Name or Initials ____________________

▼▲▼▲▼▲▼ 56:171 Operations Research ▼▲▼▲▼▲▼
▲▼▲▼▲▼▲ Final Examination Solutions ▲▼▲▼▲▼
▼▲▼▲▼▲▼ Fall 2000 ▼▲▼▲▼▲▼

• *Write your name on the first page, and initial the other pages.*
• *Answer both Parts A and B, and* **4 (out of 5)** *problems from Part C.*

		Possible
Part A:	Miscellaneous multiple choice	17
Part B:	Sensitivity analysis (LINDO)	16
Part C:	1. Discrete-time Markov chains I	15
	2. Discrete-time Markov chains II	15
	3. Continuous-time Markov chains	15
	4. Integer Programming Models	15
	5. Dynamic programming	15
	total possible:	90

▼▲▼▲▼▲▼ PART A ▼▲▼▲▼▲▼

Multiple Choice: Write the appropriate letter (a, b, c, d, etc.) : (*NOTA* = None of the above).

_a_ 1. If, in the optimal *primal* solution of an LP problem (min cx st $Ax \geq b$, $x \geq 0$), there is positive slack in constraint #1, then in the optimal dual solution,
- a. dual variable #1 must be zero
- b. dual variable #1 must be positive
- c. slack variable for dual constraint #1 must be zero
- d. dual constraint #1 must be slack
- e. *NOTA*

_c_ 2. If, in the optimal solution of the *dual* of an LP problem (min cx subject to: $Ax \geq b$, $x \geq 0$), dual variable #2 is positive, then in the optimal *primal* solution,
- a. variable #2 must be zero
- b. variable #2 must be positive
- c. slack variable for constraint #2 must be zero
- d. slack variable for constraint #2 must be positive
- e. *NOTA*

_b_ 3. If $X_{ij} > 0$ in the transportation problem, then dual variables U and V *must* satisfy
- a. $C_{ij} > U_i + V_j$
- b. $C_{ij} = U_i + V_j$
- c. $C_{ij} < U_i + V_j$
- d. $C_{ij} + U_i + V_j = 0$
- e. $C_{ij} = U_i - V_j$
- f. *NOTA*

_c_ 4. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
- a. column is 1
- b. column is 0
- c. row is 1
- d. row is 0
- e. *NOTA*

_d_ 5. In a birth/death process model of a queue, the time between arrivals is assumed to
- a. have the Beta distribution
- b. have the Poisson distribution
- c. be constant
- d. have the exponential distribution
- e. *NOTA*

_b_ 6. In an M/M/1 queue, if the arrival rate $= \lambda > \mu =$ service rate, then
- a. $\pi_0 = 1$ in steady state
- b. no steady state exists
- c. $\pi_i > 0$ for all i
- d. $\pi_0 = 0$ in steady state
- e. the queue is not a birth-death process
- f. *NOTA*

_d_ 7. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
- a. will be nonbasic
- a. will be nonfeasible
- c. will have a worse objective value
- d. will be degenerate
- e. *NOTA*

_a_ 8. An *absorbing* state of a Markov chain is one in which the probability of
- a. moving out of that state is zero
- b. moving into that state is one.
- c. moving out of that state is one.
- d. moving into that state is zero
- e. *NOTA*

The problems (9)-(12) below refer to the following LP:

$$\begin{aligned}\text{Maximize } & 3X_1 + 5X_2 \\ \text{subject to } & 2X_1 - X_2 \leq 5 \\ & X_1 - 2X_2 \geq -4 \\ & X_1 + 3X_2 \leq 15 \\ & X_1 \geq 0,\ X_2 \geq 0\end{aligned}$$

(with inequalities converted to equations:)

$$\begin{aligned}\text{Maximize } & 3X_1 + 5X_2 \\ \text{subject to } & 2X_1 - X_2 + X_3 = 5 \\ & X_1 - 2X_2 - X_5 = -4 \\ & X_1 + 3X_2 + X_4 = 15 \\ & X_j \geq 0,\ j=1,2,3,4,5\end{aligned}$$

