56:171 Operations Research
Final Examination Solution
Fall 2001

• *Write your name on the first page, and initial the other pages.*
• *Answer* ***both*** *Parts* ***A*** *and* ***B****, and select any* ***4 (out of 5)*** *problems from Part* ***C****.*

		Possible
Part A:	Miscellaneous multiple choice	21
Part B:	Sensitivity analysis (LINDO)	11
Part C:	1. Discrete-time Markov chains I	11
	2. Discrete-time Markov chains II	11
	3. Continuous-time Markov chains	11
	4. Integer Programming Models	11
	5. Stochastic dynamic programming	11
	total possible:	76

▼▲▼▲▼▲▼ PART A ▼▲▼▲▼▲▼

Multiple Choice: Write the appropriate letter (a, b, c, d, etc.) : (*NOTA* =None of the above).

_b__ 1. If $X_{ij} > 0$ in the optimal solution of a transportation problem, then dual variables U_i and V_j *must* satisfy
- a. $C_{ij} > U_i + V_j$
- b. $C_{ij} = U_i + V_j$
- c. $C_{ij} < U_i + V_j$
- d. $C_{ij} + U_i + V_j = 0$
- e. $C_{ij} = U_i - V_j$
- f. *NOTA*

_d__ 2. For a continuous-time Markov chain, let Λ be the matrix of transition probabilities. The sum of each...
- a. column is 1
- b. column is 0
- c. row is 1
- d. row is 0
- e. *NOTA*

_d__ 3. In a birth/death process model of a queue, the time between departures is assumed to
- a. have the Beta dist'n
- b. have the Poisson dist'n
- c. be constant
- d. have the exponential dist'n
- e. have the uniform dist'n
- f. *NOTA*

_c__ 4. In an M/M/1 queue, if the arrival rate = $\lambda < \mu$= service rate, then
- a. $\pi_0 = 1$ in steady state
- b. no steady state exists
- c. $\pi_i > 0$ for all i
- d. $\pi_0 = 0$ in steady state
- e. the queue is not a birth-death process
- f. *NOTA*

_d__ 5. If there is a tie in the "minimum-ratio test" of the revised simplex method, the solution in the next tableau
- a. will be nonbasic
- b. will be infeasible
- c. will have a worse objective value
- d. will be degenerate
- e. will be nonoptimal
- f. *NOTA*

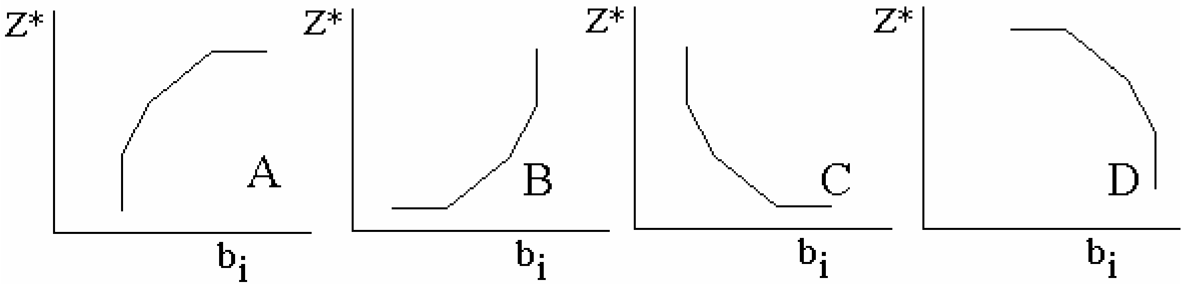
_a__ 6. An *absorbing* state of a Markov chain is one in which the probability of
- a. moving out of that state is zero
- b. moving into that state is one.
- c. moving out of that state is one.
- d. moving into that state is zero
- e. *NOTA*

_e__ 7. The number of basic variables in a solution of a transportation problem with m sources and n dest'ns is
- a. m×n
- b. m×n−1
- c. m+n+1
- d. n–m
- e. m+n–1
- f. m+n
- g. *NOTA*

_a__ 8. A balanced transportation problem is one in which
- a. sum of supplies = sum of demand
- b. cost coefficients are all 1
- c. supplies & demands all 1
- d. # sources = # destinations
- e. *NOTA*

_e_ 9. A transportation problem is a special case of assignment problem for which
- a. sum of supplies = sum of demand
- b. cost coefficients are all 1
- c. supplies & demands all 1
- d. # sources = # destinations
- e. *NOTA*

Match the four hypothetical graphs of optimal value vs right-hand-side to the appropriate combination of min/max and inequality type, by writing the correct letter (A,B,C,D) in the blanks.



