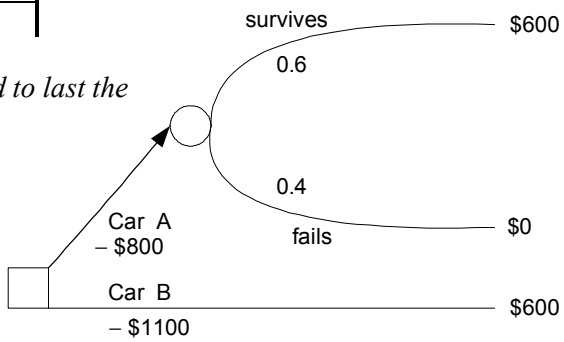
56:171 Operations Research
Quiz #8 Solution –Fall 2002

Suppose that you are in the position of having to buy a used car, and you have narrowed down your choices to two possible models: one car (A) a private sale and the other (B) is from a dealer. You must now choose between them. The cars are similar, and the only criterion is to minimize expected cost. The dealer car is more expensive, but it comes with a one-year warranty which would cover all costs of repairs. You decide that, if car A will last for 1 year, you can sell it again and recover a large part of your investment. If it falls apart, it will not be worth fixing. After test driving both cars and checking for obvious flaws, you make the following evaluation of probable resale value:

Car	Purchase price	Probability of lasting one year	Estimated resale price
A: Private	$800	0.6	$600
B: Dealer	$1100	1.0	$600

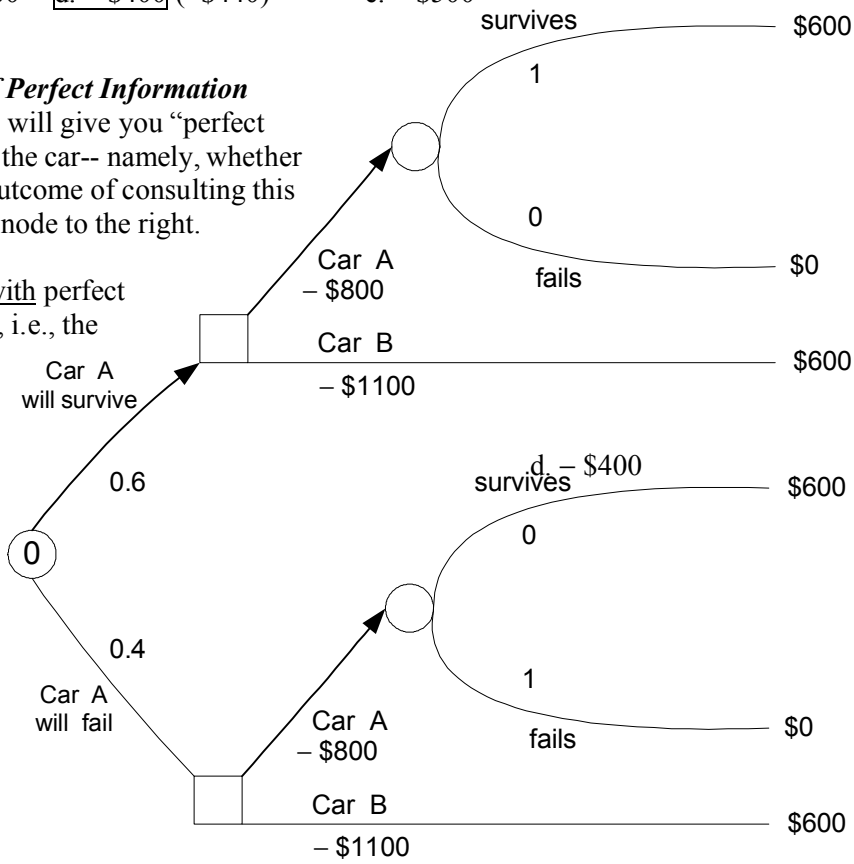
Note that car B, because of the warranty, is guaranteed to last the year and bring full resale price!



