56:171 Operations Research
Quiz #12 – Fall 2002

Determinstic Production Planning

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.
- the cost of *production* is **$7** for setup, plus **$3** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the <u>beginning</u> of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	3	1	1	3	2	1
Produce	3	0	0	3	3	0

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$2** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $ **54.00**

(b.) What should be the production quantity for Monday? __3___

(c.) What is the total cost (production + storage − salvage value) of the optimal production schedule for all six days? **$54.00**

(d.) Three values have been blanked out in the computer output, What are they?
- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A) $21.00** *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $ 40.00**
- the cost associated with the decision to produce 3 units on Monday, when there is initially one unit in stock. **(C.) $ 57.00**

(e.) Complete the last row of the table above, indicating the optimal production quantity each day.

```
 s \ x: 0      1      2      3      4  | Minimum
 0  | 999.99  10.00  11.00  12.00  13.00|  10.00
 1  |   1.00   9.00  10.00  11.00  12.00|   1.00
 2  |   0.00   8.00   9.00  10.00  11.00|   0.00
 3  |  ⁻1.00   7.00   8.00   9.00  10.00|  ⁻1.00
 4  |  ⁻2.00   6.00   7.00   8.00 999.99|  ⁻2.00
 5  |  ⁻3.00   5.00   6.00 999.99 999.99|  ⁻3.00
 6  |  ⁻4.00   4.00 999.99 999.99 999.99|  ⁻4.00
```

Stage 6 (Saturday)

```
  s \ x:  0      1      2      3      4    | Minimum
  0  | 999.99 999.99  23.00  17.00  19.00|  17.00
  1  | 999.99 [  A   ] 15.00  17.00  19.00|  15.00
  2  |  12.00  13.00  15.00  17.00  19.00|  12.00
  3  |   4.00  13.00  15.00  17.00  19.00|   4.00
  4  |   4.00  13.00  15.00  17.00  19.00|   4.00
  5  |   4.00  13.00  15.00  17.00 999.99|   4.00
  6  |   4.00  13.00  15.00 999.99 999.99|   4.00
```

Day 5 (Friday)

```
  s \ x:  0      1      2      3      4    | Minimum
  0  | 999.99 999.99 999.99  33.00  34.00|  33.00
  1  | 999.99 999.99  31.00  32.00  32.00|  31.00
  2  | 999.99  29.00  30.00  30.00  25.00|  25.00
  3  |  20.00  28.00  28.00  23.00  26.00|  20.00
  4  |  19.00  26.00  21.00  24.00  27.00|  19.00
  5  |  17.00  19.00  22.00  25.00  28.00|  17.00
  6  |  10.00  20.00  23.00  26.00 999.99|  10.00
```

Day 4 (Thursday)

```
  s \ x:  0      1      2      3      4    | Minimum
  0  | 999.99  43.00  44.00  41.00  39.00|  39.00
  1  |  34.00  42.00  39.00  37.00  39.00|  34.00
  2  |  33.00  37.00  35.00  37.00  38.00|  33.00
  3  |  28.00  33.00  35.00  36.00  32.00|  28.00
  4  |  24.00  33.00  34.00  30.00 999.99|  24.00
  5  |  24.00  32.00  28.00 999.99 999.99|  24.00
  6  |  23.00  26.00 999.99 999.99 999.99|  23.00
```

Day 3 (Wednesday)

```
  s \ x:  0      1      2      3      4    | Minimum
  0  | 999.99  49.00  47.00  49.00  47.00|  47.00
  1  |  40.00  45.00  47.00  45.00  44.00| [  B   ]
  2  |  36.00  45.00  43.00  42.00  45.00|  36.00
  3  |  36.00  41.00  40.00  43.00  45.00|  36.00
  4  |  32.00  38.00  41.00  43.00 999.99|  32.00
  5  |  29.00  39.00  41.00 999.99 999.99|  29.00
  6  |  30.00  39.00 999.99 999.99 999.99|  30.00
```

Day 2 (Tuesday)

```
  s \ x:  0      1      2      3      4    | Minimum
  0  | 999.99 999.99 999.99  63.00  59.00|  59.00
  1  | 999.99 999.99  61.00 [  C   ] 56.00|  56.00
  2  | 999.99  59.00  55.00  54.00  57.00|  54.00
  3  |  50.00  53.00  52.00  55.00  54.00|  50.00
  4  |  44.00  50.00  53.00  52.00  52.00|  44.00
  5  |  41.00  51.00  50.00  50.00  54.00|  41.00
  6  |  42.00  48.00  48.00  52.00 999.99|  42.00
```