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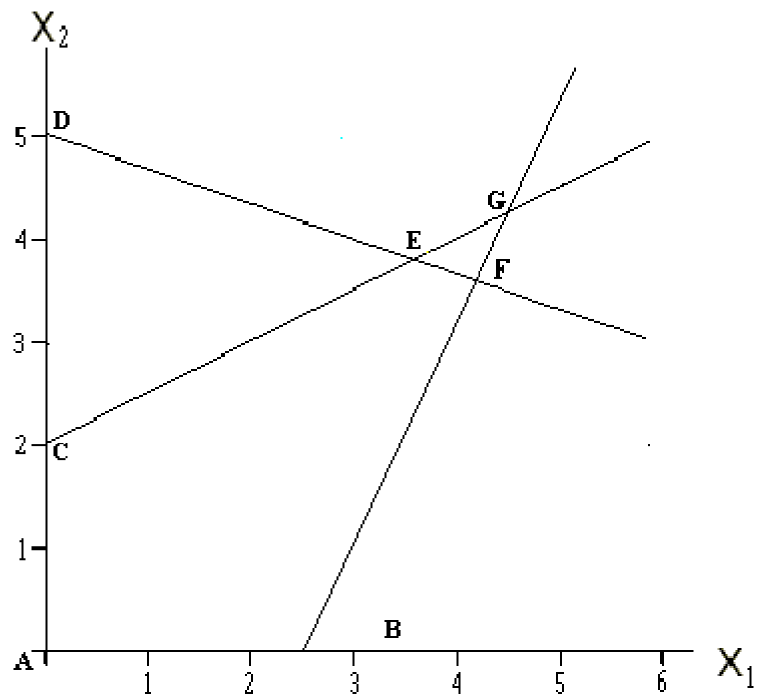


_d__ 9 .The feasible region includes (possibly with others) points:

a. E, F, & G c. D, C, & E e. D, E, & G

b. G&F d. B & F f. *NOTA*

_e__ 10. At point **F**, the basic variables include the variables

a. X_1 , X_2 & X_3 c. X_2, X_4 & X_5 e. X_1 , X_2 & X_5

b. X_1 , X_3 & X_4 d. X_1, X_2 & X_4 f. *NOTA*

__f_ 11. Which point is degenerate in the primal problem?

a. point A c. point C e. point E

b. point B d. point D f. *NOTA*

_a__ 12. The *dual* of this LP has the following constraints (not including nonnegativity or nonpositivity):

a. 2 constraints of type (≥) c. one each of type ≤ & ≥ e. one each of type ≥ & =

b. 2 constraints of type (≤) d. 2 of type ≤ and 1 of type ≥ f. *None of the above*

_a__ 13. The dual of the LP has the following types of variables:

a. two non-negative variables and one non-positive variable d. three non-positive variables

b. one non-negative and two non-positive variables e. three nonnegative variables

c. two non-negative variables and one unrestricted in sign f. *None of the above*

_e__ 14. If point F is optimal, then which dual variables must be zero, according to the *Complementary Slackness Theorem* ?

a. Y_1 and Y_2 c. Y_2 and Y_3 e. Y_2 only

b. Y_1 and Y_3 d. Y_1 only f. Y_3 only

15. The number of basic variables in a solution of a transportation problem with 5 sources and 7 destinations is __11_

_d__ 16. A balanced transportation problem is one in which

a. # sources = # destinations c. supplies & demands all 1 e. *NOTA*

b. cost coefficients are all 1 d. sum of supplies = sum of demand

_c__ 17. An assignment problem is a transportation problem for which

a. # sources = # destinations c. supplies & demands all 1 e. *NOTA*

b. cost coefficients are all 1 d. sum of supplies = sum of demand

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis in LP.

