

<><><><><> 56:171 Operations Research <><><><><>  
 Final Exam - December 16, 1992  
 <><><><><> Instructor: D.L. Bricker <><><><><>

Do one from Part One (28 pts.) , and four from Part Two (18 pts. each)

Score:	Part One:	I. _____	LINDO Analysis
		II. _____	Simplex Algorithm
	Part Two:	III. _____	Decision trees
		IV. _____	Markov chain model: Inventory System
		V. _____	Markov chain model: Manufacturing System
		VI. _____	Continuous-time Markov chain
		VII. _____	Dynamic programming
TOTAL:		_____ (of 100 possible)	

<><><><><> PART ONE <><><><><>

**I. LINDO analysis**

*Problem Statement:* McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

- i) Red Baron must contain no more than 75% of A.
- ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of \$3.35 for Diablo and \$2.85 for Red Baron. A and B cost \$1.60 and \$2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

*Define*

- $D$  = quarts of Diablo to be produced
- $R$  = quarts of Red Baron to be produced
- $A_1$  = quarts of A used to make Diablo
- $A_2$  = quarts of A used to make Red Baron
- $B_1$  = quarts of B used to make Diablo
- $B_2$  = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

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MAX      3.35 D + 2.85 R - 1.6 A1 - 1.6 A2 - 2.05 B1 - 2.05 B2
SUBJECT TO
2) - D + A1 + B1 =      0
3) - R + A2 + B2 =      0
4)  A1 + A2 <=    40
5)  B1 + B2 <=    30
6) - 0.25 D + A1 >=    0
7) - 0.5 D + B1 >=    0
8) - 0.75 R + A2 <=    0
END
    
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OBJECTIVE FUNCTION VALUE
1)      99.000000
    
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VARIABLE	VALUE	REDUCED COST
D	50.000000	0.000000
R	20.000000	0.000000
A1	25.000000	0.000000
A2	15.000000	0.000000
B1	25.000000	0.000000
B2	5.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-2.350000
3)	0.000000	-4.350000
4)	0.000000	0.750000
5)	0.000000	2.300001

6)	12.500000	0.000000
7)	0.000000	-1.999999
8)	0.000000	2.000000

RANGES IN WHICH THE BASIS IS UNCHANGED

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
D	3.350000	0.750000	0.500000
R	2.850000	0.500000	0.375000
A1	-1.600000	1.500001	0.666666
A2	-1.600000	0.666666	0.500000
B1	-2.050000	1.500001	1.000000
B2	-2.050000	1.000000	1.500001

ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	10.000000	10.000000
3	0.000000	16.666668	3.333333
4	40.000000	50.000000	10.000000
5	30.000000	10.000000	16.666664
6	0.000000	12.500000	INFINITY
7	0.000000	6.250000	5.000000
8	0.000000	2.500000	12.500000

THE TABLEAU:

ROW	(BASIS)	D	R	A1	A2
1	ART	0.000	0.000	0.000	0.000
2	A1	0.000	0.000	1.000	0.000
3	R	0.000	1.000	0.000	0.000
4	A2	0.000	0.000	0.000	1.000
5	B2	0.000	0.000	0.000	0.000
6	SLK 6	0.000	0.000	0.000	0.000
7	D	1.000	0.000	0.000	0.000
8	B1	0.000	0.000	0.000	0.000
ROW	B1	B2	SLK 4	SLK 5	SLK 6
1	0.000	0.000	0.750	2.300	0.000
2	0.000	0.000	-0.500	1.500	0.000
3	0.000	0.000	2.000	-2.000	0.000
4	0.000	0.000	1.500	-1.500	0.000
5	0.000	1.000	0.500	-0.500	0.000
6	0.000	0.000	-0.250	0.750	1.000
7	0.000	0.000	-1.000	3.000	0.000
8	1.000	0.000	-0.500	1.500	0.000
ROW	SLK 7	SLK 8	RHS		
1	2.000	2.000	99.000		
2	3.000	2.000	25.000		
3	-4.000	-4.000	20.000		
4	-3.000	-2.000	15.000		
5	-1.000	-2.000	5.000		
6	2.000	1.000	12.500		
7	4.000	4.000	50.000		
8	1.000	2.000	25.000		