__B_ 10. Min cx st $Ax \geq b$

__C_ 11. Min cx st $Ax \leq b$

_D_ 12. Max cx st $Ax \geq b$

_A_ 13. Max cx st $Ax \leq b$

_a__ 14. If, in the optimal *primal* solution of an LP problem (max cx st $Ax \geq b$, $x \geq 0$), there is *positive* slack in constraint #1, then in the optimal dual solution, where y_1 is the first dual variable,

a. $y_1 = 0$ c. slack variable for dual constraint #1 must be zero e. $y_1 < 0$
b. $y_1 > 0$ d. dual constraint #1 must be slack f. *NOTA*

_c__ 15. If, in the optimal solution of the *dual* of the LP problem: min cx subject to: $Ax \geq b$, $x \geq 0$, dual variable y_2 is nonzero, then in the optimal *primal* solution,

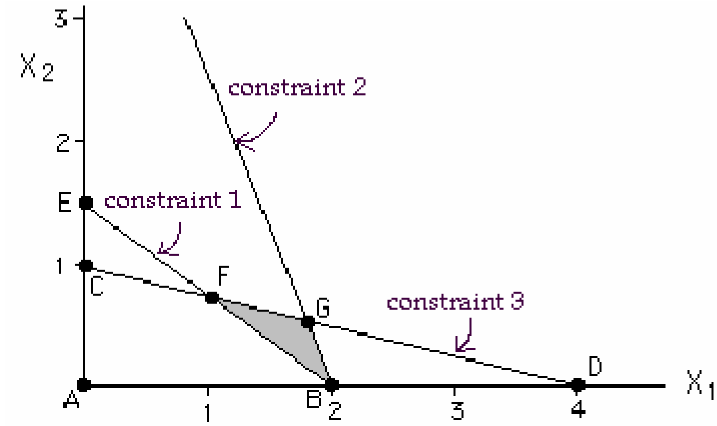
a. variable x_2 *must* be zero c. slack variable for constraint #2 must be zero
b. variable x_2 *must* be positive d. slack variable for constraint #2 must be positive e. *NOTA*

_b__ 16. Bayes' Rule is used to compute

a. the joint probability of a "state of nature" and the outcome of an experiment
b. the conditional probability of a "state of nature" given the outcome of an experiment
c. the conditional probability of the outcome of an experiment given a "state of nature"
d. NOTA

The problems below refer to the following LP:

	(with inequalities converted to equations:)
Minimize $8X_1 + 4X_2$	Minimize $8X_1 + 4X_2$
subject to $3X_1 + 4X_2 \geq 6$	subject to $3X_1 + 4X_2 - X_3 = 6$
$5X_1 + 2X_2 \leq 10$	$5X_1 + 2X_2 + X_4 = 10$
$X_1 + 4X_2 \leq 4$	$X_1 + 4X_2 + X_5 = 4$
$X_1 \geq 0, X_2 \geq 0$	$X_j \geq 0$, j=1,2, 3,4,5



_a__ 17.The feasible region includes points
- a. B, F, & G
- b. A, B, C, & F
- c. C, E, & F
- d. E, F, &G
- e. B, D, & G
- f. *NOTA*

_c__ 18. At point G, the nonbasic variables include
- a. X_2 & X_3
- b. X_3 & X_4
- c. X_4 & X_5
- d. X_1 & X_4
- e. X_3 & X_5
- f. *NOTA*

_c__ 19. The dual of the original LP (before introducing slack & surplus variables) has the following constraints (not including nonnegativity or nonpositivity constraints):
- a. 2 constraints of type (≥)
- b. one each of type ≤ & ≥
- c. 2 constraints of type (≤)
- d. one each of type ≥ & =
- e. *NOTA*

_b__ 20. The dual of the LP has the following types of variables:
- a. two non-negative variables and one non-positive variable
- b. one non-negative and two non-positive variables
- c. two non-negative variables and one unrestricted in sign
- d. three non-negative variables
- e. three non-positive variables
- f. NOTA

_e__ 21. If point F is optimal, then which dual variables must be zero, according to the *Complementary Slackness Theorem* ?
- a. Y_1 and Y_2
- b. Y_1 and Y_3
- c. Y_2 and Y_3
- d. Y_1 only
- e. Y_2 only
- f. Y_3 only

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis in LP.

