56:171 Operations Research
Quiz #4 -- 27 September 2002

"A manufacturer produces two types of plastic cladding. These have the trade names Ankalor and Beslite.

- One yard of Ankalor requires 8 lb of polyamine, 2.5 lb of diurethane and 2 lb of monomer.
- A yard of Beslite needs 10 lb of polyamine, 1 lb of diurethane, and 4 lb of monomer.
- The company has in stock 80,000 lb of polyamine, 20,000 lb of diurethane, and 30,000 lb of monomer.
- Both plastics can be produced by alternate parameter settings of the production plant, which is able to produce cladding at the rate of 12 yards per hour.
- A total of 750 production plant hours are available for the next planning period.
- The contribution to profit on Ankalor is $10/yard and on Beslite is $20/yard.
- The company has a contract to deliver at least 3,000 yards of Ankalor.

What production plan should be implemented in order to maximize the contribution to the firm's profit from this product division."

Definition of variables: A = Number of yards of Ankalor produced
B = Number of yards of Beslite produced

LP model:

1) Maximize $10 A + 20 B$ subject to
2) $8 A + 10 B \leq 80{,}000$ (lbs. Polyamine available)
3) $2.5 A + 1 B \leq 20{,}000$ (lbs. Diurethane available)
4) $2 A + 4 B \leq 30{,}000$ (lbs. Monomer available)
5) $A + B \leq 9{,}000$ (lbs. Plant capacity)
6) $A \geq 3{,}000$ (Contract)

$A \geq 0, B \geq 0$

The LINDO solution is:

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         OBJECTIVE FUNCTION VALUE
             1)   142000.000
        VARIABLE         VALUE     REDUCED COST
            A          3000.000        0.000
            B          5600.000        0.000

        ROW         SLACK OR SURPLUS  DUAL PRICES
         2)                0.000          2.000
         3)             6900.000          0.000
         4)             1600.000          0.000
         5)              400.000          0.000
         6)                0.000         -6.000

 RANGES IN WHCH THE BASIS IS UNCHANGED
                           OBJ COEFFICIENT RANGES
     VARIABLE     CURRENT    ALLOWABLE   ALLOWABLE
                    COEF      INCREASE    DECREASE
        A          10.000       6.000    INFINITY
        B          20.000     INFINITY      7.500

                          RIGHTHAND SIDE RANGES
    ROW           CURRENT       ALLOWABLE     ALLOWABLE
                    RHS          INCREASE      DECREASE
      2          80000.000       4000.000      56000.000
      3          20000.000       INFINITY       6900.000
      4          30000.000       INFINITY       1600.000
      5           9000.000       INFINITY        400.000
      6           3000.000       2000.000       1333.333
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THE TABLEAU
 ROW (BASIS)    A      B     SLK 2 SLK 3  SLK 4  SLK 5   SLK 6     RHS
  1    ART     .00    .00    2.00   .00    .00    .00    6.00       0.14E+06
  2    B       .00   1.00     .10   .00    .00    .00     .80     5600.00
  3    SLK 3   .00    .00    -.10  1.00    .00    .00    1.70     6900.00
  4    SLK 4   .00    .00    -.40   .00   1.00    .00   -1.20     1600.00
  5    SLK 5   .00    .00    -.10   .00    .00   1.00     .20      400.00
  6    A      1.00    .00     .00   .00    .00    .00   -1.00     3000.00
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*Consult the LINDO output above to answer the following questions. If there is not sufficient information in the LINDO output, answer "**NSI**".*

_b_ 1. Suppose that the company can purchase 2000 pounds of additional polyamine for $2.50 per pound. Should they make the purchase? a. yes [b. no] c. *NSI*
(since the dual variable for row [2] is only $2.00/lb.)

_a_ 2. Regardless of your answer in (4), suppose that they do purchase 2000 pounds of additional polyamine. Increasing the quantity of polyamine used in the model above is equivalent to
[a. decreasing the slack in row 2 by 2000] d. decreasing surplus in row 2 by 2000
b. increasing the surplus in row 2 by 2000 e. none of the above
c. increasing the slack in row 2 by 2000 f. NSI
(since 8A+10B+SLK2 = 80000 & 8A+10B=82000 ⇒ SLK2 = –2000.)

_c_ 3. If the company purchases 2000 pounds of additional polyamine, what is the total amount of Ankelor that they should deliver? (Choose nearest value!)
a. 2800 yards [c. 3000 yards] *unchanged!* e. 3200 yards
b. 2900 yards d. 3100 yards f. *NSI*
(substitution rate of SLK2 for A is 0, so A is unchanged as SLK2 decreases, up to ALLOWABLE INCREASE for RHS of row#2, i.e., 4000.)

_d_ 4. If the company purchases 2000 pounds of additional polyamine, what is the total amount of Beslite that they should deliver? (Choose nearest value!)
a. 5500 yards c. 5700 yards e. 5900 yards
b. 5600 yards d. 5800 yards f. *NSI*
(substitution rate of SLK2 for B is 0.10, so B increases by 0.10 for each pound decrease in SLK2)

_c_ 5. How will the decision to purchase 2000 pounds of additional polyamine change the quantity of diurethane used during the next planning period?
a. increase by 100 pounds [c. increase by 200 pounds] e. none of the above
b. decrease by 100 pounds d. decrease by 200 pounds f. *NSI*
(substitution rate of SLK2 for SLK3 is -0.10, so SLK3 decreases by 0.10lb for each pound decrease in SLK2 ⇒ 2.5A+1B *(i.e., amount of diurethane used) increases by 200 pounds.)*

_b_ 6. If the profit contribution from Beslite were to decrease from $20 to $13/yard, will the optimal values of A &/or B change? a. yes [b. no] c. *NSI*
(Since decrease of $7 is less than ALLOWABLE DECREASE which is $7.50.)

_a_ 7. If the profit contribution from Ankelor were to increase from $10 to $17/yard, will the optimal values of A &/or B change? [a. yes] b. no c. *NSI*
(Since increase of $7 is greater than ALLOWABLE INCREASE which is $6.)

_a_ 8. Suppose that the company could deliver 1000 yards less than the contracted amount of Ankalor if they were to pay a penalty of $5/yard shortage.
Should they do so? [a. yes] b. no c. *NSI*
(Since dual variable for row [6] is –$6.00, profit would decrease by $6 for every pound increase, or decrease by $6 for every pound decrease in requirements.)

_b_ 9. Is the optimal solution of this LP *degenerate*? a. yes [b. no] c. *NSI*
(No zero appears on RHS of optimal tableau.)

_b_ 10. Are there multiple optimal solutions of this LP? a. yes [b. no] c. *NSI*
(No zero appears in objective row (1) of any nonbasic column of tableau.)