Day 1 (Monday)

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Determinstic Production Planning

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.
- the cost of *production* is **$5** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the <u>beginning</u> of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	2	3	3	1	2	1
Produce	**0**	**3**	**4**	**0**	**3**	**0**

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $ **65.00**

 (b.) What should be the production quantity for Monday? __**0**__

(c.) What is the total cost (production + storage – salvage value) of the optimal production schedule for all six days? __**$65.00**__

(d.) Three values have been blanked out in the computer output, What are they?
- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)** __**$19.00**__ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B)** $ __**54.00**__
- the cost associated with the decision to produce 3 units on Monday, when there is initially one unit in stock. **(C.)** $ __**69.00**__

(e .) Complete the last row of the table above, indicating the optimal production quantity each day.

```
 s \ x:  0       1       2       3       4  | Minimum
 0 |  999.99    9.00   10.00   11.00   12.00|    9.00
 1 |    1.00    7.00    8.00    9.00   10.00|    1.00
 2 |   ‾1.00    5.00    6.00    7.00    8.00|   ‾1.00
 3 |   ‾3.00    3.00    4.00    5.00   12.00|   ‾3.00
 4 |   ‾5.00    1.00    2.00    9.00  999.99|   ‾5.00
 5 |   ‾7.00   ‾1.00    6.00  999.99  999.99|   ‾7.00
 6 |   ‾9.00    3.00  999.99  999.99  999.99|   ‾9.00
```

Stage 6
(Saturday)

```
    s \ x:   0       1       2       3       4  | Minimum
    0 |  999.99  999.99   22.00   18.00   20.00|   18.00
    1 |  999.99  [  A  ]   15.00   17.00   19.00|   15.00
    2 |   11.00   12.00   14.00   16.00   18.00|   11.00
    3 |    4.00   11.00   13.00   15.00   17.00|    4.00
    4 |    3.00   10.00   12.00   14.00   16.00|    3.00
    5 |    2.00    9.00   11.00   13.00  999.99|    2.00
    6 |    1.00    8.00   10.00  999.99  999.99|    1.00
```

Day 5 (Friday)

```
    s \ x:   0       1       2       3       4  | Minimum
    0 |  999.99   27.00   28.00   28.00   25.00|   25.00
    1 |   19.00   25.00   25.00   22.00   25.00|   19.00
    2 |   17.00   22.00   19.00   22.00   25.00|   17.00
    3 |   14.00   16.00   19.00   22.00   25.00|   14.00
    4 |    8.00   16.00   19.00   22.00  999.99|    8.00
    5 |    8.00   16.00   19.00  999.99  999.99|    8.00
    6 |    8.00   16.00  999.99  999.99  999.99|    8.00
```

Day 4 (Thursday)

```
    s \ x:   0       1       2       3       4  | Minimum
    0 |  999.99  999.99  999.99   42.00   40.00|   40.00
    1 |  999.99  999.99   39.00   37.00   39.00|   37.00
    2 |  999.99   36.00   34.00   36.00   37.00|   34.00
    3 |   28.00   31.00   33.00   34.00   32.00|   28.00
    4 |   23.00   30.00   31.00   29.00   33.00|   23.00
    5 |   22.00   28.00   26.00   30.00   34.00|   22.00
    6 |   20.00   23.00   27.00   31.00  999.99|   20.00
```

Day 3 (Wednesday)

```
    s \ x:   0       1       2       3       4  | Minimum
    0 |  999.99  999.99  999.99   57.00   58.00|   57.00
    1 |  999.99  999.99   54.00   55.00   56.00|  [  B  ]
    2 |  999.99   51.00   52.00   53.00   51.00|   51.00
    3 |   43.00   49.00   50.00   48.00   47.00|   43.00
    4 |   41.00   47.00   45.00   44.00   47.00|   41.00
    5 |   39.00   42.00   41.00   44.00   46.00|   39.00
    6 |   34.00   38.00   41.00   43.00  999.99|   34.00
```

Day 2 (Tuesday)

```
    s \ x:   0       1       2       3       4  | Minimum
    0 |  999.99  999.99   70.00   71.00   72.00|   70.00
    1 |  999.99   67.00   68.00  [  C  ]   65.00|   65.00
    2 |   59.00   65.00   66.00   62.00   64.00|   59.00
    3 |   57.00   63.00   59.00   61.00   63.00|   57.00
    4 |   55.00   56.00   58.00   60.00   59.00|   55.00
    5 |   48.00   55.00   57.00   56.00  999.99|   48.00
    6 |   47.00   54.00   53.00  999.99  999.99|   47.00
```

