▼▲▼▲▼▲▼ 56:171 Operations Research ▼▲▼▲▼▲▼
▲▼▲▼▲▼▲ Final Examination Solutions ▲▼▲▼▲▼
▼▲▼▲▼▲▼ Fall 1998 ▼▲▼▲▼▲▼

• *Write your name on the first page, and initial the other pages.*
• *Answer both Parts A and B, and* **4 (out of 5)** *problems from Part C.*

		Possible
Part A:	Miscellaneous multiple choice	20
Part B:	Sensitivity analysis (LINDO)	20
Part C:	1. Discrete-time Markov chains I	15
	2. Discrete-time Markov chains II	15
	3. Continuous-time Markov chains	15
	4. Decision analysis	15
	5. Dynamic programming	15
	total possible:	100

▼▲▼▲▼▲▼ PART A ▼▲▼▲▼▲▼

Multiple Choice: Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =None of the above).

_e_ 1. If, in the optimal *primal* solution of an LP problem (min cx st Ax≥b, x≥0), there is zero slack in constraint #1, then in the optimal dual solution,
a. dual variable #1 must be zero c. slack variable for dual constraint #1 must be zero
b. dual variable #1 must be positive d. dual constraint #1 must be slack e. *NOTA*

_c_ 2. If, in the optimal solution of the *dual* of an LP problem (min cx subject to: Ax≥b, x≥0), dual variable #2 is positive, then in the optimal *primal* solution,
a. variable #2 must be zero c. slack variable for constraint #2 must be zero
b. variable #2 must be positive d. constraint #2 must be slack e. *NOTA*

_b_ 3. If $X_{ij}>0$ in the transportation problem, then dual variables U and V *must* satisfy
a. $C_{ij} > U_i + V_j$ c. $C_{ij} < U_i + V_j$ e. $C_{ij} = U_i - V_j$
b. $C_{ij} = U_i + V_j$ d. $C_{ij} + U_i + V_j = 0$ f. *NOTA*

_c_ 4. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
a. column is 1 c. row is 1
b. column is 0 d. row is 0 e. *NOTA*

_b_ 5. In PERT, the completion time for the project is assumed to
a. have the Beta distribution c. be constant
b. have the Normal distribution d. have the exponential distribution e. *NOTA*

_b_ 6. In an M/M/1 queue, if the arrival rate = $\lambda > \mu$= service rate, then
a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i e. the queue is not a birth-death process
b. no steady state exists d. $\pi_0 = 0$ in steady state f. *NOTA*

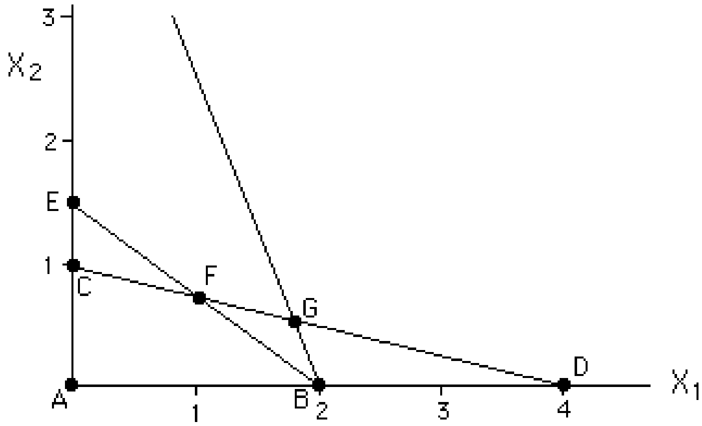
_d_ 7. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
a. will be nonbasic c. will have a worse objective value
a. will be nonfeasible d. will be degenerate e. *NOTA*

_d_ 8. An absorbing state of a Markov chain is one in which the probability of
a. moving into that state is zero c. moving into that state is one.
b. moving out of that state is one. d. moving out of that state is zero e. *NOTA*

The problems (9)-(12) below refer to the following LP:

Minimize $8X_1 + 4X_2$
subject to $3X_1 + 4X_2 \geq 6$
$5X_1 + 2X_2 \leq 10$
$X_1 + 4X_2 \leq 4$
$X_1 \geq 0, X_2 \geq 0$

(with inequalities converted to equations:)
Minimize $8X_1 + 4X_2$
subject to $3X_1 + 4X_2 - X_3 = 6$
$5X_1 + 2X_2 + X_4 = 10$
$X_1 + 4X_2 + X_5 = 4$
$X_j \geq 0$, j=1,2, 3,4,5



_a__ 9 .The feasible region includes points

a. B, F, & G
c. C, E, & F
b. A, B, C, & F
d. E, F, &G
e. *NOTA*

_d__ 10. At point F, the basic variables include the variables

a. X_2 & X_3
c. X_4 & X_5
b. X_3 & X_4
d. X_1 & X_4
e. *NOTA*

__b_ 11. Which point is degenerate in the primal problem?

a. point A
c. point C
b. point B
d. point D
e. *NOTA*

_c__ 12. The dual of this LP has the following constraints (not including nonnegativity or nonpositivity):

a. 2 constraints of type (≥)
b. one each of type ≤ & ≥
c. 2 constraints of type (≤)
d. one each of type ≥ & =
e. *None of the above*

_b__ 13. The dual of the LP has the following types of variables:

a. three non-negative variables
b. one non-negative and two non-positive variables
c. two non-negative variables and one unrestricted in sign
d. two non-negative variables and one non-positive variable
e. three non-positive variables
f. *None of the above*

