56:171 0	peration	s Res	earch
Ouiz #9 –	Novem	ber 8.	2002

			A 1 2	3 0.518 0.407	4 0.482 0.593	
ſ		L)	<u>E  </u> 1   2	1 1.851 1 0.741 1	2 ro 1.482 3 2.592 3	<u>w sum</u> .333 .333
			n 1 2 3 4	$f_{13}^{(n)}$ 0.2 0.1 0.06 0.04	6 5 8	$f_{14}^{(n)}$ 0.1 0.11 0.081 0.0571
P 1 2 3 4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		5 6 7 8 9 10	0.03 0.02 0.01 0.01 0.00 0.00	202 241 56866 09806 76864 538048	0.04001 0.028011 0.0196081 0.0137257 0.009608 0.0067256
<u>b</u>	1. Which is the matrix $\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$	Q (used in computation	of E)'	?		- 7
	a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$	b. $\begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}$	c.	0.3 0.4 0.2 0.5	0.2 0 0.1 0	.1 .2
	d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	f. N	one of the	above (N	IOTA)
<u>a</u>	2. Which is the matrix $\begin{bmatrix} 0.2 & 0.1 \end{bmatrix}$	R (used in computation $\begin{bmatrix} 0.3 & 0.4 \end{bmatrix}$	of A)	? 0.3 0.4	0.2 0	.1]
	a. 0.1 0.2	b. $\begin{bmatrix} 0.2 & 0.5 \end{bmatrix}$	с.	0.2 0.5	0.1 0	.2
	d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	f. <i>N</i>	OTA		
<u>e</u>	3. If the system begins in nearest value)	state $\#1$ , what is the pro	babil	ity that it	is absorbe	ed into state #4? (Choose
	a. 30% or less e. 50%	b. 35% f. 55%	c. 4 g. 6	0% 0%	f sta sag	d. 45% h. 65% <i>or more</i>
<u> </u>	4. If the system begins in stage) that the system exi nearest value)	sts before it is absorbed i	into o	one of the t	two absor	bing states? (Choose
	a. 1	b. 2	c. 3			d. 4
	e. 5	f. 6	g. 7	с. ·	,· 1	h. 8 or more
<u> </u>	5. For a discrete-time Ma	irkov chain, let P be the i	matri	x of transi	tion prob	abilities. The sum of
	a column is 1	c row is 1				
	b. column is 0	d. row is 0		e. NOT	<sup>T</sup> A	

Markov Chains. Consider the discrete-time Markov chain diagrammed below:

<u>a</u> 6. An absorbing state of the	f a Markov ch	ain is one i	n which the p	robability c	of	
b. moving into that	state is one.	d. movi	ng into that st	ate is zero	e. NOTA	
g 7. The minimal closed	set(s) in the a	bove Mark	ov chain =			
a. $\{1,2,3,4\}$	b. {1,2}		c. {3.4}		d. {1,2,3	,4} & {3,4}
e. {1} & {2}	f. {1,2} &	: {3,4}	g. {3} &	{4}	h. NOTA	
<u>h</u> 8. The probability that	the system rea	aches an ab	sorbing state	beginning	in state 1, is	(choose
nearest value):						
a. 0.3 b. 0.4	c. 0.5	d. 0.6	e. 0.7	f. 0.8	g. 0.9	h. 1.0
<u>b</u> 9. The recurrent state(s	s) in the above	e Markov cl	nain =			
a. 1 & 2	b. 3 & 4		c. 1, 2, 3,	& 4	d. NOTA	λ
<u>a</u> 10. The transient state	s) in the abov	e Markov c	hain =			
a. 1 & 2	b. 3 & 4		c. 1, 2, 3,	& 4	d. NOTA	λ
<u>e</u> 11. The quantity $f_{13}^{(4)}$	is					
a. the probability the	at the system,	beginning	in state 1, is i	n state 3 at s	stage n	
b. the probability the	at the system f	first visits s	tate 3 before	state 4.		
c. the expected num	ber of visits to	o state 3 du	ring the first 4	4 stages, beg	ginning in st	ate1
d. the stage in which	n the system, b	beginning i	n state 1, visit	s state 3 for	the fourth t	ime
e. the probability the	at the system,	beginning	in state 1, firs	t reaches st	ate 3 in stage	e 4
f. NOTA						
<u>b</u> 12. From which state is	the system m	ore likely t	o eventually	reach state 4	4?	
a. 1	b. 2		c. equally	likely	d. NOT	4
<u>g</u> 13. What is the probabili	ty that the sys	stem is abso	orbed into stat	te 4, starting	g in state 1, i	f the first
transition is to state 2? (	choose neare.	st value)	100/		1 4 5 0 /	
a. 30% or less	b. 35%		c. 40%		d. 45%	
e. 50%	1. 55%		g. 60%		h. 65% c	or more
$\underline{g}$ 14. The probability the	it the system,	starting in s	state 1, is in s	tate 1 after 2	2 stages? (ch	oose nearest
value)	1 100/		1.50/		1 200/	
a. 5% or less	b. 10%		c. 15%		d. 20%	
e. 25%	f. 30%	, ,	g. 35%		h. 40% c	or more
$\underline{c}$ 15. The probability that	t the system, s	starting in s	state 1, is in si	ate 4 after :	stages? (ch	oose nearest
value)	1 100/		- 150/		1 2007	
a. $5\%$ or less	D. 10%		C. 15%		a. 20%	
e. 25%	1.30%	4.0.4	g. 55%	f 0 6	11.40%	h 08
$\begin{array}{ccc} a. \ 0.1 & 0. \ 0.2 \\ \end{array}$	C. U.5	u. 0.4	e. 0.3	1. 0.0	g. 0.7	II. 0.8
$\underline{u}$ 10. In general, the stea	uysiale probat	Sinty distri	$\pi$ (II It	exisis) mus		
a. $P\pi=0$	D. Pπ=1		$C \pi P = 1$		$\alpha \pi \mathbf{P} \equiv \pi$	
D					$u. \pi - \pi$	

