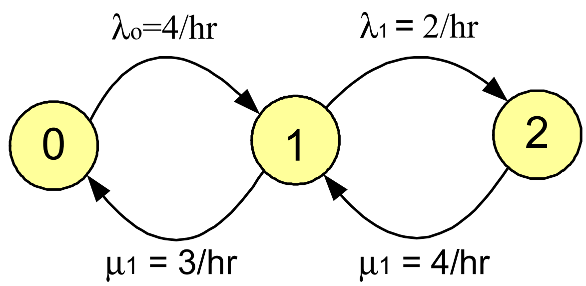
56:171 Operations Research
Quiz #10 Versions A, B, & C --Fall 2002

Version A: A machine operator has the task of keeping two machines running. Each machine runs for an average of **30** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **20** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **15** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0= 4/hr $\qquad$ λ_1 = 2/hr

μ_1=3/hr $\qquad$ μ_2= 4/hr

_b_5. Which equation is used to compute the steady-state probability π_0?

(a.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2}$

(e). $\pi_0 = \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2}\right)^{-1}$

(b.)

$$\boxed{\pi_0 = \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2}\right)^{-1} = \left(1 + \frac{4}{3} + \frac{4}{3} \times \frac{1}{2}\right)^{-1} = (3)^{-1} = \frac{1}{3} \Rightarrow \pi_1 = \frac{4}{3}\pi_0 = \frac{4}{9}, \pi_2 = \frac{2}{3}\pi_0 = \frac{2}{9}}$$

(c.) $\pi_0 = \dfrac{1 + \frac{\lambda_0}{\lambda_1}}{1 + \frac{\mu_1}{\mu_2}}$

(f.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \left(\frac{\lambda_1}{\mu_2}\right)^2$

(g.) $\pi_0 = \frac{1 + \lambda_0 + \lambda_0 \times \lambda_1}{1 + \mu_1 + \mu_1 \times \mu_2}$

(h.) *None of the above*

__f_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \pi_0$ $\qquad$ b. $\pi_1 = \frac{1}{4}\pi_0$ $\qquad$ c. $\pi_1 = \frac{1}{3}\pi_0$ $\qquad$ d. $\pi_1 = \frac{1}{2}\pi_0$

e. $\pi_1 = \frac{2}{3}\pi_0$ $\qquad$ f. $\boxed{\pi_1 = \frac{4}{3}\pi_0}$ $\qquad$ g. $\pi_1 = 2\pi_0$ $\qquad$ h. *None of the above*

__b_7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30%	b. 35% (33%)	c. 40%	d. 45%	e. 50%
f 55%	g. 60%	h. 65%	i. 70%	j. 75%

__f_ 8. The average number of machines which are running is *(choose nearest value)*:

a. 0.2	b. 0.4	c. 0.6	d. 0.8	e. 1.0
f. 1.2 (1.111)	g. 1.4	h. 1.6	i. 1.8	j. 2.0

_J__ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20%	b. 25%	c. 30%	d. 35%	e. 40%
f. 45%	g. 50%	h. 55%	i. 60%	j. 65% (67%)

_h_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5	b. 0.75	c. 1.0	d. 1.25	e. 1.5
f. 1.75	g. 2.0	h. [2.25 (2.222)]	i. 2.5	j. 2.75

_c_ 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15	b. 20	[c. 25]	d. 30	e. 35
f. 40	g. 45	h. 50	i. 55	j. 60

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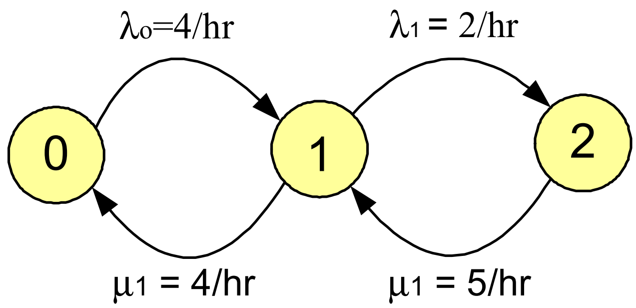
Version B: A machine operator has the task of keeping two machines running. Each machine runs for an average of **30** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **15** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **12** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0 =4/hr $\quad$ λ_1= 2/hr

μ_1 =4/hr $\quad$ μ_2 =5/hr

_a__5. Which equation is used to compute the steady-state probability π_0?

(a.) $\boxed{\pi_0 = \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2}\right)^{-1} = \left(1 + \frac{4}{4} + \frac{4}{4} \times \frac{2}{5}\right)^{-1} = \left(\frac{12}{5}\right)^{-1} = \frac{5}{12} \Rightarrow \pi_1 = \frac{5}{12}, \pi_2 = \frac{1}{6}}$

(b.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \left(\frac{\lambda_1}{\mu_2}\right)^2$

(c.) $\pi_0 = \dfrac{1 + \frac{\lambda_0}{\lambda_1}}{1 + \frac{\mu_1}{\mu_2}}$

(d.) $\pi_0 = \dfrac{1 + \lambda_0 + \lambda_0 \times \lambda_1}{1 + \mu_1 + \mu_1 \times \mu_2}$

(e.) $\pi_0 = \frac{\lambda_0}{\mu_1} \times \frac{\lambda_1}{\mu_2}$

(f.) $\pi_0 = \left(1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2}\right)^{-1}$