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:

X1 = number of **STANDARD** golf bags manufactured per quarter

X2 = number of **DELUXE** golf bags manufactured per quarter

Four operations are required, with the time per golf bag shown in the following table. *Note that the values are somewhat different from the original version of the problem.*

	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1 hr	550 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$15	

LINDO provides the following output:

```
  MAX      10 X1 + 15 X2
  SUBJECT TO
         2)   0.7 X1 + X2         <=  550
         3)   0.5 X1 + 0.8666 X2 <=  600
         4)   X1 + 0.6666 X2     <=  708
         5)   0.1 X1 + 0.25 X2   <=  135
  END

OBJECTIVE FUNCTION VALUE
        1)      8233.333

  VARIABLE         VALUE          REDUCED COST
        X1        33.333332           0.000000
        X2       526.666687           0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         13.333333
        3)       126.924011          0.000000
        4)       323.590668          0.000000
        5)         0.000000          6.666667

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1       10.000000         0.500000         4.000000
       X2       15.000000         9.999999         0.714286

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS           INCREASE         DECREASE
        2      550.000000       132.373184        10.000000
        3      600.000000         INFINITY       126.924011
        4      708.000000         INFINITY       323.590668
        5      135.000000         2.500000        45.500957
```

THE TABLEAU

ROW	(BASIS)	X1	X2	SLK 2	SLK 3	SLK 4	SLK 5	RHS
1	ART	0.00	0.00	13.333	0.00	0.00	6.667	8233.333
2	X2	0.00	1.00	-1.333	0.00	0.00	9.333	526.667
3	SLK 3	0.00	0.00	-0.511	1.00	0.00	-1.422	126.924
4	SLK 4	0.00	0.00	-2.445	0.00	1.00	7.112	323.591
5	X1	1.00	0.00	3.333	0.00	0.00	-13.333	33.333

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If the profit on STANDARD bags were to increase from $10 each to $12 each, the number of STANDARD bags to be produced would

|X| increase |__| decrease |__| remain the same |__| not sufficient info.

b. If the profit on DELUXE bags were to decrease from $15 each to $14 each, the number of DELUXE bags to be produced would

|__| increase |X| decrease |__| remain the same |__| not sufficient info.

c. The LP problem above has

|X| exactly one optimal solution |__| exactly two optimal solutions

|__| an infinite number of optimal solutions

d. If an additional 10 hours were available in the cut-&-dye department, PAR would be able to obtain an additional $_133.33_ in profits.

e. If an additional 10 hours were available in the inspect-&-pack department, PAR would be able to obtain an additional $_NSI__ in profits. *Note: 10 exceeds "ALLOWABLE INCREASE"-- increase in profit ³ 66.67.*

f. If the variable "SLK 5" were increased, this would be equivalent to

___ increasing the hours used in the inspect-&-pack department

_X_ decreasing the hours used in the inspect-&-pack department

___ none of the above

g. If the variable "SLK 5" were increased by 10, X1 would |X| increase |__| decrease by _133.3 STANDARD golf bags/quarter.

h. If the variable "SLK 5" were increased by 10, X2 would |__| increase |X| decrease by __93.3__ DELUXE golf bags/quarter.

i. If a pivot were to be performed to enter the variable **SLK5** into the basis, then according to the "minimum ratio test", the value of **SLK5** in the resulting basic solution would be approximately (choose nearest value)

__ 5 __15 __25 __35 X 45 *(45.5)* __55 __65

__ 10 __20 __30 __40 __50 __60 __70

__ not sufficient information

j. If the variable **SLK5** were to enter the basis, then the variable __SLK 4____ will leave the basis.

FYI:

Maximize	Minimize
Type of constraint i: ≤ = ≥	Sign of variable i: nonnegative unrestricted in sign nonpositive
Sign of variable j: nonnegative unrestricted in sign nonpositive	Type of constraint i: ≥ = ≤

▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

1. **Discrete-Time Markov Chains I:** The Minnesota State University admissions office has modeled the path of a student through the university as a Markov Chain:

	Freshman	Sophomore	Junior	Senior	Quits	Graduates
Freshman	0.10	0.80	0	0	0.10	0
Sophomore	0	0.10	0.85	0	0.05	0
Junior	0	0	0.12	0.80	0.08	0
Senior	0	0	0	0.10	0.05	0.85
Quits	0	0	0	0	1.00	0
Graduates	0	0	0	0	0	1.00

Each student's state is observed at the *beginning of each fall semester*. For example, if a student who is a junior at the beginning of the current fall semester has an 80% chance of becoming a senior at the beginning of the next fall semester, a 15% chance of remaining a junior, and a 5% chance of quitting. (We will assume that a student who quits never re-enrolls.)