Day 1 (Monday)

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We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.
- the cost of *production* is **$6** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the <u>beginning</u> of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	3	4	1	2	1	2
Produce	**2**	**4**	**0**	**3**	**0**	**2**

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $ **73.00**

(b.) What should be the production quantity for Monday? __**2**__

(c.) What is the total cost (production + storage – salvage value) of the optimal production schedule for all six days? $ _**73.00**_

(d.) Three values have been blanked out in the computer output, What are they?
- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)__19.00__** *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $__57.00__**
- the cost associated with the decision to produce 2 units on Monday, when there is initially one unit in stock. **(C.) $__76.00__**

(e.) Complete the last row of the table above, indicating the optimal production quantity each day.

```
 s \ x:  0       1       2       3       4   | Minimum
 0  | 999.99  999.99  14.00   15.00   16.00|   14.00
 1  | 999.99   11.00  12.00   13.00   14.00|   11.00
 2  |   2.00    9.00  10.00   11.00   12.00|    2.00
 3  |   0.00    7.00   8.00    9.00   10.00|    0.00
 4  |  ‾2.00    5.00   6.00    7.00   14.00|   ‾2.00
 5  |  ‾4.00    3.00   4.00   11.00  999.99|   ‾4.00
 6  |  ‾6.00    1.00   8.00  999.99  999.99|   ‾6.00
```

Stage 6 (Saturday)

```
  s \ x: 0      1      2      3      4   | Minimum
  0 | 999.99  24.00  25.00  20.00  22.00|  20.00
  1 |  15.00  __A__  17.00  19.00  21.00|  15.00
  2 |  13.00  14.00  16.00  18.00  20.00|  13.00
  3 |   5.00  13.00  15.00  17.00  19.00|   5.00
  4 |   4.00  12.00  14.00  16.00 999.99|   4.00
  5 |   3.00  11.00  13.00 999.99 999.99|   3.00
  6 |   2.00  10.00 999.99 999.99 999.99|   2.00
```

Day 5 (Friday)

```
  s \ x: 0      1      2      3      4   | Minimum
  0 | 999.99 999.99  34.00  33.00  35.00|  33.00
  1 | 999.99  31.00  30.00  32.00  28.00|  28.00
  2 |  22.00  27.00  29.00  25.00  28.00|  22.00
  3 |  18.00  26.00  22.00  25.00  28.00|  18.00
  4 |  17.00  19.00  22.00  25.00  28.00|  17.00
  5 |  10.00  19.00  22.00  25.00 999.99|  10.00
  6 |  10.00  19.00  22.00 999.99 999.99|  10.00
```

Day 4 (Thursday)

```
  s \ x: 0      1      2      3      4   | Minimum
  0 | 999.99  43.00  42.00  40.00  40.00|  40.00
  1 |  34.00  39.00  37.00  37.00  40.00|  34.00
  2 |  30.00  34.00  34.00  37.00  34.00|  30.00
  3 |  25.00  31.00  34.00  31.00  35.00|  25.00
  4 |  22.00  31.00  28.00  32.00 999.99|  22.00
  5 |  22.00  25.00  29.00 999.99 999.99|  22.00
  6 |  16.00  26.00 999.99 999.99 999.99|  16.00
```

Day 3 (Wednesday)

```
  s \ x: 0      1      2      3      4   | Minimum
  0 | 999.99 999.99 999.99 999.99  62.00|  62.00
  1 | 999.99 999.99 999.99  59.00  57.00|  __B__
  2 | 999.99 999.99  56.00  54.00  54.00|  54.00
  3 | 999.99  53.00  51.00  51.00  50.00|  50.00
  4 |  44.00  48.00  48.00  47.00  48.00|  44.00
  5 |  39.00  45.00  44.00  45.00  49.00|  39.00
  6 |  36.00  41.00  42.00  46.00  44.00|  36.00
```

Day 2 (Tuesday)

```
  s \ x: 0      1      2      3      4   | Minimum
  0 | 999.99 999.99 999.99  80.00  79.00|  79.00
  1 | 999.99 999.99  __C__  76.00  77.00|  76.00
  2 | 999.99  74.00  73.00  74.00  74.00|  73.00
  3 |  65.00  70.00  71.00  71.00  69.00|  65.00
  4 |  61.00  68.00  68.00  66.00  65.00|  61.00
  5 |  59.00  65.00  63.00  62.00  63.00|  59.00
  6 |  56.00  60.00  59.00  60.00 999.99|  56.00
```

Day 1 (Monday)