_a 1. Which car should you buy?
a. Car A b. Car B

_d 2. What is the expected cost for the year, after resale of the car? This is the expected value without information (about the future), denoted **EVWOI**.
(choose nearest value)
a. – $100 b. – $200 c. – $300 d. – $400 (–$440) e. – $500

Calculation of Expected Value of Perfect Information
Suppose that a "seer" of the future will give you "perfect information" before you purchase the car-- namely, whether car A will survive the year. The outcome of consulting this seer is the initial (left-most) event node to the right.



_c 3. What is the expected value with perfect information (denoted **EVWPI**), i.e., the expected value at node 0?
(nearest value)
a. – $100 b. – $200
c. – $300 (–$320) d. – $400

_a 4. What is the expected value of perfect information (**EVPI**)? *(nearest value)*
a. $100 (= $120) b. $200
c. $300 d. $400
e. $500

For $50, you may take car A to an independent mechanic, who will do a complete inspection and offer you an opinion as to whether the car will last 1 year. For various subjective reasons, you assign the following probabilities to the accuracy of the mechanic's opinion:

Given:	Mechanic says Yes	Mechanic says No
A car that will survive 1 year	90%	10%
A car that will fail in next year	30%	70%

That is, the mechanic is 90% likely to correctly identify a car that will survive the year, but only 70% likely to correctly identify a car that will fail.

Let **AS** and **AF** be the "states of nature", namely "car **A S**urvives" and "car **A F**ails" during the next year, respectively.
Let **PR** and **NR** be the outcomes of the mechanic's inspection, namely "**P**ositive **R**eport" and "**N**egative **R**eport", respectively.

Bayes' Rule states that if S_i are the states of nature and O_j are the outcomes of an experiment,

$$P\{S_i \mid O_j\} = \frac{P\{O_j \mid S_i\} \times P\{S_i\}}{P\{O_j\}}$$

$$\text{where } P\{O_j\} = \sum_i P\{O_j \mid S_i\} \times P\{S_i\}$$

The probability that the mechanic will give a postive report, i.e., $P\{PR\}$ is

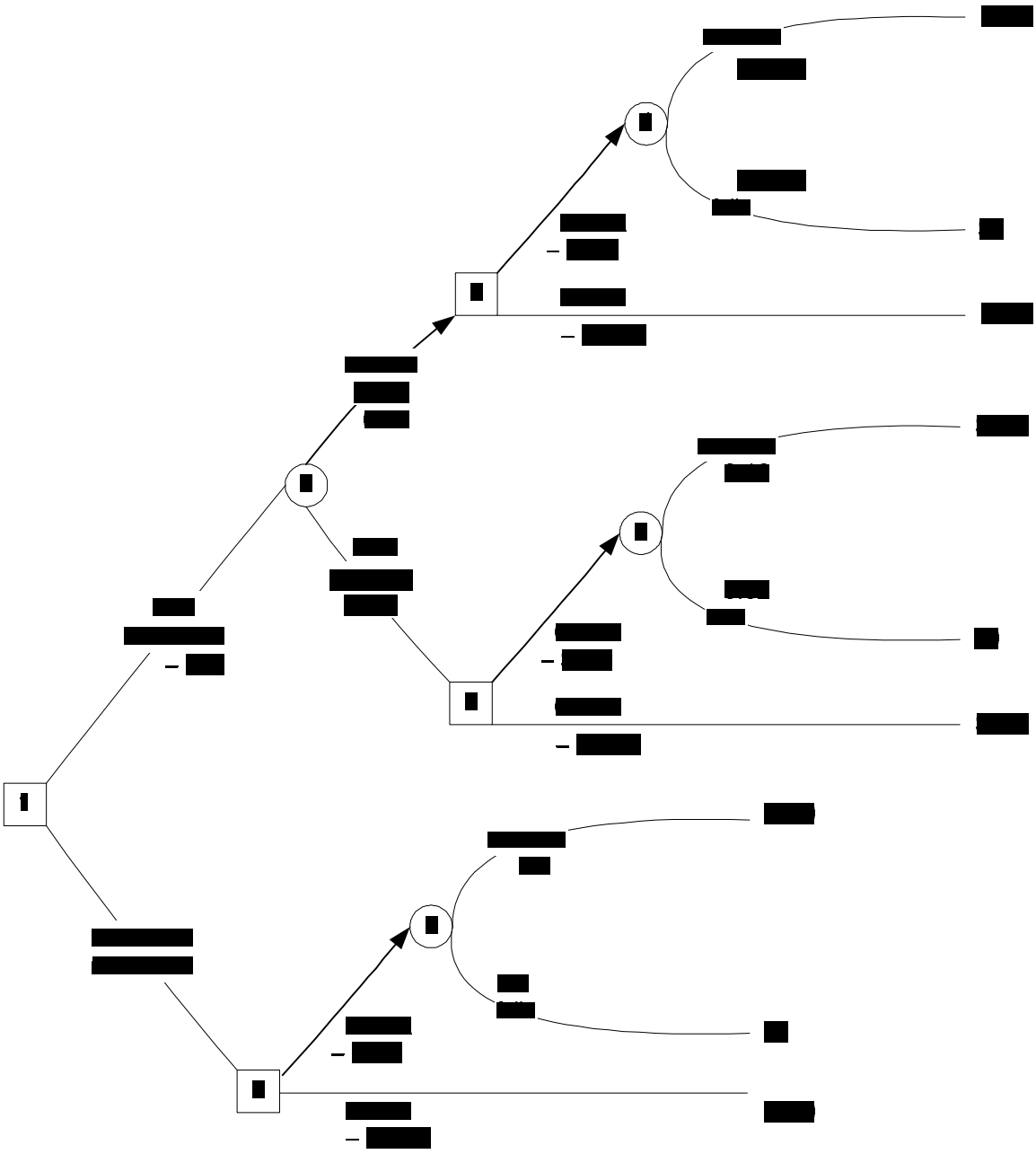
$$P\{PR\} = P\{PR \mid AS\}P\{AS\} + P\{PR \mid AF\}P\{AF\} = (0.9)(0.6) + (0.3)(0.4) = 0.66$$

