <=   7680
        4)   5 A + 10 B + 7 C + 3 D + 2 E    <=   7680
 END

: GO
```

```
              OBJECTIVE FUNCTION VALUE
     1)          1817.6000

       VARIABLE     VALUE            REDUCED COST
          A        0.000000          1.379667
          B        0.000000          0.248334
          C      512.000000          0.000000
          D        0.000000          0.075334
          E      512.000000          0.000000

    ROW      SLACK OR SURPLUS      DUAL PRICES
     2)          0.000000           0.225833
     3)          0.000000           0.010833
     4)       3072.000000           0.000000

   RANGES IN WHICH THE BASIS IS UNCHANGED

                         OBJ COEFFICIENT RANGES

VARIABLE       CURRENT          ALLOWABLE         ALLOWABLE
                COEF            INCREASE          DECREASE
       A        1.417000         1.379667          INFINITY
       B        1.430000         0.248334          INFINITY
       C        1.850000         0.092857           0.041091
       D        2.183000         0.075334          INFINITY
       E        1.700000         0.113001           0.081250

                         RIGHTHAND SIDE RANGES

     ROW                        CURRENT                    ALLOWABLE
ALLOWABLE
                                          RHS
INCREASE                        DECREASE
       2         7680.000        2671.304199          2792.727539
       3         7680.000        4388.571289          3840.000000
       4         7680.000               INFINITY      3072.000000

: TABLEAU

     ROW  (BASIS)                      A                   B
C                  D
   1       ART          1.380     0.248     0.000     0.075
   2       C            1.267     0.233     1.000     1.833
   3       E            0.267     0.733     0.000    -0.667
   4       SLK  4      -4.400     6.900     0.000    -8.500

     ROW                   E             SLK  2         SLK  3           SLK
4
   1          0.000     0.226     0.011     0.000     1817.600
   2         -0.000     0.183    -0.117     0.000      512.000
   3          1.000    -0.067     0.133     0.000      512.000
   4          0.000    -1.150     0.550     1.000     3072.000