_e__ 14. If point F is optimal, then which dual variables must be zero, according to the *Complementary Slackness Theorem* ?

a. Y_1 and Y_2
b. Y_1 and Y_3
c. Y_2 and Y_3
d. Y_1 only
e. Y_2 only
f. Y_3 only

Consider a discrete-time Markov chain with transition probability matrix :

$$\begin{bmatrix} .6 & .4 \\ .3 & .7 \end{bmatrix}$$

_a_ 15. If the system is initially in state #1, the probability that the system will be in state 2 after exactly one step is:

a. 0.4
c. 0.7
e. none of the above
b. 0.6
d. 0.52

_d_ 16. If the Markov chain in the previous problem was initially in state #1, the probability that the system will still be in state 1 after 2 transitions is

a. 0.36
c. 0
e. 0.52
b. 0.60
d. 0.48
f. *NOTA*

_c_ 17. The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation

a. $P\,\pi = 0$ c. $\pi\,(I-P) = 0$ e. *NOTA*

b. $P\,\pi = \pi$ d. $P^t\,\pi = 0$

_a_ 18. For a continuous-time Markov chain, let Λ be the matrix of transition rates. The sum of each ...

a. row is 0 c. row is 1 e. *NOTA*

b. column is 0 d. column is 1

_e_ 19. To compute the steady state distribution π of a continuous-time Markov chain, one must solve (in addition to sum of π components equal to 1) the matrix equation (where Λ^t is the transpose of Λ):

a. $\pi\,\Lambda = 1$ c. $\Lambda^T\,\pi = \pi$ e. $\pi\,\Lambda = 0$

b. $\Lambda^T\,\pi = 1$ d. $\pi\,\Lambda = \pi$ f. *NOTA*

_e_ 20. Little's Law is applicable to queues of the class(es):

a. M/M/1 c. any birth-death process e. any queue with steady state

b. M/M/c for any c d. any continuous-time Markov chain f. *NOTA*

_e_ 21. In a birth/death model of a queue,

a. time between arrivals has Poisson distribution

b. number of "customers" served cannot exceed 1

c. the distribution of the number of customers in the system has exponential distribution

d. the arrival rate is the same for all states

e. *None of the above*

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis in LP.

"A manufacturer produces two types of plastic cladding. These have the trade names Ankalor and Beslite. One yard of Ankalor requires 8 lb of polyamine, 2.5 lb of diurethane and 2 lb of monomer. A yard of Beslite needs 10 lb of polyamine, 1 lb of diurethane, and 4 lb of monomer. The company has in stock 80,000 lb of polyamine, 20,000 lb of diurethane, and 30,000 lb of monomer. Both plastics can be produced by alternate parameter settings of the production plant, which is able to produce sheeting at the rate of 12 yards per hour. A total of 750 production plant hours are available for the next planning period. The contribution to profit on Ankalor is \$10/yard and on Beslite is \$20/yard.

The company has a contract to deliver at least 3,000 yards of Ankalor. What production plan should be implemented in order to maximize the contribution to the firm's profit from this product division."

Definition of variables:

A = Number of yards of Ankalor produced

B = Number of yards of Beslite produced

LP model: 1) Maximize 10 A + 20 B subject to

2)	8 A + 10 B	≤80,000	(lbs. Polyamine available)
3)	2.5 A + 1 B	≤ 20,000	(lbs. Diurethane available)
4)	2 A + 4 B	≤ 30,000	(lbs. Monomer available)
5)	A + B	≤ 9,000	(lbs. Plant capacity)
6)	A	≥ 3,000	(Contract)
	A ≥ 0, B ≥ 0		

The LINDO solution is:

```
OBJECTIVE FUNCTION VALUE
     1)   142000.000
VARIABLE          VALUE        REDUCED COST
     A          3000.000           0.000
     B          5600.000           0.000

ROW           SLACK OR SURPLUS    DUAL PRICES
  2)                 0.000           2.000
  3)              6900.000           0.000
  4)              1600.000           0.000
  5)               400.000           0.000
  6)                 0.000          -6.000
```

RANGES IN WHCH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	10.000	6.000	INFINITY
B	20.000	INFINITY	7.500

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	80000.000	4000.000	56000.000
3	20000.000	INFINITY	6900.000
4	30000.000	INFINITY	1600.000
5	9000.000	INFINITY	400.000
6	3000.000	2000.000	1333.333

THE TABLEAU

ROW	(BASIS)	A	B	SLK 2	SLK 3	SLK 4	SLK 5
1	ART	.000	.000	2.000	.000	.000	.000
2	B	.000	1.000	.100	.000	.000	.000
3	SLK 3	.000	.000	-.100	1.000	.000	.000
4	SLK 4	.000	.000	-.400	.000	1.000	.000
5	SLK 5	.000	.000	-.100	.000	.000	1.000
6	A	1.000	.000	.000	.000	.000	.000

ROW	SLK 6	
1	6.0	0.14E+06
2	.800	5600.000
3	1.700	6900.000
4	-1.200	1600.000
5	.200	400.000
6	-1.000	3000.000

Consult the LINDO output above to answer the following questions. If there is not sufficient information in the LINDO output, answer "NSI".