_e 5. According to Bayes' theorem, the probability that car A will survive, given that the mechanic gives a positive report, i.e., $P\{AS \mid PR\}$, is *(choose nearest value)*:

a. 0.6 b. 0.65 c. 0.7 d. 0.75 e. 0.8 (0.818) f. 0.85 g. 0.9 h. 0.95

The decision tree on the next page includes your decision as to whether or not to hire the mechanic.

6. Insert P{AS|PR}, i.e., the probability that car A survives if the mechanic gives a positive report, and P{AF|PR} on the appropriate branches of the tree.



7. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value		Node	Value		Node	Value
1	**– $424**		4	**– $492**		7	**– $440**
2	– $374		5	– $500		8	$360
3	**– $308**		6	$108			

8. Should you hire the mechanic? *Circle:* [Yes] No

__e__ 9. The expected value of the mechanic's opinion is *(Choose nearest value)*:
a. $0 b. $15 c. $30 d. $45 [e. $60] ($66) f. $75 g. $90 h. $105

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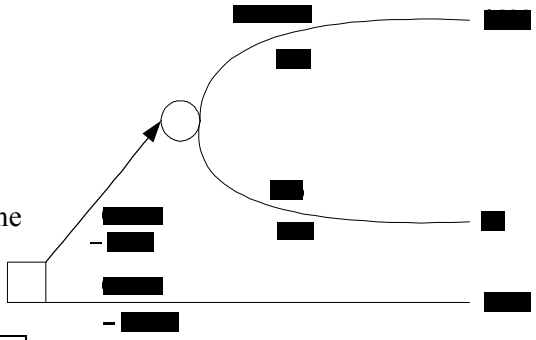
Quiz #8 – 1 November 2002

Suppose that you are in the position of having to buy a used car, and you have narrowed down your choices to two possible models: one car (A) a private sale and the other (B) is from a dealer. You must now choose between them. The cars are similar, and the only criterion is to minimize expected cost. The dealer car is more expensive, but it comes with a one-year warranty which would cover all costs of repairs. You decide that, if car A will last for 1 year, you can sell it again and recover a large part of your investment. If it falls apart, it will not be worth fixing. After test driving both cars and checking for obvious flaws, you make the following evaluation of probable resale value:

Car	Purchase price	Probability of lasting one year	Estimated resale price
A: Private	$900	0.5	$600
B: Dealer	$1100	1.0	$600

Note that car B, because of the warranty, is guaranteed to last the year and bring full resale price!