:PARARHS
ROW:3
NEW RHS VAL=15360

   VAR     VAR     PIVOT        RHS       DUAL PRICE         OBJ
   OUT     IN       ROW         VAL      BEFORE PIVOT        VAL
                               7680.00   0.108333E-01       1817.60
   C       SLK  3    2        12068.6    0.108333E-01       1865.14
   SLK 3   ART       2        12068.6    0.108372E-07       1865.14
                              15360.0     -INFINITY        INFEASIBLE
```

*ANSWER THE FOLLOWING QUESTIONS. IF NOT SUFFICIENT INFORMATION APPEARS IN THE LINDO OUTPUT, STATE THIS.*

(a.) State the optimal product mix (i.e., the optimal quantity of each product to be produced each week.) State the total profit at the optimum.
(b.) Is the optimal solution degenerate?
(c.) How much can the sales price of product A increase before the optimal production plan would change?
(d.) If we were to force one pound of product A to be produced, how would the total profit change?
(e.) According to the substitution rates in the TABLEAU output, what would be the effect of producing one pound of product A on the production of: product B? product C? product E?
(f.) If 100 minutes of machine #2 time were lost owing to breakdowns, what is the effect on the profit? Will there be a change in the optimal values of the quantities to be produced?
(g.) How much should the company be willing to pay for extra minutes on machine #1? How many minutes should they be willing to buy at this price?
(h.) Based upon the PARARHS output (and any other above), plot as much as you can of the optimal profit vs. minutes available on machine #1.
(i.) By how much must the selling price of product E drop before we need to revise the production plan?
(j.) If 100 minutes of machine #3 time were lost owing to breakdowns, what is the effect on the profit? Will there be a change in the optimal values of the quantities to be produced?

(4.) **TRANSPORTATION PROBLEM** Consider the transportation problem below.

| FROM \ TO: | D | E | F | G | supply |
|---|---|---|---|---|---|
| A | 3 | 7 | 6 | 4 | 5 |
| B | 2 | 4 | 3 | 2 | 2 |
| C | 4 | 3 | 8 | 5 | 3 |
| demand: | 3 | 3 | 2 | 2 | |

a. Use the "Northwest Corner Rule" to find an initial feasible solution.
b. What is the cost of this initialsolution?
c. Compute the dual variables.
d. Compute the reduced costs of shipments from B to D ($X_{BD}$) and C to E ($X_{CE}$).
e. Which shipment in part (d) will lower the total transportation costs?
f. What amount may be shipped along the route you selected in part (e)?

g. In this new solution, what shipments (give amounts and destinations) are sent **from B**?
h. By what amount has the transportation cost been lowered?
i. In this new solution, ___ shipments are positive, and ___ are zero, but basic. This is called a ____________ solution.

**◆ ◆ ◆ ◆ ◆ PART TWO: SELECT TWO PROBLEMS ◆ ◆ ◆ ◆ ◆**

(5.) At an intermediate step of the **simplex algorithm**, in which the objective is to be minimized, the tableau is:

| $-z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | -3 | 0 | 2 | -10 |
| 0 | 1 | 0 | 1 | 4 | 0 | -1 | 3 |
| 0 | 0 | 1 | 0 | 2 | 0 | 1 | 1 |
| 0 | -1 | 0 | 0 | -2 | 1 | 3 | 0 |

(a.) What is the basis for this tableau?
(b.) What are the current values of z and $x_1$ through $x_6$ for this basic solution?
(c.) Would increasing $x_4$ increase or decrease the objective function?
(d.) Would increasing $x_6$ increase or decrease the objective function?
(e.) What is the substitution rate of $x_4$ for $x_5$? If $x_4$ is increased by 1 unit, how is $x_5$ changed?
(e.) Is the current solution optimal? If not, perform a pivot to improve the objective function.

(6.) *Formulate an LP model for the following problem.*

- A manufacturer must plan production of a certain item over the next 4 quarters.
- Each unit of the item requires 1 man-hour of labor.
- Labor costs are \$10 per hour regularly, or \$15 per hour for overtime.
- Overtime is limited to 50% of regular time available.
- If a unit of the item is available for sale during a quarter but is not sold, an inventory carrying cost of \$2 per unit is charged.
- Other data are:

| Quarter | Man-hrs Available | Demand | Profit($) |
|---|---|---|---|
| 1 | 1000 | 1200 | 18 |
| 2 | 800 | 1100 | 16 |
| 3 | 900 | 1400 | 17 |
| 4 | 1000 | 1200 | 17 |

- Demand limits the amount which can be sold, but **need not** be satisfied.
- Profit that is specified is based upon regular-time labor costs. If overtime is used, the profits decrease accordingly.

The firm wishes to decide how much is to be produced each quarter and how much is to be sold each quarter.

*Suggested decision variables:*

$R_i$ = # of units to be produced using regular-time labor during quarter i

$O_i$ = # of units to be produced using overtime labor during quarter i

$I_i$ = # of units carried in inventory at the end of quarter i

a. Write the constraint which relates demand to production & inventory in quarter #3.

b. What is the objective function coefficient of $O_3$?

c. What is the objective function coefficient of $I_3$?

d. Write the constraint which limits the amount of overtime used in quarter #3.

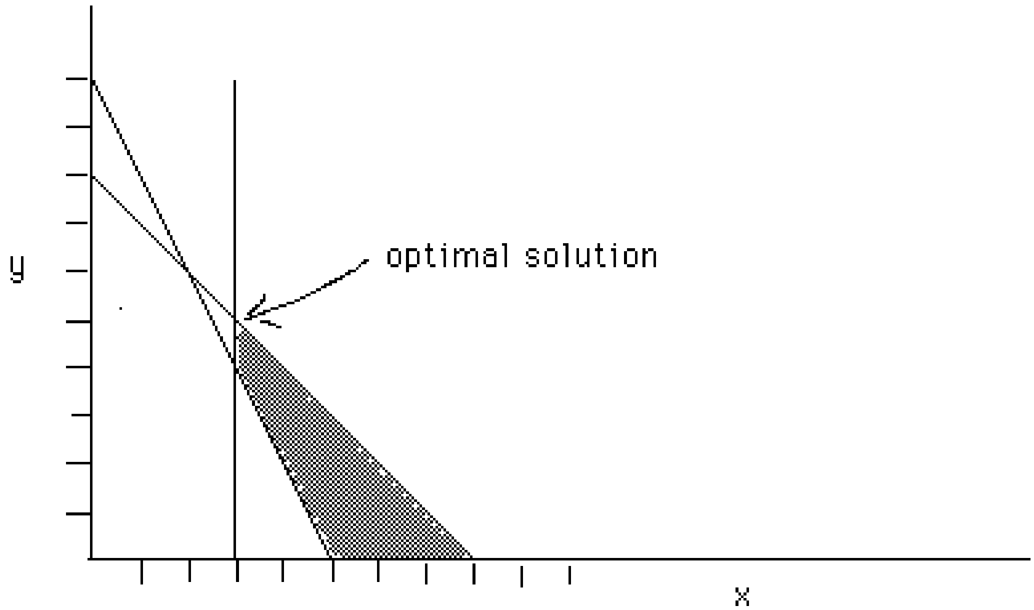
e. Write the constraint resulting from the limited man-hours available in quarter #3.

f. What is the total number of variables (excluding slack & surplus variables, etc.)?

g. What is the total number of constraints (excluding nonnegativity constraints)?

(7.) Consider the following LP and its graphical solution:

```
Maximize  x + 2y
subject to   x  + y ≤ 8
            2x + y  ≥ 10
             x ≥ 3,  y ≥ 0
```




(a.) What is the optimal solution? the optimal objective value?
(b.) By examination of the graphical solution, what must be the value of the optimal dual variable corresponding to the constraint $2x + y \geq 10$ ?
(c.) Write the dual LP for this problem.

(8.) **TRAVELING SALESMAN PROBLEM.** Five products are to be manufactured weekly on the same machine. The table below gives the cost of switching the machine from one product to another product. (Assume that this is also the cost of switching to the last product of the week to the first product to be scheduled the following week!)

| to: / from: | A | B | C | D | E |
|---|---|---|---|---|---|
| A | - | 1 | 8 | 3 | 4 |
| B | 1 | - | 8 | 2 | 3 |
| C | 1 | 3 | - | 5 | 1 |
| D | 2 | 5 | 6 | - | 5 |
| E | 5 | 3 | 7 | 6 | - |

After applying the "Hungarian Method" to the above matrix to solve the associated assignment problem (with large number, M, inserted along the diagonal), we have:

| to: / from: | A | B | C | D | E |
|---|---|---|---|---|---|
| A | M | 0 | 3 | 1 | 3 |
| B | 0 | M | 3 | 0 | 2 |
| C | 0 | 2 | M | 3 | 0 |
| D | 0 | 3 | 0 | M | 3 |
| E | 2 | 0 | 0 | 2 | M |

a. What is the solution of the assignment problem?
b. What is its cost?
c. Is it a valid product sequence? If not, why not?
d. If not a valid sequence, what bound (upper or lower) do you have on the optimal cost?