1. How many quarts of "Red Baron" are produced in the optimal solution? \_\_\_\_\_
2. How much profit does the firm make on these two products? \_\_\_\_\_
3. What *additional* amount should the firm be willing to pay to have another quart of ingredient A available? \_\_\_\_\_ What is the *total* amount the firm should be willing to pay for another quart of ingredient A? \_\_\_\_\_ How many quarts should they be willing to buy at this cost?  
\_\_\_\_\_

4. If one more quart of ingredient A were to be used, what would be the changes in
  - the "slack variable" in row #4? \_\_\_\_\_ (increase/decrease)
  - the "slack variable" in row #6? \_\_\_\_\_ (increase/decrease)
  - the quantity of A used in producing "Red Baron"? \_\_\_\_\_ (increase/decrease)
  - the quantity of A used in producing "Diablo"? \_\_\_\_\_ (increase/decrease)
  - the quantity of "Diablo" produced? \_\_\_\_\_ (increase/decrease)
  - the quantity of "Red Baron" produced? \_\_\_\_\_ (increase/decrease)
5. How much can the price of "Red Baron" increase before the composition of the current optimal product mix changes? \_\_\_\_\_
6. What is the optimal value of the objective function of the LP which is the dual of this problem? \_\_\_\_\_ . This dual LP objective is (*circle one:*) minimized / maximized.

**II. Simplex Algorithm for LP:** At an intermediate step of the simplex algorithm, in which the objective is to be **minimized**, the tableau is:

$\bar{z}$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
1	0	0	0	2	0	-1	-10
0	1	0	1	4	0	1	4
0	0	1	0	2	0	-1	3
0	-1	0	0	-2	1	3	1

1. What are the basic variables for this tableau? (*circle:*)
 

$-Z$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
------	-------	-------	-------	-------	-------	-------	-----
2. The current value of  $z$  for this basic solution is (*circle:* +10 or -10)
3. The current value of  $x_1$  for this basic solution is
 

a. 0	b. 1	c. 3	d. 4	e. 10
------	------	------	------	-------
4. The current value of  $x_2$  for this basic solution is
 

a. 0	b. 1	c. 3	d. 4	e. 10
------	------	------	------	-------
5. The current value of  $x_3$  for this basic solution is
 

a. 0	b. 1	c. 3	d. 4	e. 10
------	------	------	------	-------
6. Increasing  $x_4$  would (*circle:* increase / decrease) the objective function.
7. What is the substitution rate of  $x_4$  for  $x_5$ ?
 

a. 0	b. 1	c. -1	d. 2	e. -2
------	------	-------	------	-------
8. If  $x_4$  were increased by 2 units, the value of  $x_5$  will
 

a. not change	b. increase by 2	c. decrease by 2
d. increase by 4	e. decrease by 4	f. none of the above
9. Perform a pivot on the tableau above to improve the objective function, and write the new tableau below:

$\bar{z}$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
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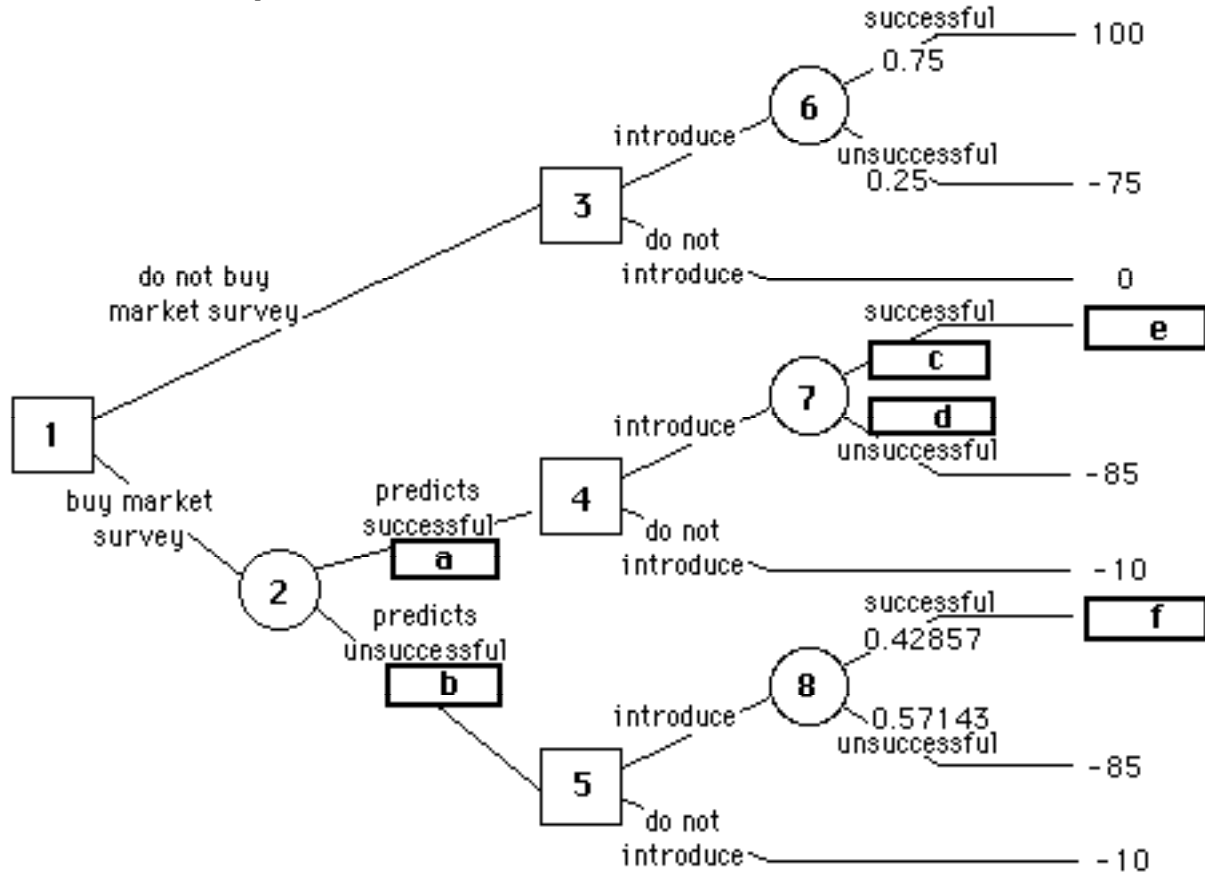
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10. What is the improvement in the objective resulting from the pivot in (9)? \_\_\_\_\_
11. If the original constraints were of type " $\leq$ " and  $X_4$ ,  $X_5$ , and  $X_6$  are slack variables, the value of the first dual variable  $\pi_1$  corresponding to the first tableau above is
- a. 0                      b. 1                      c. -1                      d. 2                      e. -2  
f. none of the above                      g. cannot be determined
12. If the original constraints were of type " $\geq$ " and  $X_4$ ,  $X_5$ , and  $X_6$  are surplus variables, the value of the first dual variable  $\pi_1$  corresponding to the first tableau above is
- a. 0                      b. 1                      c. -1                      d. 2                      e. -2  
f. none of the above                      g. cannot be determined

<><><><><> **PART TWO** <><><><><>

**III. Decision Analysis:** A company is considering the introduction of a new product. Past experience leads them to believe that it will either be very successful, or a failure, and to estimate the probability of success to be **75%**. If successful, the profit will be **100** thousand dollars. If not successful, the loss will be **75** thousand dollars. A market survey firm has offered to conduct a survey at a cost of **10** thousand dollars. This market survey firm's accuracy is estimated to be **80%** (i.e., for products which have been successful, the firm has correctly predicted success 80% of the time, and for products which have been unsuccessful, the firm has also predicted no success 80% of the time). The company has constructed the following decision tree to help them in the planning. (*Squares represent decisions, and circles represent chance nodes.*)



1. There are six numerical values missing in the decision tree (indicated by a through f in the bold rectangles). Compute these values:

- a. probability that market survey firm predicts successful product \_\_\_\_\_
- b. probability that market survey firm predicts unsuccessful product \_\_\_\_\_
- c. probability that product is successful if success is predicted \_\_\_\_\_
- d. probability that product is unsuccessful if success is predicted \_\_\_\_\_
- e. "payoff" if product is successful and market survey predicts success \_\_\_\_\_
- f. "payoff" if product is successful and market survey predicts no success \_\_\_\_\_