Problem Statement: The Classic Stone Cutter Company produces four types of stone sculptures: figures, figurines, free forms, and statues. Each product requires the following hours of work for cutting and chiseling stone and polishing the final product:

Operation	FIGURES	FIGURINES	FREE FORMS	STATUES
Cutting	30	5	45	60
Chiseling	20	8	60	30
Polishing	0	20	0	120
Profit ($/unit)	280	40	500	510

The company's current work force has production capacity sufficient to allocate 300 hours to cutting, 180 hours to chiseling, and 300 hours to polishing each week.

Define the variables :FIGURES = # of figures to be produced each week,
FIGURINES = # figurines to be produced each week,
etc.

The LINDO output for solving this problem follows:

```
 MAX     280 FIGURE + 40 FIGURINE + 500 FREEFORM + 510 STATUE
 SUBJECT TO
    2)    30 FIGURE + 5 FIGURINE + 45 FREEFORM + 60 STATUE <=   300
    3)    20 FIGURE + 8 FIGURINE + 60 FREEFORM + 30 STATUE <=   180
    4)    20 FIGURINE + 120 STATUE                         <=   300
 END
         OBJECTIVE FUNCTION VALUE
  1)        2700.00000

     VARIABLE        VALUE          REDUCED COST
     FIGURE        6.000000           0.000000
     FIGURINE      0.000000          30.000000
     FREEFORM      0.000000          70.000000
     STATUE        2.000000           0.000000

  ROW          SLACK OR SURPLUS          DUAL PRICES
      2)          0.000000               6.000000
      3)          0.000000               5.000000
      4)         60.000000               0.000000

     RANGES IN WHICH THE BASIS IS UNCHANGED

                              OBJ COEFFICIENT RANGES
     VARIABLE          CURRENT          ALLOWABLE          ALLOWABLE
                          COEF           INCREASE           DECREASE
     FIGURE          280.000000         60.000000         9.333333
     FIGURINE         40.000000         30.000000             INFINITY
     FREEFORM        500.000000         70.000000             INFINITY
     STATUE          510.000000         23.333336        89.999992

                              RIGHTHAND SIDE RANGES
  ROW                CURRENT        ALLOWABLE             ALLOWABLE
                       RHS           INCREASE              DECREASE
      2              300.000000       7.500000              30.000000
      3              180.000000      20.000000               5.000000
      4              300.000000          INFINITY           60.000000

   THE TABLEAU
  ROW  (BASIS) FIGURE   FIGURINE  FREEFORM  STATUE    SLK 2    SLK 3   SLK 4     RHS
  1    ART      0.00     30.000    70.000   0.00     6.000    5.000   0.00   2700.000
  2    FIGURE   1.00      1.100     7.500   0.00    -0.100    0.200   0.00      6.000
  3    STATUE   0.00     -0.467    -3.000   1.00     0.067   -0.100   0.00      2.000
  4    SLK   4  0.00     76.000   360.000   0.00    -8.000   12.00    1.00     60.000
```

Ignoring the restriction that the numbers of items produced per week must be integer, answer the following questions:

1. The optimal solution above is *(check as many as apply):*
   _X_ basic   __X_ feasible   ___ degenerate   _X_ unique

_c__ 2. The number of basic variables in this optimal solution (not including z, the objective value) is
a. one   b. two   c. three
d. four   e. five   f. *NOTA*

_a__ 3. In *any* basic feasible solution of this problem: *Since # products > # basic variables*
a. not every product will be included   b. exactly two products will be included
c. at least one slack variable will be >0   d. *NOTA*

_d__ 4. If it were required to make one *freeform* as a salesman's sample, the profit will decrease by *(choose the nearest value)*
a. zero   b. $25   c. $50   d. $75   e. $100
f. $125   g. $150   h. cannot be determined   i. *NOTA*

_e__ 5. If it were required to make one *freeform* as a salesman's sample, the production of statues would
a. be unchanged   b. increase by less than 1   c. decrease by less than 1
d. decrease by more than 1   e. increase by more than 1   f. cannot be determined   g. *NOTA*

_J__ 6. If it were required to make one additional statue, the profit will decrease by *(choose the nearest value)*
a. zero   b. $10   c. $20   d.$50   e. $100
f. $150   g. $200   h. $300   i. $500   j. cannot be determined