_c_ 1. How many yards of Beslite should be manufactured?
- a. 3000 yards
- b. 1600 yards
- c. 5600 yards
- d. 400 yards
- e. NSI

_c_ 2. How much of the available diurethane will be used?
- a. 6900 pounds
- b. 1600 pounds
- c. 13100 pounds
- d. 400 pounds
- e. NSI

_a_ 3. How much of the available diurethane will be unused?
- a. 6900 pounds
- b. 1600 pounds
- c. 13100 pounds
- d. 400 pounds
- e. NSI

_b_ 4. Suppose that the company can purchase 2000 pounds of additional polyamine for $2.50 per pound. Should they make the purchase? a. yes b. no c. NSI

_a_ 5. Regardless of your answer in (4), suppose that they do purchase 2000 pounds of additional polyamine. This is equivalent to
- a. decreasing the slack in row 2 by 2000
- b. increasing the surplus in row 2 by 2000
- c. increasing the slack in row 2 by 2000
- d. decreasing the surplus in row 2 by 2000
- e. none of the above
- f. NSI

_d_ 6. If the company purchases 2000 pounds of additional polyamine, what is the total amount of Beslite that they should deliver? (Choose nearest value?)
- a. 5500 yards
- b. 5600 yards
- c. 5700 yards
- d. 5800 yards
- e. 5900 yards
- f. NSI

_c_ 7. How will the decision to purchase 2000 pounds of additional polyamine change the quantity of diurethane used during the next planning period?
- a. increase by 100 pounds
- b. decrease by 100 pounds
- c. increase by 200 pounds
- d. decrease by 200 pounds
- e. none of the above
- f. NSI

Note: Purchasing an additional 2000 lbs of polyamine would force the variable SLK2 to become -2000. *According to the substitution rate* (-0.1) *of SLK2 for SLK3, SLK3 would change in the* <u>*same*</u> *direction by the amount 0.1· 2000=200 lbs which would indicate that an additional 200 lbs of diurethane is purchased. (This is, of course, conditional upon the increase of 2000 being less than the allowable increase for the RHS, which in this case is 4000.)*

_a_ 8. If the profit contribution from Beslite were to decrease to $11/yard, will the optimal solution change?
a. yes b. no c. NSI

_b_ 9. If the profit contribution from Ankelor were to increase to $15/yard, will the optimal solution change?
a. yes b. no c. NSI

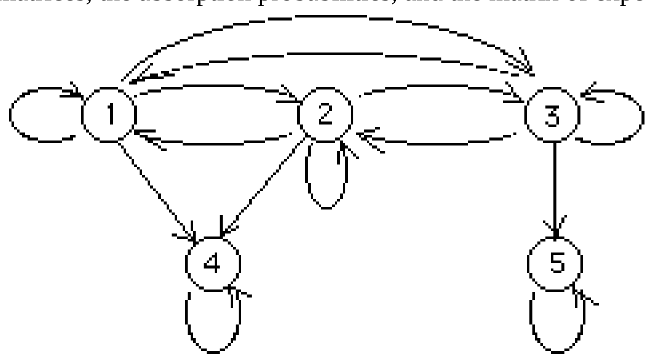
_a_ 10. Suppose that the company could deliver 1000 yards less than the contracted amount of Ankalor by paying a penalty of $5/yard shortage. Should they do so? a. yes b. no c. NSI

▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

1. **Discrete-Time Markov Chains I:** A baseball team consists of 2 stars, 13 starters, and 10 substitutes. For insurance purposes, the team owner must place a value on the players. The value of each player is defined to be the total value of the salary he will earn until retirement. At the beginning of each season, the players are classified into one of four categories:

Category 1: Substitute (earns $100,000 per year).
Category 2: Starter (earns $400,000 per year).
Category 3: Star (earns $1 million per year).
Category 4: Retired while not a star (earns no more salary).
Category 5: Retired while Star (earns no salary, but is paid $100,000/year for product endorsements).

Given that a player is a star, starter, or substitute at the beginning of the current season, the probabiliites that he will be a star, starter, substitute, or retired at the beginning of the next season are shown in the transition probability matrix P below. Also shown are a diagram of the Markov chain model of a "typical" player, several powers of P, the first-passage probability matrices, the absorption probabilities, and the matrix of expected number of visits.



$$P = \begin{bmatrix} 0.5 & 0.15 & 0.05 & 0.3 & 0 \\ 0.2 & 0.5 & 0.2 & 0.1 & 0 \\ 0.15 & 0.3 & 0.45 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 0.5 & 0.15 & 0.05 & 0.3 & 0 \\ 0.2 & 0.5 & 0.2 & 0.1 & 0 \\ 0.15 & 0.3 & 0.45 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.2875 & 0.165 & 0.077 & 0.465 & 0.005 \\ 0.23 & 0.34 & 0.2 & 0.21 & 0.02 \\ 0.2025 & 0.307 & 0.27 & 0.075 & 0.145 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad F^{(2)} = \begin{bmatrix} 0.0375 & 0.09 & 0.055 & 0.165 & 0.005 \\ 0.13 & 0.09 & 0.11 & 0.11 & 0.02 \\ 0.1275 & 0.157 & 0.067 & 0.075 & 0.045 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.188 & 0.148 & 0.082 & 0.567 & 0.012 \\ 0.213 & 0.264 & 0.169 & 0.313 & 0.04 \\ 0.203 & 0.265 & 0.193 & 0.166 & 0.172 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad F^{(3)} = \begin{bmatrix} 0.0258 & 0.0528 & 0.044 & 0.1027 & 0.007 \\ 0.0905 & 0.0495 & 0.066 & 0.103 & 0.02 \\ 0.0963 & 0.0843 & 0.041 & 0.0915 & 0.027 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.136 & 0.127 & 0.076 & 0.639 & 0.020 \\ 0.184 & 0.215 & 0.139 & 0.403 & 0.056 \\ 0.183 & 0.220 & 0.150 & 0.253 & 0.191 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad F^{(4)} = \begin{bmatrix} 0.018 & 0.030 & 0.031 & 0.071 & 0.008 \\ 0.064 & 0.027 & 0.041 & 0.090 & 0.016 \\ 0.070 & 0.045 & 0.026 & 0.087 & 0.019 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 4 & 5 \end{array} \\ \begin{bmatrix} 0.93103 & 0.068966 \\ 0.86207 & 0.13793 \\ 0.72414 & 0.27586 \end{bmatrix} \end{array} \qquad E = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 2.6959 & 1.2226 & 0.68966 \\ 1.7555 & 3.3542 & 1.3793 \\ 1.6928 & 2.163 & 2.7586 \end{bmatrix} \end{array}$$