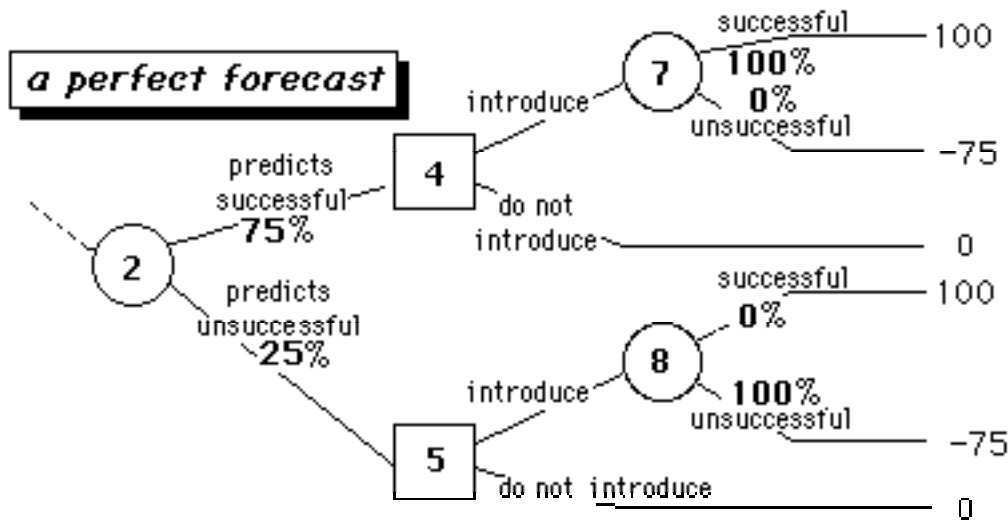
2. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	_____	5	-10	2	46.25
7	_____	4	_____	1	_____
6	56.25	3	_____		

3. Should the company buy the market survey? \_\_\_\_\_

4. What is the expected value of the market survey? \_\_\_\_\_

5. What would be the expected value of "perfect information", i.e., a prediction which is 100% accurate, so that the portion of the tree containing nodes 2, 4, 5, etc., would appear as below? \_\_\_\_\_



**4. Markov Chain Model--(s,S) Inventory System:** Consider the following inventory system for a certain spare part for a company's 2 production lines. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions:

**Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

$$P = \begin{bmatrix} 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.5 & \mathbf{A} & 0 & 0 \\ 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \end{bmatrix}, P^2 = \begin{bmatrix} 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.06 & 0.15 & 0.23 & 0.35 & 0.21 \\ 0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.046 & 0.185 & 0.328 & 0.315 & 0.126 \\ 0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \end{bmatrix}, P^4 = \begin{bmatrix} 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\ 0.058 & 0.203 & 0.316 & 0.296 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \end{bmatrix}$$

$$\sum_{n=1}^4 P^n = \begin{bmatrix} 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\ 0.2262 & 0.9219 & 1.4477 & 1.0781 & 0.3261 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \end{bmatrix}$$

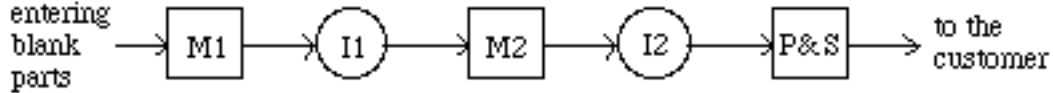
$$M = \begin{bmatrix} 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 12.692 & 2.754 & 3.153 & 4 & 9.795 \\ 15 & 3.396 & 2.307 & 3.514 & 10.816 \\ 15.769 & 4.641 & 3.076 & 2.571 & \mathbf{B} \end{bmatrix}$$

i	$\pi_i$
-	0.063
1	0.215
2	0.317
3	0.284
4	0.119
5	

1. the value  $P_{5,4}$  is
  - a.  $P\{\text{demand}=0\}$
  - b.  $P\{\text{demand}=1\}$
  - c.  $P\{\text{demand}=2\}$
  - d.  $P\{\text{demand}=3\}$