_c__ 7. If the profit of free forms were to be $600 per unit,
a. the profit would be unchanged   b. the profit would increase by $100
c. the production of free forms should increase   d. *NOTA*

_f__ 8. If ten additional hours of *chiseling* were available, the profit would increase by *(choose the nearest value)*
a. ≤ $10   b. $10   c. $20   d. $30
e. $40   f. ≥ $50   g. cannot be determined

_d__ 9. If ten additional hours of *chiseling* were available, the number of *figures* would
a. be unchanged   b. increase by 1   c. decrease by 1   d. increase by 2
e. decrease by 2   f. increase by > 2   g. decrease by >2   h. *NOTA*

_c__ 10. The number of variables in the dual of this LP problem (not including variable z for objective row) is
a. one   b. two   c. three
d. four   e. five   f. *NOTA*

_a__ 11. The sign restrictions on the dual variables are
a. all nonnegative   b. all nonpositive   c. some nonpositive, some nonnegative
d. no sign restrictions   e. *NOTA*

_b__ 12. The value of the second variable in the optimal dual solution
a. is zero   b. is positive   c. is negative
d. cannot be determined   e. *NOTA*

_b__ 11. The value of the optimal objective value of the dual problem is
a. zero   b. 2700   c. −2700
d. cannot be determined   e. *NOTA*

FYI:

Maximize	Minimize
Type of constraint i: ≤ = ≥	Sign of variable i: nonnegative unrestricted in sign nonpositive
Sign of variable j: nonnegative unrestricted in sign nonpositive	Type of constraint i: ≥ = ≤

▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

1. **Discrete-Time Markov Chains I:** A XYZ is a telemarketing firm which purchases lists of potential customers, and models its contact with customers as a Discrete-time Markov chain with 6 states:
   1. New customer with no history
   2. During most recent call, customer's expressed interest was low
   3. During most recent call, customer's expressed interest was medium
   4. During most recent call, customer's expressed interest was high
   5. Sale was completed during most recent call.
   6. Sale was lost during most recent call (customer asked not to be contacted again!)

$$P = \begin{bmatrix} 0 & 0.25 & 0.2 & 0.15 & 0.1 & 0.3 \\ 0 & 0.2 & 0.2 & 0.1 & 0.05 & 0.45 \\ 0 & 0.15 & 0.25 & 0.35 & 0.15 & 0.1 \\ 0 & 0.15 & 0.3 & 0.3 & 0.2 & 0.05 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Based on a history of past phone calls, the transition matrix to the left has been estimated, and the A and E matrices below were computed.

Each call made by the sales representative costs XYZ an average of $1, and XYZ receives $10 for each sale completed.

_e_ 1. The number of *transient* states in this Markov chain model is
a. 0 b. 1 c. 2 d. 3 e. 4 f. 5 g. 6 h. *NOTA*

_c_ 2. The number of *absorbing* states in this Markov chain model is
a. 0 b. 1 c. 2 d. 3 e. 4 f. 5 g. 6 h. *NOTA*

_c_ 3. The number of *recurrent* states in this Markov chain model is
a. 0 b. 1 c. 2 d. 3 e. 4 f. 5 g. 6 h. *NOTA*

4. The closed sets of states in this Markov chain model are *(circle all that apply!)*
a. {1 } b. {4 } c. {1,2,3,4 } d. {2,3,4 } e. {2 } [f. {5 }]
g. {1,2,3,4 } h. {3,4 } i. {3 } [j. {6 }] [k. {5,6}] [l. {1,2,3,4,5,6}]

5. The *minimal* closed sets of states in this Markov chain model are *(circle all that apply!)*
a. {1 } b. {4 } c. {1,2,3,4 } d. {2,3,4 } e. {2 } [f. {5 }]
g. {1,2,3,4 } h. {3,4 } i. {3 } [j. {6 }] k. {5,6} l. {1,2,3,4,5,6}

_e_ 6. How many calls are made (on average) to each potential customer? *(choose nearest answer)*
a. 1 c. 2 e. 3 *(2.845)* g. 4 i. 5
b. 1.5 d. 2.5 f. 3.5 h. 4.5 j. >5

_f_ 7. What percentage of potential customers will eventually make a purchase? *(choose nearest answer)*
a. 10% b. 15% c. 20% d. 25% e. 30% f. 35% *(35.27%)*
g. 40% h. 45% i. 50% j. 55% k. 60% l. ≥65%