Powers of P:

P^2\	1	2	3	4	5	6
1	0.01	0.16	0.68	0	0.15	0
2	0	0.01	0.187	0.68	0.123	0
3	0	0	0.0144	0.176	0.1296	0.68
4	0	0	0	0.01	0.055	0.935
5	0	0	0	0	1	0
6	0	0	0	0	0	1

P^3\	1	2	3	4	5	6
1	0.001	0.024	0.2176	0.544	0.213	0
2	0	0.001	0.03094	0.2176	0.172	0.578
3	0	0	0.00173	0.02912	0.139	0.829
4	0	0	0	0.001	0.055	0.943
5	0	0	0	0	1	0
6	0	0	0	0	0	1

P^4\	1	2	3	4	5	6
1	0.0001	0.0032	0.0465	0.228	0.259	0.462
2	0	0.0001	0.0045	0.046	0.185	0.762
3	0	0	0.0002	0.004	0.141	0.854
4	0	0	0	0.0001	0.055	0.944
5	0	0	0	0	1	0
6	0	0	0	0	0	1

Absorption Probabilities A:

\	5	6
1	0.2792	0.7207
2	0.1891	0.8108
3	0.1414	0.8585
4	0.05555	0.9444

Expected # of Visits E=

\	1	2	3	4
1	1.111	0.987	0.953	0.847
2	0	1.111	1.073	0.953
3	0	0	1.136	1.010
4	0	0	0	1.111

Select the **nearest** available numerical choice in answering the questions below.

_e_ 1. The number of *transient* states in this Markov chain model is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

_c_ 2. The number of *absorbing* states in this Markov chain model is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

_c_ 3. The number of *recurrent* states in this Markov chain model is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

4. The closed sets of states in this Markov chain model are (circle all that apply!)

a. {1 } d. {4 } g. {1,2,3,4 } j. {2,3,4 }
b. {2 } **[e. {5 }]** h. {1,2,3,4 } k. {3,4 }
c. {3 } **[f. {6 }]** **[i. {5,6}]** **[l. {1,2,3,4,5,6}]**

5. The *minimal* closed sets of states in this Markov chain model are (circle all that apply!)

a. {1 } d. {4 } g. {1,2,3,4 } j. {2,3,4 }
b. {2 } **[e. {5 }]** h. {1,2,3,4 } k. {3,4 }
c. {3 } **[f. {6 }]** i. {5,6} l. {1,2,3,4,5,6}

Suppose that at the beginning of the Fall '00 semester, Joe Cool was a *Freshman.*

_g_ 6. What is the probability that Joe is a junior in Fall 2002? (choose nearest answer)

a. 10% c. 30% e. 50% g. 70% ***(68%)*** i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100% k. *Not sufficient info.*

_e_ 7. What is the probability that Joe is a senior in Fall 2003? (choose nearest answer)

a. 10% c. 30% e. 50% ***(54.4%)*** g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100% k. *Not sufficient info*

_g_ 8. What is the probability that Joe *eventually* graduates? (choose nearest answer)

a. 10% c. 30% e. 50% g. 70% ***(72%)*** i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100% k. *Not sufficient info*

_c_ 9. What is the expected length of his academic career, in years? (choose nearest answer)

a. 3 year c. 4 years e. 4.5 years g. 5 years i. 5.5 years *Not sufficient info*
b. 3.75 years d. 4.25 years f. 4.75 years h. 5.25 years j. ≥5.75 years

Note: 1.111+0.9876+0.9539+0.8479=3.8994 *This is less than 4 years because the drop-outs bring down the average length of stay!*

_*e*_ 10. What fraction of students graduate in *exactly* four years? (choose nearest answer)

a. ≤25% c. 35% e. 45% ***(46.254%)*** g. 55% i. 65% k. 75% m. 85% o. *Not sufficient info*
b. 30% d. 40% f. 50% h. 60% j. 70% l. 80% n. ≥90%