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	2		А	3	4	
			1	0.542	0.458	
	$\langle \mathbf{I} \rangle$		2	0.417	0.583	
(		1	E	1	2 :	row sum
(	-(•) 🛋 📋 (•)-		1	2.083	1.25	3.333
	$\mathbf{X} \setminus \mathbf{A}$		2	0.833	2.5	3.333
			n	$f_{13}^{(n)}$		$f_{14}^{(n)}$
			1	0.2		0.1
			2	0.1	1	0.1
			3	0.0	71	0.076
			4	0.0	485	0.0544
P	1 2 3 4		5	0.0	3371	0.03832
1	0.4 0.3 0.2 0.1		6	0.0	23549	0.026872
2	0.2 0.5 0.1 0.2		7	0.0	16474	7 0.01882
3	0 0 1 0		8	0.03	11530	4 0.0131759
4	0 0 0 1		9	0.00	08070	88 0.00922353
			10	0.00	05649	54 0.00645655
a	1. Which is the matrix	R (used in computation	of A)	)?		
	$\begin{bmatrix} 0 & 2 & 0 \end{bmatrix}$		Í	04 03	0.2	0.1]
	a. 0.2 0.1	b.	c.	0.1 0.5	0.2	0.1
				0.2 0.5	0.1	0.2
	$\begin{bmatrix} 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$				
	d.	e.   • • • • • •	f. A	<i>Vone of the</i>	above	(NOTA)
				v		
b	2. Which is the matrix	Q (used in computation	of E)	)?		
	$\begin{bmatrix} 0 & 2 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 4 & 0 3 \end{bmatrix}$	Í	04 03	02	01]
	a. 0.1 0.1	b.	c.	•••••	0	
				0.2 0.5	0.1	0.2
	$\begin{bmatrix} 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$				
	d.	e.	f. A	<i>None of the</i>	above	(NOTA)
				Ū.		
d	3. If the system begins in	1 state #1, what is the pro-	babi	lity that it	is abso	rbed into state #4? (Choose
	nearest value)	, I		J		× ×
	a. 30% or less	b. 35%	c. 4	10%		d. 45%
	e 50%	f 55%	σf	50%		h 65% or more
c	4 If the system begins in	state $\#1$ what is the ext	ecte	d number (	of stage	es (including the initial
<u> </u>	stage) that the system evil	sta before it is absorbed i	into	a number (	wo ob	sorbing states? (Choose
	stage) that the system exit				.w0 a03	soluting states? (Choose
	<u>nearesi</u> value)	1.0	2	,		1.4
	a. 1	b. 2	c. 3	5		d. 4
	e. 5	t. 6	g. 7	/		h. 8 or more
<u>b</u>	5. For a discrete-time Mar	kov chain, let P be the n	natrix	x of transiti	on pro	babilities. The sum of
	each					
	a. column is 1	b. row is 1				
	c. column is 0	d. row is 0		e. NOT	A	

*Markov Chains*. Consider the discrete-time Markov chain diagrammed below:

## Version B--SOLUTIONS

<u>d</u> 6. An absorbing state of	a Markov chain is one ir	which the probability of	
a. moving into that st	ate is zero b. movin	g out of that state is one.	NOT
c. moving into that si	tate is one. $d.$ movin	g out of that state is zero	e. NOTA
$\underline{c}$ /. The minimal closed s	set(s) in the above Marko	v  chain =	$1 (1 2 2 4) \oplus (2 4)$
a. $\{1,2,3,4\}$	$\begin{array}{c} b. \{1,2\} \\ f. (1,2) \\ f. (2,4) \end{array}$	c. $\{3\} \& \{4\}$	a. $\{1,2,3,4\}$ & $\{3,4\}$
e. $\{1\} \notin \{2\}$	I. $\{1,2\} \ll \{3,4\}$	g. {3.4}	n. NOIA
<u><u>n</u> 8. The probability that t</u>	ne system reaches an abs	sording state, beginning in	i state 1, is (choose
nearest value).	a 0.5 d 0.6	207 f 08	a 0.0 h 1.0
a. $0.5$ $0.0.4$	in the above Markov ch	1.0.7	g. 0.9 II. 1.0
$\underline{a}$ 9. The transferit state(s)	h 2 & A	$a_{\rm HI} = 0.123 \ \text{\& } 4$	4 NOTA
a. $1 \propto 2$ b 10 The recurrent state(	$0.5 \times 4$ s) in the above Markov c	$1, 2, 3, \alpha 4$	u. NOTA
$\underline{0}$ 10. The recurrent state(	b $3 \& 4$	c 1 2 3 & 4	d NOTA
a. $1 \ll 2$	0. 5 & 4	$c. 1, 2, 3, \alpha +$	u. 10111
<u>a</u> 11. The quantity $J_{13}$ is	S		
a. the probability that	t the system, beginning in	n state 1, first reaches stat	e 3 in stage 4
b. the probability tha	t the system, beginning i	n state 1, is in state 3 at st	age 4
c. the expected numb	per of visits to state 3 dur	ing the first 4 stages, begi	nning in state1
d. the probability tha	t the system first visits st	ate 3 before state 4.	
e. the stage in which	the system, beginning in	state 1, visits state 3 for t	he fourth time
t. NOTA		. 11 1	
<u>b</u> 12. From which state is	the system more likely to	eventually reach state 4	
	b. 2	c. equally likely	d. NOTA
$\underline{c}$ 13. What is the probability $\underline{c}$	ity that the system is abso	orbed into state 3, starting	; in state 1, if the first
transition is to state $2?$ (a	choose nearest value)	a 400/	1 150/
a. $50\%$ or less	D. 33%	c. 40%	(1.45%)
e. 30%	1. JJ%0	g. 00%	n. 65% or more
$\underline{b}$ 14. The probability that	the system, starting in s	late 1, 15 III state 1 after 2	stages? (choose hearest
$\frac{50}{2}$ or lass	b 10%	0 15%	d 20%
a. $370 \text{ or ress}$	0.1070	c. 1370	u. $20\%$
f 15 The probability that	1. JU/0 the system starting in st	g. 5570 ata 1 jis in stata 1 aftar 3 s	11. 40/0 01 more
<u>1</u> 15: The probability that	the system, starting in st		stages! (choose neuresi
a 5% or less	h 10%	c 15%	d 20%
e. 25%	f 30%	σ 35%	h 40% or more
e 16 In general the stead	vstate probability distrib	ution $\pi$ (if it exists) must	satisfy
$\underline{\underline{v}}$ 10. In general, the stead	h $P\pi=1$	$c \pi P=1$	d $P\pi = \pi$
$a. T \mathcal{N} = \mathbf{\pi}$	f $\pi P=0$	$\sigma NOTA$	u. 1 // //
0. //1 //	1. //1 0	5. 10111	