Select the **nearest** available numerical choice in answering the questions below.

_d_ 1. The number of *transient* states in this Markov chain model is
a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

_c_ 2. The number of *absorbing* states in this Markov chain model is
a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

_c_ 3. The number of *recurrrent* states in this Markov chain model is
a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

4. The closed sets of states in this Markov chain model are (circle all that apply!)
a. {1} $\boxed{\text{d. } \{4\}}$ g. {1,2,3} j. {3,4,5}
b. {2} $\boxed{\text{e. } \{5\}}$ h. {1,2,3,4} k. {2,3,4,5}
c. {3} f. {1,2} i. {4,5} $\boxed{\text{l. } \{1,2,3,4,5\}}$

5. The *minimal* closed sets of states in this Markov chain model are (circle all that apply!)
a. {1} $\boxed{\text{d. } \{4\}}$ g. {1,2,3} j. {3,4,5}
b. {2} $\boxed{\text{e. } \{5\}}$ h. {1,2,3,4} k. {2,3,4,5}
c. {3} f. {1,2} i. {4,5} l. {1,2,3,4,5}

Suppose that at the beginning of the 1998 season, Joe Blough was a *Starter* (category #2).

_d_ 6. What is the probability that Joe is a star in 1999? (choose nearest answer)
a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

_d_ 7. What is the probability that Joe is a star in 2000? (choose nearest answer)
a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

_b_ 8. What is the probability that Joe *first* becomes a star in 2000? (choose nearest answer)
a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

_c_ 9. What is the probability that Joe *eventually* becomes a star before he retires? (choose nearest answer)
a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

_f_ 10. What is the expected length of his playing career, in years? (choose nearest answer)
a. 1 year c. 3 years e. 5 years g. 7 years i. 9 years
b. 2 years d. 4 years f. 6 years h. 8 years j. *NOTA*

_c_ 11. What fraction of players who achieve "stardom" retire while still a star? (choose nearest answer)
a. 10% c. 30% e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100%

_f_ 12. What is Joe's expected earnings (in $millions) for the remainder of his career? (choose nearest answer)
a. .5 c 1.5 e. 2.5 g. 3.5 i. 4.5
b. 1 d. 2 f. 3 h. 4 j. 5 or more

2. Discrete-time Markov Chain II: *(Model of Inventory System)* Consider the following inventory system for a certain spare part for a company's 2 production lines, costing $10 each. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to restock the shelf at the end of each day (after any needed spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** the shelf is empty or contains a single part. The annual holding cost of the part is 20% of the value.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: **Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

Suppose that the shelf is full Sunday p.m. (after restocking):

_h_ 1. What is the expected number of days until the next stockout occurs? *(choose nearest answer)*

a. 2 c. 6 e. 10 g. 14 i. 18
b. 4 d. 8 f. 12 h. 16 j. 20 or more

_b_ 2. What is the probability that the shelf is full Wednesday p.m. (before restocking) *(choose nearest answer)*

a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50% or more

_d_ 3. What is the expected number of times during the next five days that the shelf is restocked? *(choose nearest answer)*

a. 0.25 c. 0.75 e. 1.25 g. 1.75 i. 2.25
b. 0.5 d. 1 f. 1.5 h. 2 j. 2.5 *or more*

_h_ 4. How frequently do stockouts occur? Once every *(choose nearest answer)*

a. 2 days c. 6 days e. 10 days g. 14 days i. 18 days
b. 4 days d. 8 days f. 12 days h. 16 days j. 20 *or more*

_b_ 5. How many stockouts per year can be expected? *(choose nearest answer)*

a. 10 c. 20 e. 30 g. 40 i. 50
b. 15 d. 25 f. 35 h. 45 j. 55 *or more*

_d_ 6. What is the company's average inventory cost per year for this part? *(choose nearest answer)*

a. \$1 c. \$3 e. \$5 g. \$7 i. \$9
b. \$2 d. \$4 f. \$6 h. \$8 j. \$10 *or more*

_a_ 7. The number of transient states in this Markov chain is

a. 0 c. 2 e. 4 g. *None of the above*
b. 1 d. 3 f. 5

_f_ 8. The number of recurrent states in this Markov chain is

a. 0 c. 2 e. 4 g. *None of the above*
b. 1 d. 3 f. 5

The transition probability matrix and its first five powers:

P =

0	0	0.2	0.5	0.3
0	0	0.2	0.5	0.3
0.2	0.5	0.3	0	0
0	0.2	0.5	0.3	0
0	0	0.2	0.5	0.3

P^2 =

0.04	0.2	0.37	0.3	0.09
0.04	0.2	0.37	0.3	0.09
0.06	0.15	0.23	0.35	0.21
0.1	0.31	0.34	0.19	0.06
0.04	0.2	0.37	0.3	0.09

P^3 =

0.074	0.245	0.327	0.255	0.099
0.074	0.245	0.327	0.255	0.099
0.046	0.185	0.328	0.315	0.126
0.068	0.208	0.291	0.292	0.141
0.074	0.245	0.327	0.255	0.099

P^4 =

0.0654	0.2145	0.3092	0.2855	0.1254
0.0654	0.2145	0.3092	0.2855	0.1254

```
0.0656  0.227   0.3273  0.273   0.1071
0.0582  0.2039  0.3167  0.2961  0.1251
0.0654  0.2145  0.3092  0.2855  0.1254
```

P^5 =

0.06184	0.2117	0.31657	0.2883	0.12159
0.06184	0.2117	0.31657	0.2883	0.12159
0.06546	0.21825	0.31463	0.28175	0.11991
0.06334	0.21757	0.3205	0.28243	0.11616
0.06184	0.2117	0.31657	0.2883	0.12159

$$\sum_{n=1}^{5} P^n =$$

0.24124	0.8712	1.52277	1.6288	0.73599
0.24124	0.8712	1.52277	1.6288	0.73599
0.43706	1.28025	1.49993	1.21975	0.56301
0.28954	1.13947	1.7682	1.36053	0.44226
0.24124	0.8712	1.52277	1.6288	0.73599

The mean first passage time matrix:

15.7692	4.64151	3.07692	2.57143	8.36735
15.7692	4.64151	3.07692	2.57143	8.36735
12.6923	2.75472	3.15385	4	9.79592
15	3.39623	2.30769	3.51429	10.8163
15.7692	4.64151	3.07692	2.57143	8.36735

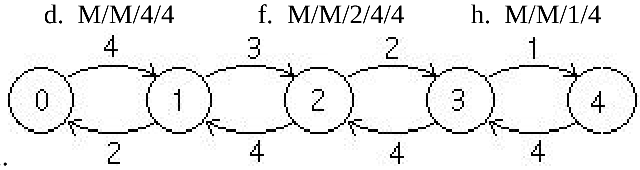
Steady-state distribution:

State	i	P{i}
SOH=0	1	0.0634146
SOH=1	2	0.215447
SOH=2	3	0.317073
SOH=3	4	0.284553
SOH=4	5	0.119512

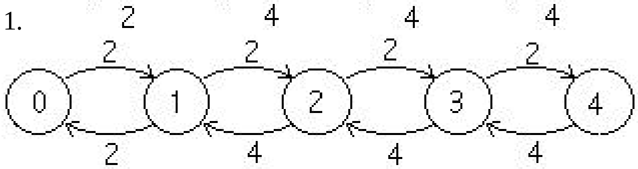
3. Birth/Death Model of a Queue:

For each birth/death process below, pick the classification of the queue and write it in the blank to the left:

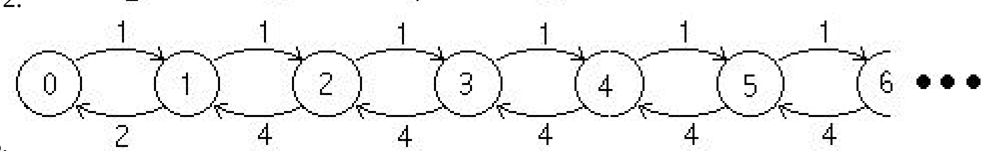
a. M/M/1	c. M/M/4	e. M/M/2/2	g. M/M/2	i. M/M/1/4/4	k. M/M/4/2
b. M/M/2	d. M/M/4/4	f. M/M/2/4/4	h. M/M/1/4	j. M/M/2/4	l. *NOTA*



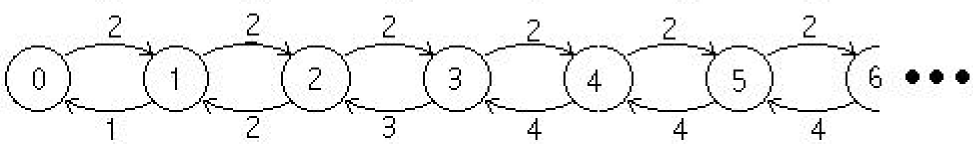
M/M/2/4/4 1.



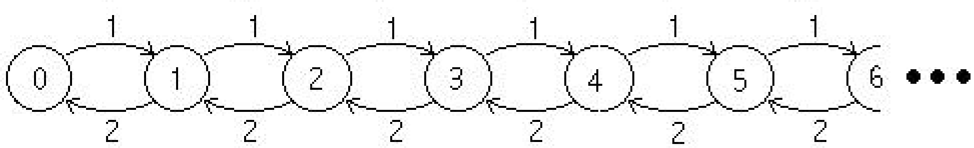
M/M/2/4 2.



M/M/2 3.



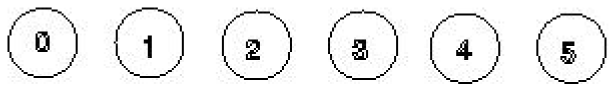
M/M/4 4.