_J_ 8. If in the most recent call, the customer expressed high interest in the product, what is the probability that he/she will eventually make a purchase? *(choose nearest answer)*
a. 10% b. 15% c. 20% d. 25% e. 30% f. 35%
g. 40% h. 45% i. 50% j. 55% *(56.23%)* k. 60% l. ≥65%

_d_ 9. Determining the E matrix requires
a. computing a determinant b. computing eigenvectors c. computing product of 2 matrices
d. inverting a matrix e. summing four matrices f. *NOTA*

_g_10. What is the value of each name on the list of potential customers? *(That is, what is the most that XYZ should be willing to pay per name?) Note: Expected profit* = $0.3527 \times \$10 - 2.845 \times \$1 = \$0.682$
a. $0.10 c. $0.30 e. $0.50 g. $0.70 i.$0.90 k. $1.10
b. $0.20 d. $0.40 f. $0.60 h. $0.80 j. $1.00 l. ≥ $1.20

A	5	6
1	0.3527	0.6473
2	0.2615	0.7385
3	0.5147	0.4853
4	0.5623	0.4377

E	1	2	3	4	Row Sum
1	1	0.5545	0.6649	0.626	2.845
2	0	1.455	0.5887	0.5022	2.545
3	0	0.5455	1.887	1.022	3.455
4	0	0.5455	0.9351	1.974	3.455

2. Discrete-time Markov Chains II: Consider an **(s,S) inventory system** in which the number of items on the shelf is checked at the end of each day. To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 6) **if** there are **s** or fewer parts on the shelf. *(That is, it is an (s,S) inventory system, with S=6.)* The demand is random.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions:

_d__ 1. The value of **s**, the *reorder point,* is
a. zero b. one c. two d. three e. four f. five g. six h. *NOTA*

_g__ 2. the value $P_{3,3}$ is
a. P{demand=0} c. P{demand=1} e. P{demand=2} g. P{demand=3}
b. P{demand≤1} d. P{demand≥1} f. P{demand≥2} h. NOTA

_g__ 3. the value $P_{0,3}$ is
a. P{demand=0} c. P{demand=1} e. P{demand=2} g. P{demand=3}
b. P{demand≤1} d. P{demand≥1} f. P{demand≥2} h. NOTA

_a_ 4. the value $P_{6,6}$ is
a. P{demand=0} c. P{demand=1} e. P{demand=2} g. P{demand=3}
b. P{demand≤1} d. P{demand≥1} f. P{demand≥2} h. NOTA

_h__ 5. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value): Note:* $m_{6,0} = 18.79$
a. ≤4 b. 6 c. 8 d. 10 e. 12
f. 14 g. 16 h. 18 i. 20 j. more than 22

_d__ 6. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value): Note:* $p_{6,6}^{(4)} = 0.07788$
a. 5% b. 6% c. 7% d. 8% e. 9%
f. 10% g. 11% h. 12% i. 13% j. ≥14%

_J__ 7. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is *(select nearest value): Note:* 0.05345+0.08064+0.1501+0.2188=0.50299
a. 5% b. 10% c. 15% d. 20% e. 25%
f. 30% g. 35% h. 40% i. 45% j. ≥50%

_e__ 8. What is the probability that the *first* stockout occurs Thursday night, if the shelf is full Monday morning,? *(select nearest value): Note:* $f_{6,0}^{(4)} = 0.04812$
a. 1% b. 2% c. 3% d. 4% e. 5%
f. 6% g. 7% h. 8% i. 9% j. ≥10%

Markov Chain model of (s,S) inventory system

Transition Probability Matrix

	0	1	2	3	4	5	6
0	0.01656	0.03609	0.09022	0.1804	0.2707	0.2707	0.1353
1	0.01656	0.03609	0.09022	0.1804	0.2707	0.2707	0.1353
2	0.01656	0.03609	0.09022	0.1804	0.2707	0.2707	0.1353
3	0.01656	0.03609	0.09022	0.1804	0.2707	0.2707	0.1353
4	0.1429	0.1804	0.2707	0.2707	0.1353	0	0
5	0.05265	0.09022	0.1804	0.2707	0.2707	0.1353	0
6	0.01656	0.03609	0.09022	0.1804	0.2707	0.2707	0.1353

Steady State Distribution

i	state	P{i}
0	SOH=zero	0.05323
1	SOH=one	0.08033
2	SOH=two	0.1496
3	SOH=three	0.2183
4	SOH=four	0.2384
5	SOH=five	0.1816
6	SOH=six	0.0785

