56:171 Operations Research
Quiz #5 Version **A** Solution -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	|12 **0 & basic**	|8 **7**	|9	|10	|11 **2**	9
B	|10	|11 *nonbasic*	|12	|11	|14 **7**	7
C	|9 **4**	|7	|11 *nonbasic*	|14 **1**	|8	5
D	|13 *nonbasic*	|12	|13 **2**	|12 **5**	|12	7
E	|8 *nonbasic*	|9	|10 **3**	|9	|10	3
Demand=	4	7	5	6	9	

_a_1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?

a. X_{A1} c. X_{B2} e. X_{E1}
b. X_{D1} d. X_{C3} f. None of above

_d_2. If we choose to assign dual variable $U_D=5$, what must be the value of dual variable V_3?

a. 0 c. 6 e. 11
b. 3 d. 8 f. None of above

_e_3. If, in addition to $U_D=5$, we determine that $V_2=-2$, then the reduced cost of X_{D2} is

a. 0 c. +5 e. +9
b. −3 d. −8 f. None of above

_a_4. If we initially assign $U_D=0$ (rather than $U_D=5$) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.

a. True b. False c. Cannot be determined

_e_5. Suppose $\mathbf{X}_{E1}$ enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?

a. $\mathbf{X}_{A2}$ c. $\mathbf{X}_{C4}$ e. $\mathbf{X}_{E3}$
b. $\mathbf{X}_{C1}$ d. $\mathbf{X}_{D3}$ f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**0**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the nine zeroes are "covered".

_c_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be
- a. 7
- b. 8
- c. 9
- d. 10
- e. 11
- f. None of the above

Consider the reduced assignment problem shown below:

\	1	2	3	4	5
1	0	0	3	1	3
2	1	2	0	2	0
3	4	3	5	5	0
4	1	0	5	3	5
5	0	2	1	0	11

_a_ 2. A zero-cost solution of the problem with this cost matrix must have
- a. $X_{11}=1$
- b. $X_{42}=0$
- c. $X_{12}=1$
- d. $X_{25}=1$
- e. $X_{54}=0$
- f. None of the above

Part 3. True(+) or False(o) ?

_+_ 1. The transportation problem is a special case of a linear programming problem.
_o_ 2. The Hungarian algorithm can be used to solve a transportation problem.
_+_ 3. Every basic feasible solution of an assignment problem must be degenerate.
_o_ 4. If the current basic solution of a transportation problem is degenerate, no improvement of the objective function will occur in the next iteration of the (transportation) simplex method.
_o_ 5. The Hungarian algorithm is the simplex method specialized to the assignment problem.
_+_ 6. At each iteration of the Hungarian method, the original cost matrix is replaced with a new cost matrix having the same optimal assignment.

56:171 Operations Research
Quiz #5 Version B -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	|12 **0 & basic**	|8 **7**	|9	|10	|11 **2**	9
B	|10	|11 *nonbasic*	|12	|11	|14 **7**	7
C	|9 **4**	|7	|11 *nonbasic*	|14 **1**	|8	5
D	|13 *nonbasic*	|12	|13 **2**	|12 **5**	|12	7
E	|8 *nonbasic*	|9	|10 **3**	|9	|10	3
Demand=	4	7	5	6	9	

_c_1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?

a. XB2 c. XA1 e. XE1
b. XC3 d. XD1 f. None of above

_d_2. If we choose to assign dual variable U_D=5, what must be the value of dual variable V_3?

a. 0 c. 6 e. 11
b. 3 d. 8 f. None of above

_e_3. If, in addition to U_D=5, we determine that $V_2 = -2$, then the reduced cost of X_{D2} is

a. 0 c. +5 e. +9
b. −3 d. −8 f. None of above

_a_4. If we initially assign U_D=0 (rather than U_D=5) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.

a. True b. False c. Cannot be determined

_e_5. Suppose X_{E1} enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?

a. XC1 c. XA2 e. XE3
b. XC4 d. XD3 f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**1**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the nine zeroes are "covered".

_c_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be

a. 7 c. 9 e. 11
b. 8 d. 10 f. None of the above

Consider the reduced assignment problem shown below:

\	1	2	3	4	5
1	0	0	3	1	3
2	1	2	0	2	0
3	4	3	5	5	0
4	1	0	5	3	5
5	0	2	1	0	11

_a_ 2. A zero-cost solution of the problem with this cost matrix must have

a. $X_{11}=1$ c. $X_{12}=1$ e. $X_{54}=0$
b. $X_{42}=0$ d. $X_{25}=1$ f. None of the above

Part 3. True(+) or False(o) ?

_+_ 1. The assignment problem is a special case of an transportation problem.
_o_ 2. The number of basic variables in a n×n assignment problem is 2n.
_o_ 3. The dual variables of the transportation problem are uniquely determined at each iteration of the simplex method.
_+_ 4. The transportation simplex method can be applied to solution of an assignment problem.
_o_ 5. The optimal dual variables of the transportation problem obtained at the final iteration must be nonnegative.
_o_ 6. A "balanced" transportation problem has an equal number of sources and destinations.

56:171 Operations Research
Quiz #5 Version C -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	|12 **0 & basic**	|8 **7**	|9	|10	|11 **2**	9
B	|10	|11 *nonbasic*	|12	|11	|14 **7**	7
C	|9 **4**	|7	|11 *nonbasic*	|14 **1**	|8	5
D	|13 *nonbasic*	|12	|13 **2**	|12 **5**	|12	7
E	|8 *nonbasic*	|9	|10 **3**	|9	|10	3
Demand=	4	7	5	6	9	

_d_ 1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?

a. X_{D1} b. X_{C3} c. X_{E1}
d X_{A1} e. X_{B2} f. None of above

_e_ 2. If we choose to assign dual variable U_D=5, what must be the value of dual variable V_3?

a. 0 b. 6 c. 11
d. 3 e. 8 f. None of above

_c_ 3. If, in addition to U_D=5, we determine that V_2= −2 , then the reduced cost of XD2 is

a. 0 b. +5 c. +9
d. −3 e. −8 f. None of above

_a_ 4. If we initially assign U_D=0 (rather than U_D=5) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.

a. True b. False c. Cannot be determined

_c_ 5. Suppose X_{E1} enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?

a. X_{A2} b. X_{C4} c. X_{E3}
d. X_{C1} e. X_{D3} f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**1**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the **eight** zeroes are "covered".

_b_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be

a. 7 b. 9 c. 11
d. 8 e. 10 f. None of the above

Consider the reduced assignment problem shown below:

```
\| 1  2  3  4  5
1| 0  0  3  1  3
2| 1  2  0  2  0
3| 4  3  5  5  0
4| 1  0  5  3  5
5| 0  2  1  0 11
```

_a_ 2. A zero-cost solution of the problem with this cost matrix must have

a. $X_{11}=1$ b. $X_{12}=1$ c. $X_{54}=0$
d. $X_{42}=0$ e. $X_{25}=1$ f. None of the above

Part 3. True(+) or False(o) ?

_o_ 1. The Hungarian algorithm can be used to solve a transportation problem.
_o_ 2. The number of basic variables in a n×n assignment problem is 2n.
_o_ 3. If the current basic solution of a transportation problem is degenerate, no improvement of the objective function will occur in the next iteration of the (transportation) simplex method.
_o_ 4. The Hungarian algorithm is the simplex method specialized to the assignment problem.
_+_ 5. At each iteration of the Hungarian method, the original cost matrix is replaced with a new cost matrix having the same optimal assignment.
_+_ 6. A "balanced" transportation problem has an equal number of supply and demand.