(h.) *None of the above*

_e_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \frac{1}{4}\pi_0$	b. $\pi_1 = \frac{1}{3}\pi_0$	c. $\pi_1 = \frac{1}{2}\pi_0$	d. $\pi_1 = \frac{2}{3}\pi_0$
e. $\boxed{\pi_1 = \pi_0}$	f. $\pi_1 = 2\pi_0$	g. $\pi_1 = 3\pi_0$	h. *None of the above*

_c_7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30%	b. 35%	c. [40% (41.66%)]	d. 45%	e. 50%

f 55% g. 60% h. 65% i. 70% j. 75%

__f_ 8. The average number of machines which are running is *(choose nearest value)*:

a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0
f. 1.2 (1.25) g. 1.4 h. 1.6 i. 1.8 j. 2.0

_i_ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20% b. 25% c. 30% d. 35% e. 40%
f. 45% g. 50% h. 55% i. 60% (58.33%) j. 65%

_i_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5 b. 0.75 c. 1.0 d. 1.25 e. 1.5
f. 1.75 g. 2.0 h. 2.25 i. 2.5 j. 2.75

_b_ 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15 b. 20 (18) c. 25 d. 30 e. 35
f. 40 g. 45 h. 50 i. 55 j. 60

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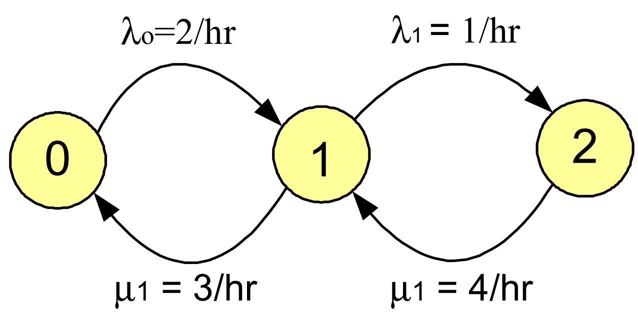
Version C: A machine operator has the task of keeping two machines running. Each machine runs for an average of **60** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **20** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **15** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0 =2/hr λ_1 =1/hr
μ_1 =3/hr μ_2=4/hr

_f_5. Which equation is used to compute the steady-state probability π_0?

(a.) $$\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_1}{\mu_2}\right)^{-1}$$

(d.) $$\pi_0 = \frac{\lambda_0}{\mu_1}\times\left(\frac{\lambda_1}{\mu_2}\right)^2$$

(b.) $$\pi_0 = \frac{1+\dfrac{\lambda_0}{\lambda_1}}{1+\dfrac{\mu_1}{\mu_2}}$$

(e.) $$\pi_0 = \frac{1+\lambda_0+\lambda_0\times\lambda_1}{1+\mu_1+\mu_1\times\mu_2}$$

(c.) $$\pi_0 = \frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}$$

(f). $$\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}\right)^{-1} = \left(1+\frac{2}{3}+\frac{2}{3}\times\frac{1}{4}\right)^{-1} = \left(\frac{11}{6}\right)^{-1} = \frac{6}{11} \Rightarrow \pi_1 = \frac{2}{3}\pi_0 = \frac{4}{11}, \pi_2 = \frac{1}{6}\pi_0 = \frac{1}{11}$$

(h.) *None of the above*

__e_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \pi_0$ b. $\pi_1 = \frac{1}{4}\pi_0$ c. $\pi_1 = \frac{1}{3}\pi_0$ d. $\pi_1 = \frac{1}{2}\pi_0$

e. $\boxed{\pi_1 = \frac{2}{3}\pi_0}$ f. $\pi_1 = 2\pi_0$ g. $\pi_1 = 3\pi_0$ h. *None of the above*

_f__7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30% b. 35% c. 40% d. 45% e. 50%
f [55% (54.54%)] g. 60% h. 65% i. 70% j. 75%

_f__ 8. The average number of machines which are running is *(choose nearest value)*:

a. 1.0 b. 1.1 c. 1.2 d. 1.3 e. 1.4
f. [1.5 (1.4545)] g. 1.6 h. 1.7 i. 1.8 j. 1.9

_f__ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20% b. 25% c. 30% d. 35% e. 40%
f. [45% (45.45%)] g. 50% h. 55% i. 60% j. 65%

_e_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5 b. 0.75 c. 1.0 d. 1.25 e. [1.5 (1.4545)]
f. 1.75 g. 2.0 h. 2.25 i. 2.5 j. 2.75

b or c 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15 [b. 20 (22.5) c. 25] d. 30 e. 35
f. 40 g. 45 h. 50 i. 55 j. 60

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The following are common to versions A, B, & C:

__d_ 12. For a continuous-time Markov chain, let Λ be the matrix of transition probabilities. The sum of each...

a. column is 1 c. row is 1
b. column is 0 [d. row is 0] e. *NOTA*

__d_ 13. In a birth/death process model of a queue, the time between departures is assumed to

a. have the Beta dist'n c. be constant e. have the uniform dist'n
b. have the Poisson dist'n d. have the exponential dist'n f. *NOTA*

__c_ 14. In an M/M/1 queue, if the arrival rate = $\lambda < \mu$ = service rate, then

a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i e. the queue is not a birth-death process
b. no steady state exists d. $\pi_0 = 0$ in steady state f. *NOTA*

True (+) or false (o)?

_+_ 15. The continuous-time Markov chain on the previous page is a birth/death process.

_o_ 16. Little's Law for queues is valid only if the queue is a birth/death process.

_+_ 17. According to Little's Law, the average arrival rate is the ratio of average number of customers in the system to the average time per customer, i.e., λ = L/W.

_+_ 18. Little's Law for queues is valid for *every* queue which is a continuous-time Markov chain. *Note: it is valid for other queues as well!*