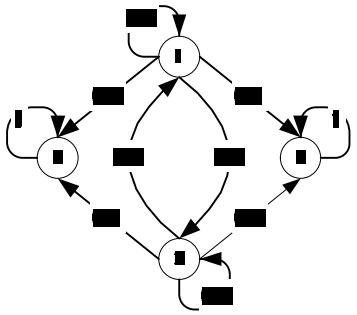
56:171 Operations Research
Quiz #9 – November 8, 2002

Markov Chains. Consider the discrete-time Markov chain diagrammed below:



P	1	2	3	4
1	0.3	0.4	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

A	3	4
1	0.518	0.482
2	0.407	0.593

E	1	2	row sum
1	1.851	1.482	3.333
2	0.741	2.592	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.1	0.11
3	0.066	0.081
4	0.0458	0.0571
5	0.03202	0.04001
6	0.02241	0.028011
7	0.0156866	0.0196081
8	0.0109806	0.0137257
9	0.0076864	0.009608
10	0.00538048	0.0067256

_b_ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_a_ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *NOTA*

_e_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #4? (Choose nearest value)

a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

_c_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? *(Choose nearest value)*

a. 1 b. 2 c. 3 d. 4
e. 5 f. 6 g. 7 h. 8 *or more*

__c_ 5. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...

a. column is 1 c. row is 1
b. column is 0 d. row is 0 e. *NOTA*

__a_ 6. An absorbing state of a Markov chain is one in which the probability of
a. moving out of that state is zero c. moving out of that state is one.
b. moving into that state is one. d. moving into that state is zero e. *NOTA*

__g_ 7. The minimal closed set(s) in the above Markov chain =
a. {1,2,3,4} b. {1,2} c. {3.4} d. {1,2,3,4} & {3,4}
e. {1} & {2} f. {1,2} & {3,4} g. {3} & {4} h. *NOTA*

_h__ 8. The probability that the system reaches an absorbing state, beginning in state 1, is *(choose nearest value):*
a. 0.3 b. 0.4 c. 0.5 d. 0.6 e. 0.7 f. 0.8 g. 0.9 h. 1.0

__b_ 9. The recurrent state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__a_ 10. The transient state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__e_ 11. The quantity $f_{13}^{(4)}$ is
a. the probability that the system, beginning in state 1, is in state 3 at stage n
b. the probability that the system first visits state 3 before state 4.
c. the expected number of visits to state 3 during the first 4 stages, beginning in state1
d. the stage in which the system, beginning in state 1, visits state 3 for the fourth time
e. the probability that the system, beginning in state 1, first reaches state 3 in stage 4
f. *NOTA*

_b__ 12. From which state is the system more likely to eventually reach state 4?
a. 1 b. 2 c. equally likely d. *NOTA*

_g_ 13. What is the probability that the system is absorbed into state 4, starting in state 1, if the first transition is to state 2? *(choose nearest value)*
a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

__g_ 14. The probability that the system, starting in state 1, is in state 1 after 2 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*

__c_ 15. The probability that the system, starting in state 1, is in state 4 after 3 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*
a. 0.1 b. 0.2 c. 0.3 d. 0.4 e. 0.5 f. 0.6 g. 0.7 h. 0.8

__d_ 16. In general, the steadystate probability distribution π (if it exists) must satisfy
a. $P\pi=0$ b. $P\pi=1$ c. $\pi P=1$ d. $\pi P=\pi$
e. $P\pi=\pi$ f. $\pi P=0$ g. *NOTA*

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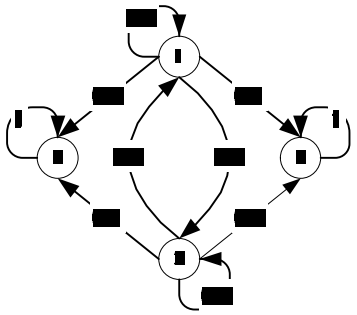

P	1	2	3	4
1	0.4	0.3	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

A	3	4
1	0.542	0.458
2	0.417	0.583

E	1	2	row sum
1	2.083	1.25	3.333
2	0.833	2.5	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.11	0.1
3	0.071	0.076
4	0.0485	0.0544
5	0.03371	0.03832
6	0.023549	0.026872
7	0.0164747	0.01882
8	0.0115304	0.0131759
9	0.00807088	0.00922353
10	0.00564954	0.00645655

_a_ 1. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_b_ 2. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_d_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #4? (Choose nearest value)

a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

_c_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? *(Choose nearest value)*

a. 1 b. 2 c. 3 d. 4
e. 5 f. 6 g. 7 h. 8 *or more*

_b_ 5. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...

a. column is 1 b. row is 1
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a. moving into that state is zero b. moving out of that state is one.
c. moving into that state is one. d. moving out of that state is zero e. *NOTA*

__c_ 7. The minimal closed set(s) in the above Markov chain =
a. {1,2,3,4} b. {1,2} c. {3} & {4} d. {1,2,3,4} & {3,4}
e. {1} & {2} f. {1,2} & {3,4} g. {3.4} h. *NOTA*

__h_ 8. The probability that the system reaches an absorbing state, beginning in state 1, is *(choose nearest value):*
a. 0.3 b. 0.4 c. 0.5 d. 0.6 e. 0.7 f. 0.8 g. 0.9 h. 1.0

__a_ 9. The transient state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

_b__ 10. The recurrent state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

_a__ 11. The quantity $f_{13}^{(4)}$ is
a. the probability that the system, beginning in state 1, first reaches state 3 in stage 4
b. the probability that the system, beginning in state 1, is in state 3 at stage 4
c. the expected number of visits to state 3 during the first 4 stages, beginning in state1
d. the probability that the system first visits state 3 before state 4.
e. the stage in which the system, beginning in state 1, visits state 3 for the fourth time
f. *NOTA*

__b_ 12. From which state is the system more likely to eventually reach state 4?
a. 1 b. 2 c. equally likely d. *NOTA*

__c_ 13. What is the probability that the system is absorbed into state 3, starting in state 1, if the first transition is to state 2? *(choose nearest value)*
a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

__b_ 14. The probability that the system, starting in state 1, is in state 1 after 2 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*

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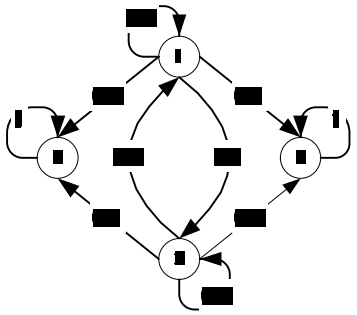

P	1	2	3	4
1	0.6	0.1	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

A	3	4
1	0.611	0.389
2	0.444	0.556

E	1	2	row sum
1	2.778	0.555	3.333
2	1.111	2.222	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.13	0.08
3	0.087	0.06
4	0.0593	0.0436
5	0.04087	0.03116
6	0.028353	0.022068
7	0.0197447	0.01555
8	0.0137803	0.010926
9	0.00962985	0.00766456
10	0.00673434	0.00537174

_b_ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_a_ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

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b. $P\pi=\pi$ d. $\pi P=0$ f. $\pi P=\pi$ g. *NOTA*