M/M/1 5.

Customers arrive at a grocery checkout lane in a Poisson process at an average rate of one every two minutes if there are 2 or fewer customers already in the checkout lane, and one every four minutes if there are already 3 in the lane. If there are already 4 in the lane, no additional customers will join the queue. The service times are exponentially distributed, with an average of 1.5 minutes if there are fewer than 3 customers in the checkout lane. If three or four customers are in the lane, another clerk assists in bagging the groceries, so that the average service time is reduced to 1 minute. The average arrival rate at this checkout lane is 0.46/minute.

6. Draw the diagram for this birth/death process, indicating the birth & death rates.



$\lambda 0 = \lambda 1 = \lambda 2 = 0.5$/minute, $\lambda 3$=0.25/minute, $\lambda i = 0$ for i>3
$\mu 1 = \mu 2 = 2/3$ per minute, $\mu 3 = \mu 4 = 1$/minute

7. Write the expression which is used to evaluate $1/\pi_0$:

$$\frac{1}{p_0} = 1 + \frac{1/2}{2/3} + \left(\frac{1/2}{2/3}\right)^2 + \left(\frac{1/2}{2/3}\right)^2 \times \frac{1}{2} + \left(\frac{1/2}{2/3}\right)^2 \times \frac{1}{2} \times \frac{1}{4}$$
$$= 1 + 0.75 + (0.75)^2 + (0.75)^2(0.5) + (0.75)^2(0.5)(0.25) = 2.664$$

_b_ 8. What fraction of the time will the second clerk be busy at this checkout lane? *(Choose nearest value.)*

a. 10%	c. 20%	e. 30%	g. 40%
b. 15%	d. 25%	f. 35%	h. 45% or more

_g_ 9. What fraction of the time will this checkout lane be empty? *(Choose nearest value.)*

a. 5%	c. 15%	e. 25%	g. 35%	i. 45%
b. 10%	d. 20%	f. 30%	h. 40%	j. 50% or more

_e_ 10. What is the average number of customers <u>waiting</u> at this checkout lane (not including the customer being served)? *(Choose nearest value.)*

a. 0.1	c. 0.3	e. 0.5	g. 0.7	i. 0.9
b. 0.2	d. 0.4	f. 0.6	h. 0.8	j. 1 *or more*

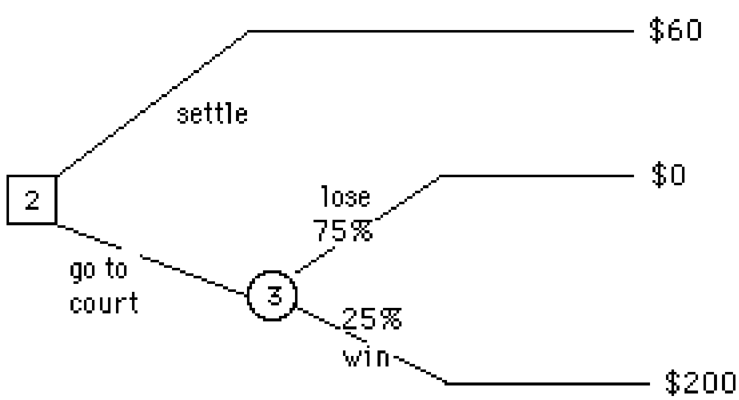
_d_ 11. What is the average time in minutes that a customer spends <u>waiting</u> at this checkout lane (not including the time being served)? *(Choose nearest value.)*

a. 0.25	c. 0.75	e. 1.25	g. 1.75
b. 0.5	d. 1	f. 1.5	h. 2 *or more*

The steadystate distribution (and the <u>C</u>umulative <u>D</u>istribution <u>F</u>unction) is

i	πi	CDF
0	0.3753	0.3753
1	0.2815	0.6569
2	0.2111	0.8680
3	0.1056	0.9736
4	0.0264	1.0000

4. Decision Trees: General Custard Corporation is being sued by Sue Smith. Sue can settle out of court and win $60,000, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive $200,000, and if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