First Visit Probabilities: Stage 4

	0	1	2	3	4	5	6
0	0.04812	0.06725	0.1008	0.1095	0.105	0.0928	0.05493
1	0.04812	0.06725	0.1008	0.1095	0.105	0.0928	0.05493
2	0.04812	0.06725	0.1008	0.1095	0.105	0.0928	0.05493
3	0.04812	0.06725	0.1008	0.1095	0.105	0.0928	0.05493
4	0.0425	0.05846	0.08435	0.1011	0.1245	0.1228	0.0657
5	0.04651	0.06382	0.09163	0.09886	0.105	0.1091	0.06395
6	0.04812	0.06725	0.1008	0.1095	0.105	0.0928	0.05493

First Visit Probabilities: Stage 5

	0	1	2	3	4	5	6
0	0.04536	0.06151	0.08476	0.08461	0.07658	0.07581	0.05285
1	0.04536	0.06151	0.08476	0.08461	0.07658	0.07581	0.05285
2	0.04536	0.06151	0.08476	0.08461	0.07658	0.07581	0.05285
3	0.04536	0.06151	0.08476	0.08461	0.07658	0.07581	0.05285
4	0.04048	0.05392	0.07127	0.07875	0.09079	0.09686	0.05639
5	0.04384	0.05834	0.07691	0.07617	0.07658	0.08837	0.05907
6	0.04536	0.06151	0.08476	0.08461	0.07658	0.07581	0.05285

4-th Power of P

	0	1	2	3	4	5	6
0	0.05345	0.08064	0.1501	0.2188	0.2383	0.1809	0.07788
1	0.05345	0.08064	0.1501	0.2188	0.2383	0.1809	0.07788
2	0.05345	0.08064	0.1501	0.2188	0.2383	0.1809	0.07788
3	0.05345	0.08064	0.1501	0.2188	0.2383	0.1809	0.07788
4	0.0526	0.07947	0.1483	0.2172	0.2387	0.1836	0.08023
5	0.05336	0.0805	0.1499	0.2185	0.2383	0.1812	0.07821
6	0.05345	0.08064	0.1501	0.2188	0.2383	0.1809	0.07788

5-th Power of P

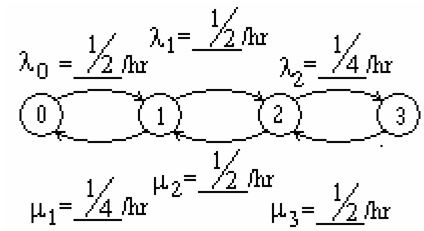
	0	1	2	3	4	5	6
0	0.05319	0.08028	0.1495	0.2183	0.2384	0.1817	0.0786
1	0.05319	0.08028	0.1495	0.2183	0.2384	0.1817	0.0786
2	0.05319	0.08028	0.1495	0.2183	0.2384	0.1817	0.0786
3	0.05319	0.08028	0.1495	0.2183	0.2384	0.1817	0.0786
4	0.05333	0.08048	0.1498	0.2185	0.2384	0.1812	0.07819
5	0.05321	0.0803	0.1496	0.2183	0.2384	0.1816	0.07856
6	0.05319	0.08028	0.1495	0.2183	0.2384	0.1817	0.0786

Mean First Passage Time Matrix

	0	1	2	3	4	5	6
0	18.79	12.45	6.683	4.58	3.695	4.851	12.74
1	18.79	12.45	6.683	4.58	3.695	4.851	12.74
2	18.79	12.45	6.683	4.58	3.695	4.851	12.74
3	18.79	12.45	6.683	4.58	3.695	4.851	12.74
4	16.84	11.01	5.748	4.303	4.195	6.008	13.9
5	18.19	11.85	6.152	4.216	3.695	5.508	14.26
6	18.79	12.45	6.683	4.58	3.695	4.851	12.74

3. Birth/Death Model of a Queue: Two mechanics work in an auto repair shop, with a **maximum capacity** of **3** cars, so that any cars arriving when there are already 3 in the shop are turned away. Each mechanic works individually, completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution). (If there is only one car in the shop, only one mechanic works on it, while the other takes a break.) Cars arrive randomly, according to a Poisson process, at the rate of one every **two** hours when there are no waiting cars in the shop, but one every **four** hours when both mechanics are busy. (If 3 cars are in the shop, of course, no cars will enter the shop.)