2. Discrete-time Markov Chains II:

Consider an **(s,S) inventory system** in which the number of items on the shelf is checked at the end of each day. The demand distribution is as follows:

n=	0	1	2
P{D=n}	0.2	0.5	0.3

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf. *(That is, it is an (s,S) inventory system, with s=2 and S=4.)*

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions:

P =

\	0	1	2	3	4
0	0	0	0.3	0.5	0.2
1	0	0	0.3	0.5	0.2
2	0.3	0.5	0.2	0	0
3	0	0.3	0.5	0.2	0
4	0	0	0.3	0.5	0.2

P^2 =

\	0	1	2	3	4
0	0.09	0.3	0.37	0.2	0.04
1	0.09	0.3	0.37	0.2	0.04
2	0.06	0.1	0.28	0.4	0.16
3	0.15	0.31	0.29	0.19	0.06
4	0.09	0.3	0.37	0.2	0.04

P^3 =

\	0	1	2	3	4
0	0.111	0.245	0.303	0.255	0.086
1	0.111	0.245	0.303	0.255	0.086
2	0.084	0.26	0.352	0.24	0.064
3	0.087	0.202	0.309	0.298	0.104
4	0.111	0.245	0.303	0.255	0.086

P^4 =

\	0	1	2	3	4
0	0.0909	0.228	0.3207	0.272	0.0884
1	0.0909	0.228	0.3207	0.272	0.0884
2	0.1056	0.248	0.3128	0.252	0.0816
3	0.0927	0.2439	0.3287	0.2561	0.0786
4	0.0909	0.228	0.3207	0.272	0.0884

P^5 =

\	0	1	2	3	4
0	0.0962	0.2419	0.3223	0.2580	0.0814
1	0.0962	0.2419	0.3223	0.2580	0.0814
2	0.0938	0.2320	0.3191	0.2680	0.0870
3	0.0986	0.2412	0.3183	0.2588	0.0830
4	0.0962	0.2419	0.3223	0.2580	0.0814

$$\sum_{n=1}^{5} P^n =$$

\	0	1	2	3	4
0	0.388	1.015	1.616	1.485	0.495
1	0.388	1.015	1.616	1.485	0.495
2	0.643	1.340	1.463	1.160	0.392
3	0.428	1.297	1.746	1.202	0.325
4	0.388	1.015	1.616	1.485	0.495

First Passage Probabilities

n	$f_{4,0}^{(n)}$
1	0.0
2	0.09
3	0.111
4	0.0828
5	0.07623

Mean First Passage Times

\	0	1	2	3	4
0	10.408	4.192	2.653	2.75	11.953
1	10.408	4.192	2.653	2.75	11.953
2	7.755	2.822	3.122	4	13.203
3	10	3.014	2.245	3.83	13.984
4	10.408	4.192	2.653	2.75	11.953

Steady State Distribution

i	name	πi
0	SOH 0	0.09607
1	SOH 1	0.23856
2	SOH 2	0.32026
3	SOH 3	0.26144
4	SOH 4	0.08366

_c__ 1. the value $P_{4,2}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

_b__ 2. the value $P_{0,3}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

c 3. the value $P_{2,0}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

d 4. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*

a. 2.5 b. 5 c. 7.5 d. 10 e. 12.5
f. 15 g. 17.5 h. 20 i. more than 20

e 5. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value):*

a. 5% b. 6% c. 7% d. 8% e. 9% ***(8.8%)***
f. 10% g. 11% h. 12% i. 13% j. ≥14%

f 6. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is *(select nearest value):*

a. 5% b. 10% c. 15% d. 20% e. 25%
f. 30% ***(31.89%)*** g. 35% h. 40% i. 45% j. ≥50%

f 7. If the shelf is full Monday morning, the *expected* number of nights that the shelf is restocked during the next five nights is *(select nearest value):*

a. 0.25 b. 0.5 c. 0.75 d. 1 e. 1.25
f. 1.5 ***(1.403)*** g. 1.75 h. 2 i. 2.25 j. ≥2.5

f 8. How frequently will the shelf be restocked? *(select nearest value): once every* ***2.988*** *days*

a. 0.5 days b. 1 days c. 1.5 days d. 2 days e. 2.5 days
f. 3 days g. 3.5 days h. 4 days i. 4.5 days j. ≥5 days

e 9. What is the probability of a stockout Thursday night? *(select nearest value):*