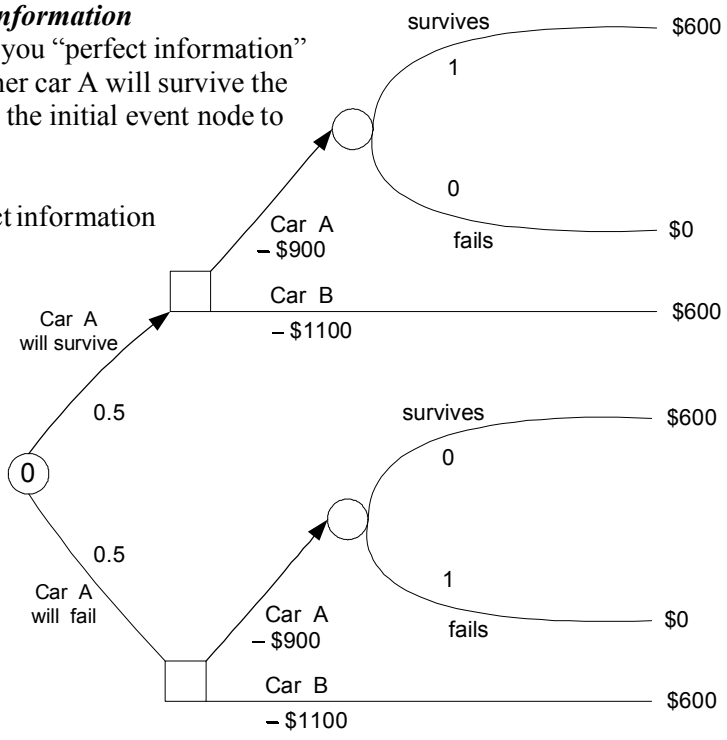


_b 1. Which car should you buy?
a. Car A b. Car B

_e 2. What is the expected cost for the year, after resale of the car? This is the expected value without information (about the future), denoted **EVWOI**.
(choose nearest value)
a. – $100 b. – $200 c. – $300 d. – $400 e. – $500

Calculation of Expected Value of Perfect Information

Suppose that a "seer" of the future will give you "perfect information" before you purchase the car-- namely, whether car A will survive the year. The outcome of consulting this seer is the initial event node to the right.



_d 3. What is the expected value with perfect information (denoted **EVWPI**)? *(nearest value)*
a. – $100 b. – $200
c. – $300 d. – $400

_a 4. What is the expected value of perfect information (**EVPI**)?
a. – $100 b. – $200
c. – $300 d. – $400
e. – $500

For $50, you may take car A to an independent mechanic, who will do a complete inspection and offer you an opinion as to whether the car will last 1 year. For various subjective reasons, you assign the following probabilities to the accuracy of the mechanic's opinion:

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$$\text{where } P\{O_j\} = \sum_i P\{O_j \mid S_i\} \times P\{S_i\}$$