- e.  $P\{\text{demand}=1\}$  f. none of the above
2. the value  $P_{2,4}$  is  
 a.  $P\{\text{demand}=0\}$  b.  $P\{\text{demand}=1\}$   
 c.  $P\{\text{demand}=2\}$  d.  $P\{\text{demand}=1\}$   
 e.  $P\{\text{demand}=1\}$  f. none of the above
3. the value  $P_{3,1}$  is  
 a.  $P\{\text{demand}=0\}$  b.  $P\{\text{demand}=1\}$   
 c.  $P\{\text{demand}=2\}$  d.  $P\{\text{demand}=1\}$   
 e.  $P\{\text{demand}=1\}$  f. none of the above
4. the numerical value A in the matrix above is  
 a. 0 b. 0.1  
 c. 0.2 d. 0.3  
 e. 0.4 f. 0.5
5. the numerical value B in the mean-first-passage time matrix (M) above is (*select nearest value*)  
 a. 1 b. 2  
 c. 4 d. 6  
 e. 8 f. 10
6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is (*select nearest value*):  
 a. 2 b. 5  
 c. 10 d. 15  
 e. 20 f. more than 20
7. If the shelf is full Monday morning, the probability that the shelf is full Wednesday night is (*select nearest value*):  
 a. 7% b. 8%  
 c. 9% d. 10%  
 e. 11% f. more than 12%
8. If the shelf is full Monday morning, the probability that the shelf is restocked Wednesday night is (*select nearest value*):  
 a. 10% b. 15%  
 c. 20% d. 25%  
 e. 30% f. more than 30%
9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (*select nearest value*):  
 a. 0.6 b. 0.7  
 c. 0.8 d. 0.9  
 e. more than once but less than twice f. more than 2
10. The number of transient states in this Markov chain model is  
 a. zero b. 1  
 c. 2 d. 5  
 e. none of the above
11. The number of recurrent states in this Markov chain model is  
 a. zero b. 1  
 c. 2 d. 5  
 e. none of the above
12. The number of absorbing states in this Markov chain model is  
 a. zero b. 1  
 c. 2 d. 5  
 e. none of the above
13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?  
 a.  $\pi_1 = 0.2\pi_3$   
 b.  $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$   
 c.  $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$   
 d.  $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$   
 e.  $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

**5. Discrete-time Markov chain--Multistage Manufacturing System:** Consider a system of 2 machines, with inspection of a part after each machining operation:



Relevant data are:

Operations	Time Rqmt. (man-hrs)	Operating Cost (\$/hr.)	Scrap rate %	% sent back for rework
Machine 1	1	10	10	
Inspection 1	0.25	10	5	5
Machine 2	1	15	5	
Inspection 2	0.5	15	15	5
Pack & Ship	0.5	10		

The manufacturing system is modeled as a discrete-time Markov chain with 6 states and the transition probability matrix:

$$P = \begin{bmatrix} Q & R \\ O & I \end{bmatrix} = \begin{bmatrix} 0 & 0.9 & 0 & 0 & 0 & 0.1 \\ 0.05 & 0 & 0.9 & 0 & 0 & 0.05 \\ 0 & 0 & 0 & 0.95 & 0 & 0.05 \\ 0 & 0 & 0.05 & 0 & 0.8 & 0.15 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

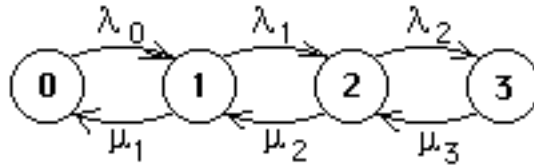
$$P^5 = \begin{bmatrix} 0 & 0.001823 & 0 & 0.07118 & 0.6156 & 0.3114 \\ 0.0001013 & 0 & 0.005777 & 0 & 0.7473 & 0.2469 \\ 0 & 0 & 0 & 0.002143 & 0.7961 & 0.2018 \\ 0 & 0 & 0.0001128 & 0 & 0.8398 & 0.1601 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = (I - Q)^{-1} = \begin{bmatrix} 1.047 & 0.9424 & 0.8905 & 0.8459 \\ 0.05236 & 1.047 & 0.9894 & 0.9399 \\ 0 & 0 & 1.05 & 0.9974 \\ 0 & 0 & 0.05249 & 1.05 \end{bmatrix}, A = ER = \begin{bmatrix} 0.6768 & 0.3232 \\ 0.7519 & 0.2481 \\ 0.7979 & 0.2021 \\ 0.8399 & 0.1601 \end{bmatrix}$$