1. Complete the transition rates (in #/hr) for this system.



__d_ 2. What is the name of the distribution of the time between arrivals when the shop is empty?

a. Markov b. Poisson c. Uniform d. Exponential
e. Normal f. Weibull g. None of the above

3. Perform the computation to determine the steady-state distribution:

$$\frac{1}{\pi_0} = 1 + \frac{1/2}{1/4} + \frac{1/2}{1/4} \times \frac{1/2}{1/2} + \frac{1/2}{1/4} \times \frac{1/2}{1/2} \times \frac{1/4}{1/2} = 1 + 2 + 2 + 1 = 6$$

State i	0	1	2	3
π_i =	$1 \times \pi_0$ = 1/6	__2_ $\times \pi_0$ = 1/3	__2__ $\times \pi_0$ = 1/3	__1__ $\times \pi_0$ = 1/6

_b__ 4. The steady-state probability that the shop is empty is *(choose nearest value):*

a. 10% b. 20% c. 30% d. 40%
e. 50% f. 60% g. 70% h. >80%

_c__ 5. In steady state, the fraction of the day that *exactly one* car will be in the shop is *(choose nearest value):*

a. 10% b. 20% c. 30% d. 40%
e. 50% f. 60% g. 70% h. >80%

_c__ 6. In steady state, the average number of cars in the shop is *(choose nearest value):*

a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3

The average arrival rate in steady state is approximately one every 3 hours, i.e., 0.3333/hour.

_b__ 7. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*

a. 4 hours b. 4.5 hours c. 5 hours d. 5.5 hours
e. 6 hours f. 6.5 hours g. 7 hours h. >7.5 hours

8. The Markov chain model diagrammed above is *(select all that apply):*

a. a discrete-time Markov chain b. a Birth-Death process c. a Poisson process
d. a continuous-time Markov chain e. an M/M/2 queue f. an M/M/2/3/3 queue
g. an M/M/3 queue h. an M/M/2/3 queue

4. Integer Programming Model Formulation Part I. You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (#1 and #2). The length and type of each song are given in the table below:

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Ballad & Hit	2
5	Ballad	4
6	Hit	3
7	neither ballad nor hit	5
8	Ballad & hit	4

Define the variables

Y_i = 1 if song #i is on side 1;
0 otherwise (i.e., if on side 2)

For each restriction, choose a linear constraint from the list (a) through (i) below.

_k_ 1. Side #2 must have at least 2 ballads
_d_ 2. If song #3 is on side 1, then song #5 must be on side 2
_c_ 3. The number of hit songs on side 2 should be no more than 3
_J_ 4. If song 3 is on side 1, then both songs 1 & 2 must be on side 2.

a. $Y_2+Y_4+Y_6+Y_8 \geq 3$	b. $Y_2+Y_4+Y_6+Y_8 \leq 3$	c. $Y_2+Y_4+Y_6+Y_8 \geq 1$	d. $Y_3+Y_5 \leq 1$
e. $Y_1+Y_2-2Y_3 \leq 0$	f. $Y_1+Y_2-Y_3 \leq 2$	g. $Y_1+Y_3+Y_4+Y_5+Y_8 \leq 2$	h. $Y_3+Y_5 \geq 1$
i. $Y_1+Y_2-2Y_3 \geq 0$	j. $Y_1+Y_2+2Y_3 \leq 2$	k. $Y_1+Y_3+Y_4+Y_5+Y_8 \leq 3$	l. $Y_3 \leq Y_5$
m. $Y_3 \geq Y_5$	n. $Y_1+Y_2+Y_3 \leq 2$	o. $Y_1+Y_3+Y_4+Y_5+Y_8 \geq 2$	p. *NOTA*

Part II. Comquat owns four production plants at which personal computers are produced. In order to use a plant to produce computers, a fixed cost must be paid to set up the production line in that plant.

Define the variables for an *integer LP model*:

Y_i = 1 if the production line has been set up at plant #i
0 otherwise
X_i = # of computers produced at plant #i

For each restriction, choose a constraint from the list (a) through (k) below.

_b_ 5. Computers are to be produced at no more than 3 plants.
_r_ 6. If the production line at plant 1 is set up, then that plant can produce up to 5000 computers; otherwise, none can be produced at that plant.
_d_ 7. The production lines at plants 1 and 2 cannot both be set up.
_g_ 8. The total production must be at least 20,000 computers.
_f_ 9. If the production line at plant 2 is set up, that plant must produce at least 5000 computers.
_p_ 10. If the production line at plant 1 is not set up, then the production line at plant 2 cannot be set up.