__a__ 1. What is the decision which maximizes the expected value?

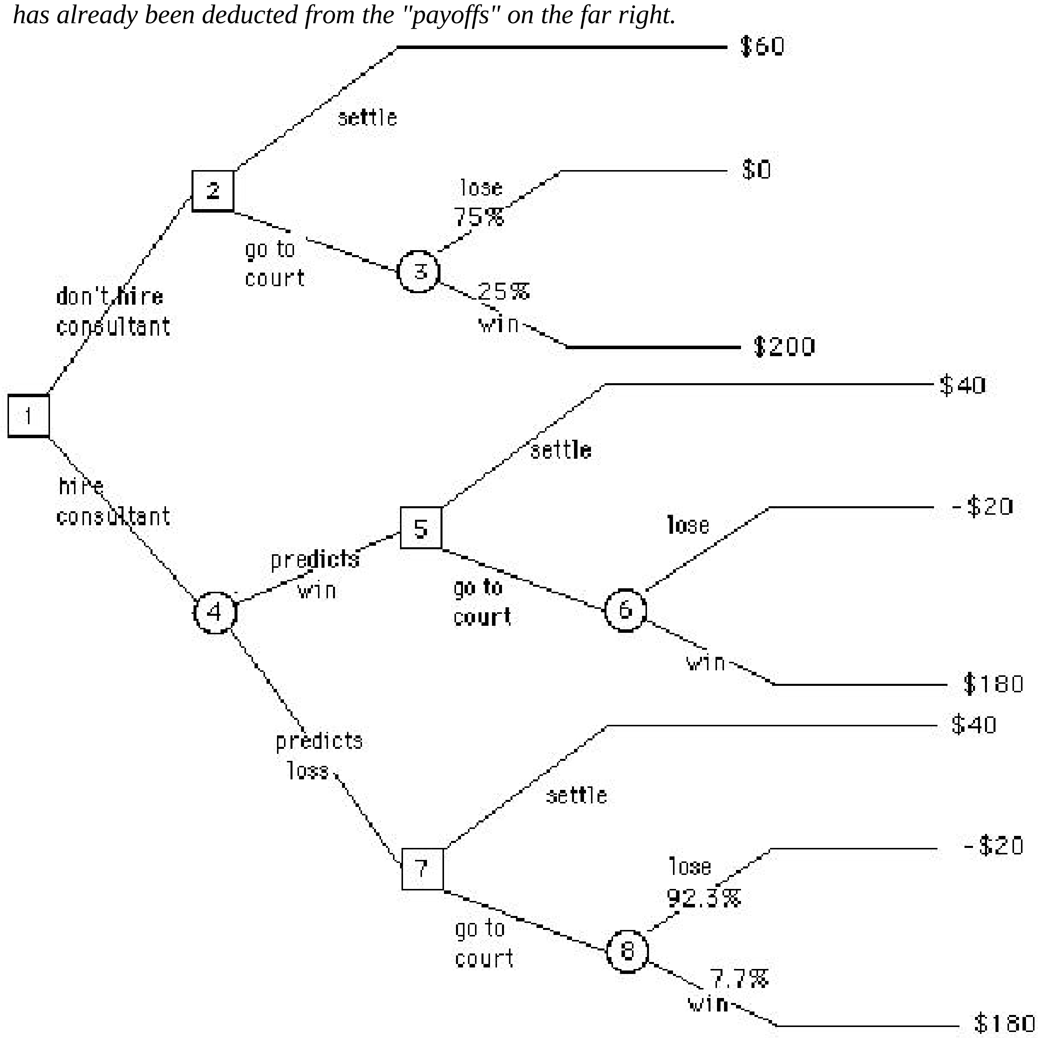
a. settle b. go to court

For \$20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant predicts the correct outcome 80% of the time.

2. Complete the following blanks

P{W} *(prior probability)*	<u>0.25</u>
P{L} *(prior probability)*	<u>0.75</u>
$P\{PW \mid W\}$	<u>0.8</u>
$P\{PL \mid W\}$	<u>0.2</u>
$P\{PW \mid L\}$	<u>0.2</u>
$P\{PL \mid L\}$	<u>0.8</u>
P{PW}	<u>0.35</u> $= P\{PW \mid W\}\, P\{W\} + P\{PW \mid L\}\, P\{L\} = (0.8)(0.25) + (0.2)(0.75)$
P{PL}	<u>0.65</u>
$P\{W \mid PW\}$	<u>0.5714</u> according to Bayes' theorem: $\dfrac{P\{PW \mid W\}\, P\{W\}}{P\{PW\}} = \dfrac{(0.8)(0.25)}{0.35} = \dfrac{2}{0.35}$

The decision tree below includes Sue's decision as to whether or not to hire the consultant. *Note that the consultant's fee has already been deducted from the "payoffs" on the far right.*



3. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	<u>-4.6</u>	5	94.286	2	<u>60</u>
7	40	4	59	1	<u>60</u>______

6	94.286	3	50

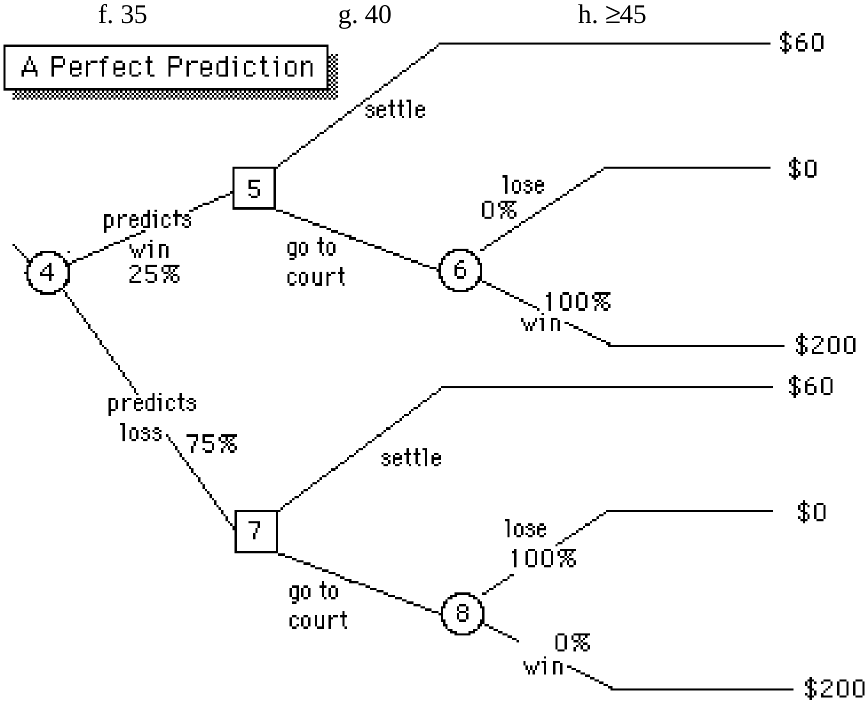
4. Should Sue hire the consultant? *Circle:* Yes $\boxed{\text{No}}$