- The number of transient states in this Markov chain is
  - zero
  - two
  - four
  - six
  - None of the above
- The number of recurrent states in this Markov chain is
  - zero
  - two
  - four
  - six
  - None of the above
- The percent of parts which must be scrapped is (*choose nearest value*):
  - 15%
  - 20%
  - 25%
  - 30%
  - 35%
  - more than 40%
- The expected number of blanks which must be processed in order to produce 100 parts is (*choose nearest value*):
  - 110
  - 120
  - 130
  - 140
  - 150
  - more than 160
- The probability that a part will be successfully completed if it must be reworked on the first machine is (*choose nearest value*):
  - 60%
  - 65%
  - 70%
  - 75%
  - 80%
  - less than 60%
- The expected number of man-hours at the first inspection station in order to successfully produce 100 parts is (*choose nearest value*):
  - 20
  - 25
  - 30
  - 35
  - 40
  - more than 45 man-hours
- The probability that a part is successfully completed with no reworking is (*choose nearest value*):
  - 45%
  - 50%
  - 55%
  - 60%
  - 65%
  - more than 70%



**6. BIRTH/DEATH MODEL:** A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.



- The Markov chain model diagrammed above is (select one or more):
  - a discrete-time Markov chain
  - a continuous-time Markov chain
  - a Birth-Death process
  - an M/M/1 queue
  - an M/M/3 queue
  - an M/M/1/3 queue
  - an M/M/1/3/3 queue
  - a Poisson process
- The value of  $\lambda_2$  is
  - 1/hr.
  - 2/hr.
  - 3/hr.
  - 4/hr.
  - 0.5/hr.
  - none of the above
- The value of  $\mu_2$  is
  - 1/hr.
  - 2/hr.
  - 3/hr.
  - 0.5/hr.
  - 0.5/hr.
  - none of the above
- The value of  $\lambda_0$  is
  - 1/hr.
  - 2/hr.
  - 3/hr.
  - 0.5/hr.
  - 0.5/hr.
  - none of the above
- The steady-state probability  $\pi_0$  is computed by computing
  - $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{1}{0.4}$
  - $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{3}{4} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2} \times \frac{1}{4} \approx \frac{1}{0.451}$
  - $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \approx \frac{1}{0.366}$
  - $\frac{1}{\pi_0} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.753}$
  - $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.496}$
  - none of the above
- The operator will be busy what fraction of the time? (choose nearest value)
  - 40%
  - 45%
  - 50%
  - 60%
  - 65%
  - 75%
- What fraction of the time will the operator be busy but with no machine waiting to be serviced? (choose nearest value)
  - 10%
  - 20%
  - 30%
  - 40%
  - 50%
  - 60%
- Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? (select nearest value)
  - 0.1 hr. (i.e., 6 min.)
  - 0.15 hr. (i.e., 9 min.)
  - 0.2 hr. (i.e., 12 min.)
  - 0.25 hr. (i.e., 15 min.)
  - 0.3 hr. (i.e., 18 min.)
  - greater than 0.33 hr. (i.e., >20 min.)
- What will be the utilization of this group of 3 machines? (choose nearest value)
  - 50%
  - 55%
  - 60%
  - 65%
  - 70%
  - greater than 75%

**7. Dynamic Programming:** Consider a production/inventory system with the following characteristics:

- Maximum inventory level is 8
- Storage costs are \$1/week per unit in inventory at the beginning of the week
- Maximum production level is 6/week
- Setup cost for production is \$10 in each week during which production is scheduled
- Marginal production costs (costs in excess of setup cost) are \$2 per unit
- Demand in each of the next 6 weeks is assumed to be known and must be satisfied:
 

Week #	1	2	3	4	5	6
Stage #	6	5	4	3	2	1
Demand:	4	4	3	2	3	1
- Anything produced during a certain week (plus anything in inventory at the beginning of the week) may be used to satisfy demand during that week, while anything in excess of the maximum inventory level (8) at the end of the week, after demand is satisfied, is discarded)
- At the end of the 6 weeks, a salvage value of \$3 per unit remaining in inventory is recovered.
- Initially, the inventory contains 2 units.