Constraints:

a. $Y_2 \leq 5000X_2$	b. $Y_1+Y_2+Y_3+Y_4 \leq 3$	c. $Y_1+Y_2+Y_3+Y_4 \geq 3$	d. $Y_1+Y_2 \leq 1$
e. $Y_2 \leq 5000X_2$	f. $X_2 \geq 5000Y_2$	g. $X_1+X_2+X_3+X_4 \geq 20000$	h. $Y_1+Y_2 \geq 1$
i. $X_2 \leq 5000Y_2$	j. $Y_2 \geq 5000X_2$	k. $Y_1 Y_2 = 0$	l. $Y_1 \leq Y_2$
m. $Y_1 \leq 5000X_1$	n. $X_1+X_2+X_3+X_4 \leq 2$	o. $5000 X_1Y_1 \geq 1$	p. $Y_1 \geq Y_2$
q. $Y_1 \geq 5000X_1$	r. $X_1 \leq 5000Y_1$	s. $X_1 \geq 5000Y_1$	t. *NOTA*

5. Stochastic Production Planning Production must be planned for the next four weeks.
Other data:
Production cost is \$10 for setup, plus \$5 per unit produced, up to a maximum of 3 units.
Storage cost: \$1 per unit stored (based upon **beginning**-of-day stock), up to a maximum of 5 units in storage
Shortage cost: \$20 per unit short
Salvage value: \$2 per unit in stock remaining in storage Saturday night
Initial inventory: No units are in stock at the beginning of the first week.
Demand D is randomly distributed, with P{D=0} = P{D=2} = 25%, P{D=1}=50%
A dynamic programming model was used to compute the optimal production quantities for each week in order to minimize the expected cost. *Note that the recursion is **backward**, so that stage 1 is the final week, and in stage 4 there are 4 weeks remaining to be planned.*

1. What is the minimum expected total cost of the 4-week schedule, given **no initial inventory**? _48.45_
2. Complete the computation of the missing element in the table for stage 1 below:
   Computation: __0__ *(storage)* + __0__ *(shortage)* + ___15_ *(production)*
   + 0.25× (−2) + 0.5×_0_+0.25× (20+15) *(expected remaining cost)*
   = _23.25_____
3. Complete the computation of the missing element in the table for stage 4 below:
   Computation: __3__ *(storage)* + __0__ *(shortage)* + __15___ *(production)*
   + 0.25× 10.06 + 0.5× 15.59 +0.25× 23.08 *(expected remaining cost)*
   = __34.08____
4. What is the optimal production quantity during the first week? __3__
5. Suppose that during the first week, the demand is 2 units. What should then be the second week's production quanity? __3__
6. If, as in (5) the first week's demand is 2, what is the optimal expected cost of the last 3 weeks of the planning period? __32.06___

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Stage 1---

s \ x:	0	1	2	3	Min
-1	999.99	61.25	48.25	43.00	43.00
0	26.25		18.00	21.00	18.00
1	9.25	14.00	17.00	20.00	9.25
2	0.00	13.00	16.00	19.50	0.00
3	-1.00	12.00	15.50	20.00	-1.00
4	-2.00	11.50	16.00	21.00	-2.00

Stage 2---

s \ x:	0	1	2	3	Min
-1	999.99	71.75	62.06	54.13	54.13
0	36.75	37.06	29.13	27.06	27.06
1	23.06	25.13	23.06	25.00	23.06
2	11.13	19.06	21.00	25.25	11.13
3	5.06	17.00	21.25	26.00	5.06
4	3.00	17.25	22.00	27.00	3.00

Stage 3---

s \ x:	0	1	2	3	Min
-1	999.99	82.36	72.83	66.08	66.08
0	47.36	47.83	41.08	37.59	37.59
1	33.83	37.08	33.59	32.06	32.06
2	23.08	29.59	28.06	30.52	23.08
3	15.59	24.06	26.52	31.00	15.59
4	10.06	22.52	27.00	32.00	10.06

Stage 4---

s \ x:	0	1	2	3	Min
-1	999.99	93.96	83.33	76.20	76.20
0	58.96	58.33	51.20	48.45	48.45
1	44.33	47.20	44.45	42.08	42.08
2	33.20	40.45	38.08	38.45	33.20
3	26.45		34.45	38.06	26.45
4	20.08	30.45	34.06	39.06	20.08