_f__ 5. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤16 b. 18 c. 20 d. 22
e. 17 f. 19 g. 21 h. ≥23

_f___ 6. What would be the expected value of "perfect information" which is *given* to Sue at no cost, i.e., a prediction which is 100% accurate, so that the portion of the tree containing nodes 4, 5, 6, 7, etc., would appear as below? *(Choose nearest value, in thousands of $)*

a. ≤10 b. 15 c. 20 d. 25
e. 30 f. 35 g. 40 h. ≥45



5. Dynamic Programming. *Match Problem.* Suppose that there are 15 matches originally on the table, and you are challenged by your friend to play this game. Each player must pick up either 1, 2, 3, or 4 matches, with the player who picks up the last match paying $1.

Define **F(i)** to be the **minimal cost** to you (either $1 or $0) if
- it is your turn to pick up matches, and
- **i** matches remain on the table.

Thus, F(1) = 1, since you are forced to pick up the last match; F(2) = 0 (since you can pick up one match, forcing your opponent to pick up the last match), etc.

1. What is the value of F(3)? __0__
2. What is the value of F(4)? __0__
3. What is the value of F(6)? __1__
4. What is the value of F(15)? __0__

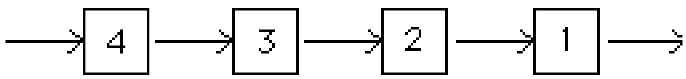
__a__5. If you are allowed to decide whether you or your friend should take the first turn, what is your optimal decision?

a. You take first turn c. You are indifferent about this choice
b. Friend takes first turn d. You refuse to play the game

Consider the following *zero-one knapsack problem*, with a capacity of 8 units of weight:

item #	Weight	Value
1	4	7
2	3	6
3	1	1
4	2	3

The Dynamic Programming approach used to solve this problem imagines that the items are considered in the order: first the decision is made whether to include item 4, second-- whether to include item 3, etc. *(although the computations are done in a backward fashion, starting with item 1(stage 1) and ending with item 4 (stage 4)):*



Dynamic programming output for this problem is given below:

---Stage 1---

s \ x:	0	1	State	Optimal Values	Optimal Decisions	Resulting State
0	.00	¯992.00	0	.00	0	0
1	.00	¯992.00	1	.00	0	1
2	.00	¯992.00	2	.00	0	2
3	.00	¯992.00	3	.00	0	3
4	.00	7.00	4	7.00	1	0
5	.00	7.00	5	7.00	1	1
6	.00	7.00	6	7.00	1	2
7	.00	7.00	7	7.00	1	3
8	.00	7.00	8	7.00	1	4

---Stage 2---

s \ x:	0	1	State	Optimal Values	Optimal Decisions	Resulting State
0	.00	¯993.00	0	.00	0	0
1	.00	¯993.00	1	.00	0	1
2	.00	¯993.00	2	.00	0	2
3	.00	6.00	3	6.00	1	0
4	7.00	6.00	4	7.00	0	4
5	7.00	6.00	5	7.00	0	5
6	7.00	6.00	6	7.00	0	6
7	7.00	13.00	7	13.00	1	4
8	7.00	13.00	8	13.00	1	5

---Stage 3---

s \ x:	0	1	State	Optimal Values	Optimal Decisions	Resulting State
0	.00	¯998.00	0	.00	0	0
1	.00	1.00	1	1.00	1	0
2	.00	1.00	2	1.00	1	1
3	6.00	1.00	3	6.00	0	3
4	7.00	7.00	4	7.00	0	4
5	7.00	8.00			1	3
6	7.00	8.00	5	8.00	1	4
7	13.00	8.00	6	8.00	1	5
8	13.00	14.00	7	13.00	0	7
			8	14.00	1	7

---Stage 4---

s \ x:	0	1	State	Optimal Values	Optimal Decisions	Resulting State
0	.00	¯996.00	0	.00	0	0
1	1.00	¯996.00	1	1.00	0	1
2	1.00	3.00	2	3.00	1	0
3	6.00	4.00	3	6.00	0	3
4	7.00	4.00	4	7.00	0	4
5	8.00	9.00	5	9.00	1	3
6	8.00	**a**	6	**b**	**c**	**d**
7	13.00	11.00	7	13.00	0	7
8	14.00	11.00	8	14.00	0	8

6. Complete the blank entries in Stage 4, that is, if there is a capacity of 6 pounds,
   - what is the value which can be obtained if item 4 is included in the knapsack? **a** = 6 = 3 + f3(6-2) = 3+7 = 10
   - what is the maximum value which can be obtained if there is a capacity of 6 pounds? **b** = 10
   - what is the optimal decision at stage 4, if the remaining capacity available to be filled is 6 pounds? **c** = 1 units of item #4
   - If 6 pounds of capacity remains available in the knapsack and **c** units of item #4 are included, the remaining capacity is **d** = 4 pounds.

7. Actually, there is an available capacity of 8 pounds when we are at stage 4 (i.e., considering whether or not to include item #4). Trace your way through the tables above to obtain the optimal solution:
   - Maximum value possible is $ 14.00
   - Include _1_ units of item #1,
     _1_ units of item #2,
     _1_ units of item #3,
     _0_ units of item #4