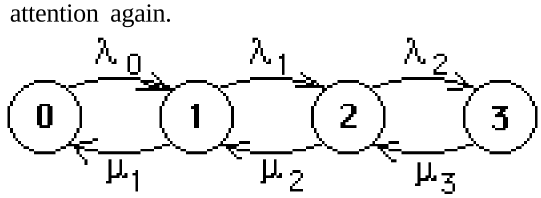
a. 5% b. 6% c. 7% d. 8% e. 9%
f. 10% g. 11% h. 12% i. 13% j. ≥14%

10. Circle one or more of the following equations which are among those solved to compute the steady state probability distribution:

a. $p_0 = 0.3p_2$
b. $p_4 = 0.2p_2 + 0.5p_3 + 0.3p_4$
c. $p_3 = 0.3p_0 + 0.5p_1 + 0.2p_2$
d. $p_2 = 0.3p_0 + 0.3p_1 + 0.2p_2 + 0.5p_3 + 0.3p_4$
e. $p_4 = 0.3p_2 + 0.5p_3 + 0.2p_4$
f. $p_0 + p_1 + p_2 + p_3 + p_4 = 1$

3. Birth/Death Model of a Queue:

A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.



b,d,f 1. The Markov chain model diagrammed above is *(select one or more):*

a. a discrete-time Markov chain b. a continuous-time Markov chain c. a Poisson process
d. a Birth-Death process e. an M/M/1 queue f. an M/M/1/3/3 queue
g. an M/M/3 queue h. an M/M/1/3 queue

a 2. The value of λ_2 is

a. 1/hr. b. 2/hr. c. 3/hr d. 4/hr
e. 0.25/hr. f. 0.5/hr. g. *none of the above*

d 3. The value of μ_2 is

a. 1/hr. b. 2/hr. c. 3/hr d. 4/hr
e. 0.25/hr. f. 0.5/hr. g. *none of the above*

_c_ 4. The value of λ_0 is

a. 1/hr. b. 2/hr. c. 3/hr d. 4/hr
e. 0.25/hr. f. 0.5/hr. g. *none of the above*

_b_ 5. The steady-state probability π_0 is computed by solving

a. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \approx \frac{1}{0.366}$ b. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{3}{4} \approx \frac{1}{0.451}$

c. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \approx \frac{1}{0.4}$ d. $\frac{1}{\pi_0} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.753}$

e. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.496}$ f. *none of the above*

_d_ 6. The operator will be busy what fraction of the time? *(choose nearest value)* $1 - p_0 = 1 - 0.451$

a. 40% b. 45% c. 50% d. 55%
e. 60% f. 65% g. 70% h. ≥75%

_c_ 7. What fraction of the time will the operator be busy but with no machine waiting to be serviced? *(choose nearest value)* $\pi_1 = 0.75\pi_0 = 0.75(0.451)$

a. 10% b. 20% c. 30% d. 40%
e. 50% f. 60% g. 70% h. ≥80%

_a_ 8. Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? *(select nearest value)*

a. 0.1 hr. (i.e.,6 min.) b. 0.15 hr. (i.e., 9 min.)
c. 0.2 hr. (i.e., 12 min.) d. 0.25 hr. (i.e., 15 min.)
e. 0.3 hr. (i.e., 18 min.) f. greater than 0.33 hr. (i.e., >20 min.)

Solution: $p_0 = 0.451, p_1 = 0.338, p_2 = 0.169, p_3 = 0.0423$

Average arrival rate = $\bar{l} = 3p_0 + 2p_1 + 1p_2 + 0p_3 = 3(0.451) + 2(0.338) + 0.169 = 2.2/\text{hr}$

Average # of machines waiting $= L_q = 0p_0 + 0p_1 + 1p_2 + 2p_3 = 1(0.169) + 2(0.423) = 0.25$

By Little's Law, $L_q = \bar{l}\, W_q \Rightarrow W_q = \frac{0.25}{2.2/hr} = 0.114hr = 6.8\min$