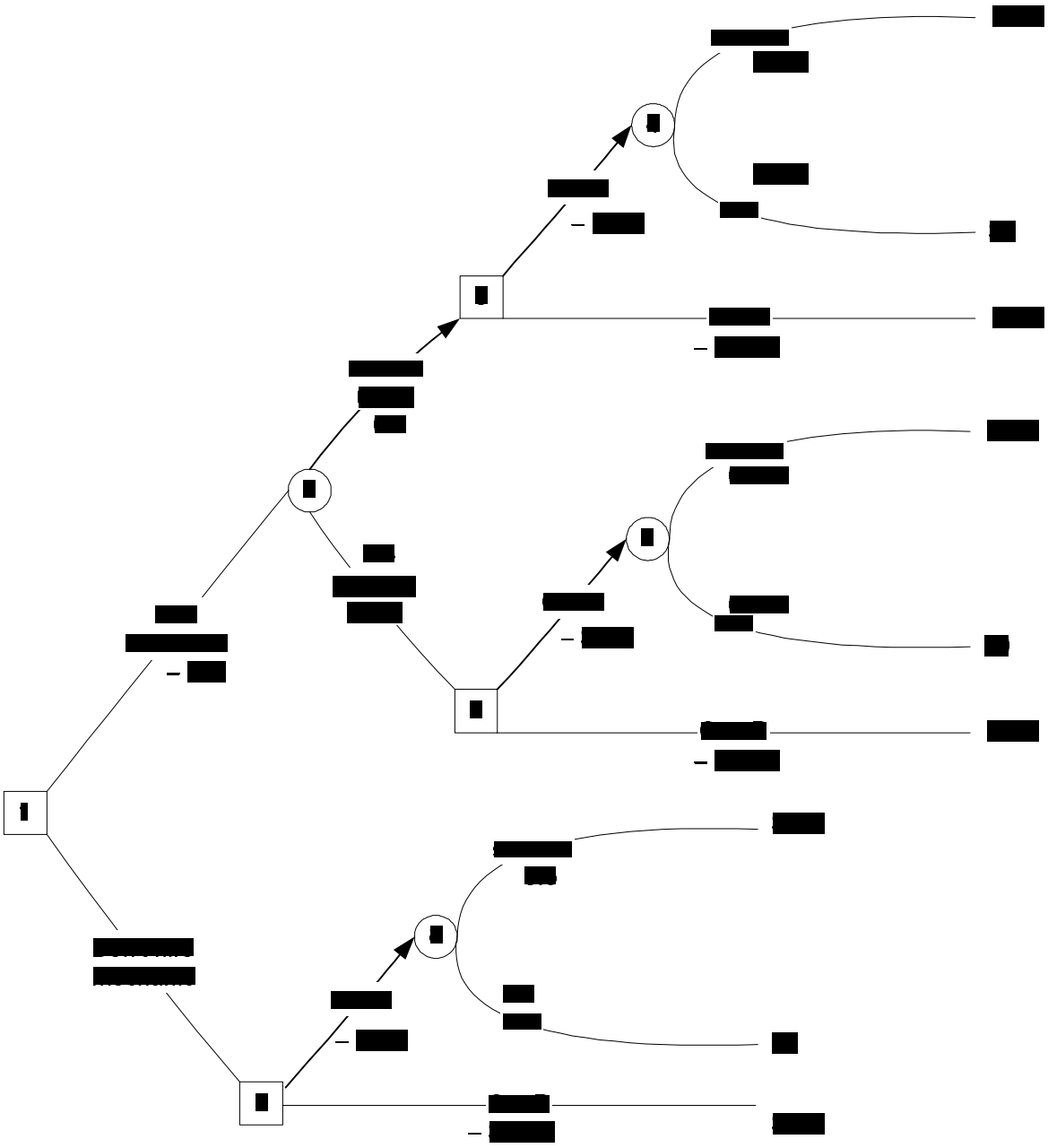
The probability that the mechanic will give a postive report, i.e., $P\{PR\}$ is

$$P\{PR\} = P\{PR \mid AS\}P\{AS\} + P\{PR \mid AF\}P\{AF\} = (0.9)(0.5) + (0.3)(0.5) = 0.6$$

_d 5. According to Bayes' theorem, the probability that car A will survive, given that the mechanic gives a positive report, i.e., $P\{AS \mid PR\}$, is *(choose nearest value)*:

a. 0.6 b. 0.65 c. 0.7 d. 0.75 e. 0.8 f. 0.85 g. 0.9

6. The decision tree below includes your decision as to whether or not to hire the mechanic. Insert P{AS|PR} and P{AF|PR} on the appropriate branches.



7. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value		Node	Value		Node	Value
1	**– $500**		4	**– $450**		7	**– $500**
2	– $470		5	– $500		8	$300
3	**– $450**		6	$75			

8. Should you hire the mechanic? *Circle:* Yes [No]

____ 9. The expected value of the mechanic's opinion is *(Choose nearest value)*:
a. $0 b. $15 c. $30 d. $45 e. $60 f. $75 g. $90 h. $105