A dynamic programming model is defined for the problem of minimizing total cost over the 6-week period. The state variable is the beginning-of-the-week inventory level. The optimal value function  $f_n(S)$  is the minimum cost of satisfying the demand of the last  $n$  weeks, if current inventory level is  $S$ . *Note that stages are numbered in reverse-chronological order, i.e., the first week is stage #6, and the last week is stage #1.*

The table of intermediate computational results ("details") at stage 4 is:

Stage 4

s \ x:	0	1	2	3	4	5	6
0	9999.99	9999.99	9999.99	43.00	44.00	41.00	43.00
1	9999.99	9999.99	42.00	43.00	40.00	42.00	44.00
2	9999.99	41.00	42.00	39.00	41.00	43.00	39.00
3	30.00	41.00	38.00	40.00	A	38.00	36.00
4	30.00	37.00	39.00	41.00	37.00	35.00	37.00
5	26.00	38.00	40.00	36.00	34.00	36.00	38.00
6	27.00	39.00	35.00	33.00	35.00	37.00	9999.99
7	28.00	34.00	32.00	34.00	36.00	9999.99	9999.99
8	23.00	31.00	33.00	35.00	9999.99	9999.99	9999.99

The tables of optimal values & decisions at each stage are:

Stage 6:			
State	Optimal Values	Optimal Decisions	Resulting State
0	73.00	5	1
1	72.00	4	1
2	69.00	6	4
3	68.00	5	4
4	63.00	0	0
5	58.00	0	1
6	58.00	0	2
7	58.00	0	3
8	53.00	0	4

Stage 5:			
State	Optimal Values	Optimal Decisions	Resulting State
0	59.00	4	0
1	53.00	6	3
2	52.00	5	3
3	51.00	4	3
4	45.00	6	5
5	45.00	0	0
6	45.00	0	1
7	45.00	0	2
8	37.00	0	3
9	38.00	0	4

Stage 4:			
State	Optimal Values	Optimal Decisions	Resulting State
0	41.00	5	2
1	40.00	4	2
2	39.00	3	2
		6	5
3	30.00	0	0
4	<b>B</b>	0	<b>C</b>
5	26.00	0	2
6	27.00	0	3
7	28.00	0	4
8	23.00	0	5

Stage 3:			
State	Optimal Values	Optimal Decisions	Resulting State
0	27.00	6	4
1	26.00	5	4
2	21.00	0	0
3	21.00	0	1
4	21.00	0	2
5	15.00	0	3
6	11.00	0	4
7	11.00	0	5
8	11.00	0	6

Stage 2:			
State	Optimal Values	Optimal Decisions	Resulting State
0	19.00	4	1
1	18.00	3	1
2	17.00	2	1
3	<b>D</b>	0	0
4	5.00	0	1
5	4.00	0	2
6	3.00	0	3
7	2.00	0	4
8	1.00	0	5

Stage 1:			
State	Optimal Values	Optimal Decisions	Resulting State
0	7.00	6	5
1	1.00	0	0
2	-1.00	0	1
3	-3.00	0	2
4	-5.00	0	3
5	-7.00	0	4
6	-9.00	0	5
7	-11.00	0	6
8	-13.00	0	7

- The value **A** in the table above is
  - 39
  - 40
  - 41
  - 42
  - 43
  - none of the above
- The value **B** in the table above is
  - 30
  - 31
  - 32
  - 33
  - 34
  - none of the above
- The value **C** in the table above is
  - 0
  - 1
  - 2
  - 3
  - 4
  - none of the above
- The optimal production quantity for week #1 (stage #6) is
  - 0
  - 2
  - 4
  - 5
  - 6
  - none of the above
- The optimal production quantity for week #2 (stage #5) is
  - 0
  - 2
  - 4
  - 5
  - 6
  - none of the above
- In the optimal plan, the inventory at the beginning of the second week is
  - 0
  - 2
  - 4
  - 5
  - 6
  - none of the above
- The optimal production quantity for week #6 (stage #1) is
  - 0
  - 2
  - 4
  - 5
  - 6
  - none of the above
- The total cost of satisfying the demand during the 6 weeks (if initial inventory is 2), is
  - 0
  - 2
  - 4
  - 5
  - 6
  - none of the above