_d_ 9. What will be the utilization of this group of 3 machines? *(choose nearest value)*

a. 60% b. 65% c. 70% d. 75%
e. 80% f. 85% g. 90% h. ≥95%

Solution: Average # machines operating $= 3p_0 + 2p_1 + 1p_2 + 0p_3 = 2.2$

and so the utilization is $\frac{2.2}{3} = 73.3\%$

4. Integer Programming Model Formulation Part I. You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (#1 and #2). The length and type of each song are given in the table below:

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Ballad & Hit	2
5	Ballad	4
6	Hit	3
7	neither ballad nor hit	5
8	Ballad & hit	4

Define the variables

Y_i = 1 if song #i is on side 1;
0 otherwise (i.e., if on side 2)

Thus, $1-Y_i$ = 1 if song #i is on side 2;
0 otherwise (i.e., if on side 1)

Name or Initials ___________________

For each restriction, choose a linear constraint from the list (a) through (i) below.

_g_ 1. Side #2 must have at least 3 ballads ⇒ *side 1 can have no more than 2 of the 5 ballads*
_c_ 2. Side #1 must have at least 2 hit songs
_d_ 3. If song #2 is on side 1, then song #3 must be on side 2
_c_ 4. The number of hit songs on side 2 should be no more than 2⇒ *side 1 can have at least 2 of the 4 hits*
_f_ 5. If both songs 1 & 2 are on side 1, then song 3 must be on side 2.

a. $Y_2+Y_4+Y_6+Y_8 \geq 3$	b. $Y_2+Y_4+Y_6+Y_8 \leq 2$	c. $Y_2+Y_4+Y_6+Y_8 \geq 2$	d. $Y_2+Y_3 \leq 1$
e. $Y_1+Y_2-Y_3 \leq 2$	f. $Y_1+Y_2+Y_3 \leq 2$	g. $Y_1+Y_3+Y_4+Y_5+Y_8 \leq 2$	h. $Y_2+Y_3 \geq 1$
i. $Y_1+Y_2-Y_3 \geq 2$	j. $Y_1+Y_2+Y_3 \leq 1$	k. $Y_1+Y_3+Y_4+Y_5+Y_8 \leq 3$	l. *None of the above*

Part II. Comquat owns four production plants at which personal computers are produced. In order to use a plant to produce computers, a fixed cost must be paid to set up the production line in that plant.

Define the variables:

Y_i = 1 if the production line has been set up at plant #i
0 otherwise

X_i = # of computers produced at plant #i

For each restriction, choose a constraint from the list (a) through (l) below.

_b_ 7. Computers are to be produced at no more than 3 plants.
_i_ 8. If the production line at plant 2 is set up, then that plant can produce up to 8000 computers; otherwise, none can be produced at that plant.
_d_ 9. The production lines at plants 2 and 3 cannot both be set up.
_g_ 10. The total production must be at least 20,000 computers.
_f_ 11. If the production line at plant 2 is set up, that plant must produce at least 2000 computers.
_k_ 12. If the production line at plant 2 is not set up, then the production line at plant 3 cannot be set up.

Constraints:

a. $Y_2 \leq 8000X_2$	b. $Y_1+Y_2+Y_3+Y_4 \leq 3$	c. $Y_1+Y_2+Y_3+Y_4 \geq 3$	d. $Y_2+Y_3 \leq 1$
e. $Y_2 \leq 2000X_2$	f. $X_2 \geq 2000Y_2$	g. $X_1+X_2+X_3+X_4 \geq 20000$	h. $Y_2+Y_3 \geq 1$
i. $X_2 \leq 8000Y_2$	j. $Y_2 \leq Y_3$	k. $Y_2 \geq Y_3$	l. *None of the above*

5. Deterministic Production Planning Production must be planned for each day of the next week (Sunday through Saturday). The following shipments have been planned for each day:

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Required	1	3	1	4	2	2	1

Other data:

Production cost is $3 for setup, plus $1 per unit produced, up to a maximum of 3 units.
Storage cost: $1 per unit stored (based upon **beginning**-of-day stock), up to a maximum of 5 units in storage
Salvage value: $2 per unit in stock remaining in storage Saturday night
Initial inventory: 2 units are in stock Sunday morning.