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			A     3     4       1     0.611     0.389       2     0.444     0.556
(		<b>'</b> )	E         1         2 row sum           1         2.778         0.555         3.333           2         1.111         2.222         3.333
			n $f_{13}^{(n)}$ $f_{14}^{(n)}$ 1 0.2 0.1 2 0.13 0.08 3 0.087 0.06 4 0.0503 0.0436
P             1             2             3             4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
<u>b</u>	1. Which is the matrix	Q (used in computation	of E)?
	a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$	b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$	$\mathbf{c} \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$
	d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$e. \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	f. None of the above (NOTA)
<u>a</u>	2. Which is the matrix	R (used in computation of	of A)?
	a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$	b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$	$\begin{array}{c} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{array}$
	$\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$
	$\begin{bmatrix} 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$	
<u> </u>	3. If the system begins in	state #1, what is the pro	bability that it is absorbed into state #4? (Choose
	a. 30% or less	b. 35%	c. 40% d. 45%
	e. 50%	f. 55%	g. 60% h. 65% or more
<u> </u>	4. If the system begins in stage) that the system exit	state #1, what is the exp sts before it is absorbed i	pected number of stages (including the initial into one of the two absorbing states? <i>(Choose</i>
	<u>neuresi</u> vaiue) a 1	h 2	c 3 d 4
	e. 5	f. 6	g. 7 h. 8 or more
<u> </u>	5. For a discrete-time Mar each	kov chain, let P be the m	natrix of transition probabilities. The sum of
	a. column is 1	c. row is 1	
	b. column is 0	d. row is 0	e. NOTA

Markov Chains. Consider the discrete-time Markov chain diagrammed below:

<u>c</u>	_6. An absorbing state of	a Markov chain is one i	n which the probability of	of
	a. moving out of that	state is one c. movi	ng out of that state is zero	0. NOT (
	b. moving into that st	ate is one. d. $movi$	ng into that state is zero	e. NOIA
<u>e</u>	$_{-}$ /. The minimal closed s	et(s) in the above Mark	ov chain =	$\sim (1 2 2 4) \otimes (2 4)$
	a. $\{1, 2, 3, 4\}$ b. $(1)$ & $(2)$	C. $\{1,2\}$	e. $\{5\} \ll \{4\}$	g. $\{1,2,3,4\} \propto \{3,4\}$
h	0. $\{1\} \propto \{2\}$ 8 The requirement state(s)	$\begin{array}{c} \text{u. } \{1,2\} & \{3,4\} \\ \text{in the above Markov a} \end{array}$	1. $\{3.4\}$	II. NOTA
0	$2 \circ 1 \otimes 2$	h 2 & 1	$a_1 2 3 8 4$	4 NOTA
9	a. 1 $\propto 2$ Q The transient state(s)	$0.5 \times 4$	$1, 2, 3, \alpha 4$	u. NOTA
<u>a</u>	$2^{9}$ . The transient state(s)	h $3 \& 1$	c = 1 + 2 + 3 + 8 + 4	d NOTA
h	10 The probability that	the system reaches an $\epsilon$	bsorbing state beginnin	$\alpha$ in state 1 is (choose
<u> </u>	_ 10. The probability that nearest value).	the system reaches an a	iosoronig state, beginnin	g in state 1, is (choose
	a 03 b 04	c 0.5 d 0.6	e 07 f 08	g 0 9 h 1 0
d	11 The quantity $f^{(4)}$ is			8
<u> </u>	_ 11. The quality $f_{13}$ is	<b>)</b>		
	a. the probability that	the system, beginning	in state 1, is in state 3 at	stage 4
	b. the probability that	t the system first visits s	state 3 before state 4.	ainning in state 1
	c. the expected numb	er of visits to state 5 du	ring the first 4 stages, be	ginning in state i
	a. the grobability that	the system, beginning	n state 1, mist reaches si	ate 5 III stage 4
	f $NOTA$	the system, beginning h	Il state 1, visits state 5 10	t the fourth time
h	12 From which state is t	the system more likely.	to eventually reach state.	49
_0_	a 1	h 2	c equally likely	d NOTA
f	13. What is the probabili	ty that the system is abs	sorbed into state 4. startir	in state 1, if the first
_ <del>_</del>	transition is to state 2? (a	choose nearest value)		
	a. 30% or less	b. 35%	c. 40%	d. 45%
	e. 50%	f. 55%	g. 60%	h. 65% or more
h	14. The probability that	the system, starting in	state 1, is in state 1 after	2 stages? (choose nearest
	value)		·	
	a. 5% or less	b. 10%	c. 15%	d. 20%
	e. 25%	f. 30%	g. 35%	h. 40% or more
<u>e</u>	_15. The probability that	the system, starting in a	state 1, is in state 4 after	3 stages? (choose nearest
	value)			
	a. 5% <i>or less</i>	b. 10%	c. 15%	d. 20%
	e. 25%	f. 30%	g. 35%	h. 40% or more
<u>_f</u>	16. In general, the steady	ystate probability distrib	oution $\pi$ (if it exists) mus	st satisfy
	a. Pπ=0	c. Pπ=1	e. $\pi P=1$	
	b. Pπ=π	d. πP=0	f. $\pi P = \pi$	g. NOTA