A dynamic programming model was used to compute the optimal production quantities for each day in order to meet the shipment schedule.
The stages were numbered in a **backward** fashion, i.e., n= 7(Sunday), 6(Monday), … 1(Saturday).

Several values have been blanked in the tables of computations-- compute them:
A = ___4______ = cost of last day (Saturday) if stock is zero and 1 is produced.
B = __11______ = cost of Thursday through Saturday if on Thursday morning, 2 are in stock and 2 are produced.
C = __16______ = minimal cost of Wednesday through Saturday if 4 are in stock
D = ___0______ = optimal production quantity on Wednesday if 4 are in stock
E = ___0______ = resulting stock on Thursday if 4 are in stock Wednesday & optimal quantity is produced

What is the minimum total cost of the schedule? ___32___

Complete the table below with the inventory levels and production quantities if the optimal schedule is used.

Day	SUN	MON	TUES	WED	THURS	FRI	SAT	final value
Inventory	2	1	0	1	0	0	0	2
Prod'n qty	0	2	2	3	2	2	3	

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Name or Initials ____________________

---Stage 1 (Saturday)---

s \ x:	0	1	2	3
0	9999	_A_	3	2
1	1	3	2	1
2	0	2	1	0
3	-1	1	0	-1
4	-2	0	-1	9999
5	-3	-1	9999	9999

---Stage 2 (Friday)---

s \ x:	0	1	2	3
0	9999	9999	7	7
1	9999	7	7	7
2	4	7	7	7
3	4	7	7	7
4	4	7	7	7
5	4	7	7	9999

---Stage 3 (Thursday)---

s \ x:	0	1	2	3
0	9999	9999	12	13
1	9999	12	13	11
2	9	13	_B_	12
3	10	11	12	13
4	8	12	13	14
5	9	13	14	9999

---Stage 4 (Wednesday)---

s \ x:	0	1	2	3
1	9999	9999	9999	19
2	9999	9999	19	19
3	9999	19	19	18
4	16	19	18	20
5	16	18	20	19

---Stage 5 (Tuesday)---

s \ x:	0	1	2	3
0	9999	9999	24	25
1	9999	24	25	25
2	21	25	25	24
3	22	25	24	25
4	22	24	25	9999
5	21	25	9999	9999

---Stage 6 (Monday)---

s \ x:	0	1	2	3
0	9999	9999	9999	30
1	9999	9999	30	31
2	9999	30	31	29
3	27	31	29	31
4	28	29	31	32
5	26	31	32	32

---Stage 7---

s \ x:	0	1	2	3
2	32	35	34	36

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Stage 7 (**Sunday**)

State	Optimal Values	Optimal Decisions	Resulting State
2	32	0	1

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Stage 6 (**Monday**):

State	Optimal Values	Optimal Decisions	Resulting State
0	30	3	0
1	30	2	0
2	29	3	2
3	27	0	0
4	28	0	1
5	26	0	2

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Stage 5 (**Tuesday**):

State	Optimal Values	Optimal Decisions	Resulting State
0	24	2	1
1	24	1	1
2	21	0	1
3	22	0	2
4	22	0	3
5	21	0	4

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Stage 4 (**Wednesday**):

State	Optimal Values	Optimal Decisions	Resulting State
1	19	3	0
2	19	2	0
3	18	3	2
4	_C_	_D_	_E_
5	16	0	1

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Stage 3 (**Thursday**):

State	Optimal Values	Optimal Decisions	Resulting State
0	12	2	0
1	11	3	2
2	9	0	0
3	10	0	1
4	8	0	2
5	9	0	3

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Stage 2 (**Friday**):

State	Optimal Values	Optimal Decisions	Resulting State
0	7	2	0
1	7	1	0
2	4	0	0
3	4	0	1
4	4	0	2
5	4	0	3

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Stage 1 (**Saturday**):

State	Optimal Values	Optimal Decisions	Resulting State
0	2	3	2
1	1	0	0
2	0	0	1
3	-1	0	2
4	-2	0	3

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5    -3     0          4
```