56:171 Operations Research
Homework #8 Solution -- Fall 2002

1. **Decision Analysis** (an exercise from *Operations Research: a Practical Introduction*, by M. Carter & C. Price)
Suppose that you are in the position of having to buy a used car, and you have narrowed down your choices to two possible models: one car is a private sale and the other is from a dealer. You must now choose between them. The cars are similar, and the only criterion is to minimize expected cost. The dealer car is more expensive, but it comes with a one-year warranty which would cover all costs of repairs. You decide that, if the car will last for 1 year, you can sell it again and recover a large part of your investment. If it falls apart, it will not be worth fixing. After test driving both cars and checking for obvious flaws, you make the following evaluation of probably resale value:

Car	Purchase price	Probability of lasting one year	Estimated resale price
A: Private	$800	0.3	$600
B: Dealer	$1500	0.9	$1000

a. Which car would you buy?

b. What is the expected value of perfect information (EVPI)?

Suppose you have the opportunity to take car A to an independent mechanic, who will charge you $50 to do a complete inspection and offer you an opinion as to whether the car will last 1 year. For various subjective reasons, you assign the following probabiliities to the accuracy of the mechanic's opinion:

Given:	Mechanic says Yes	Mechanic says No
A car that will last 1 year	70%	30%
A car that will not last 1 year	10%	90%

(For example, if a car that will last 1 year is taken to the mechanic, there is 70% probability that he will give you the opinion that it will last a year.)

c. Assuming that you must buy one of these two cars, formulate this problem as a decision tree problem.

First we use Bayes' Rule to compute the posterior probabilities of survival & failure of car A, given the mechanic's report::

$$P\{S_i \mid O_j\} = \frac{P\{O_j \mid S_i\} \times P\{S_i\}}{P\{O_j\}}$$

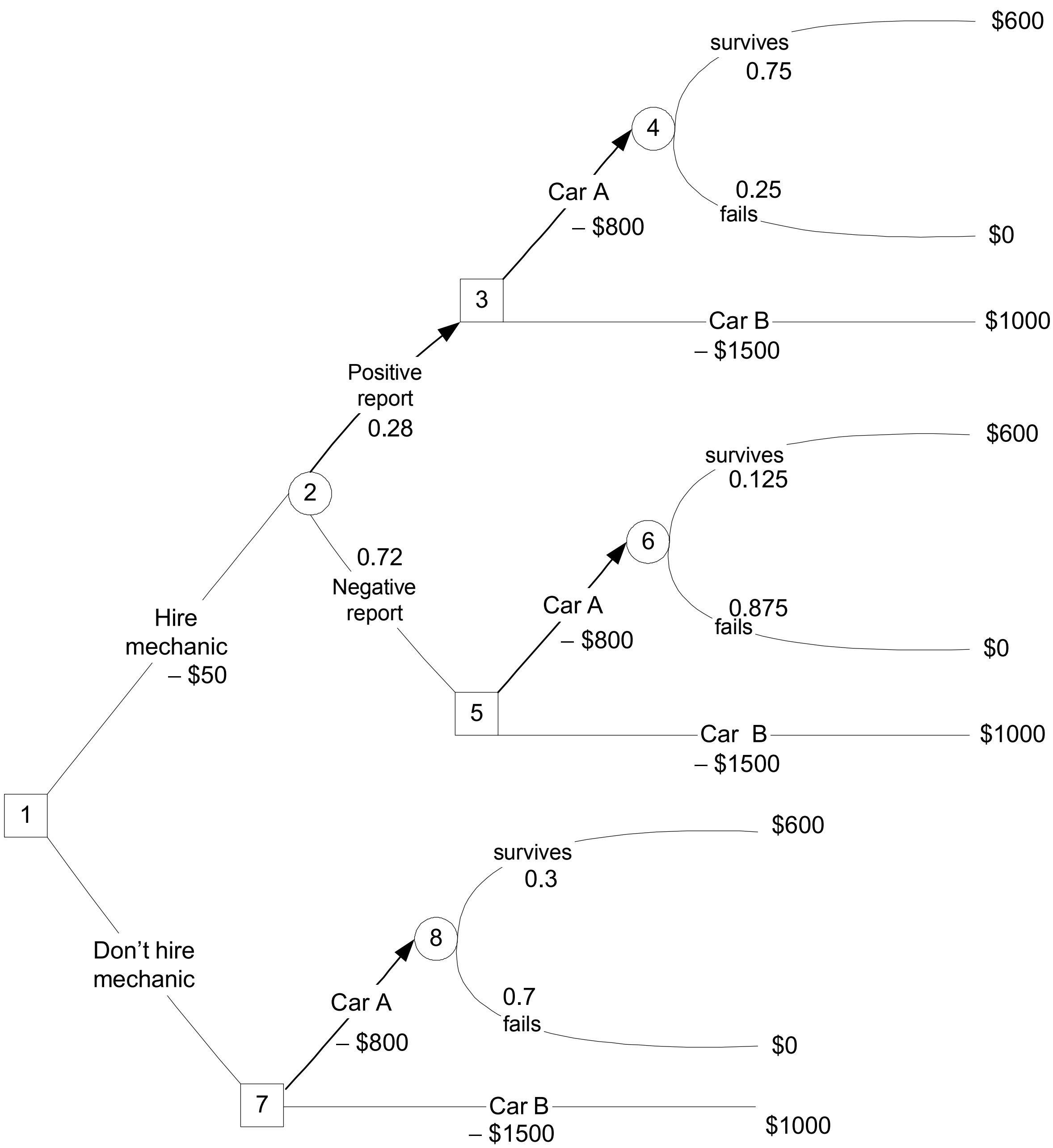
where $P\{O_j\} = \sum_i P\{O_j \mid S_i\} \times P\{S_i\}$

Thus, for example, the probability that the mechanic will give a positive report is 28%.
If he does, car A is 75% likely to survive. If, on the other hand, he gives a negative report (with probability 72%) the care is 87.5% likely to fail.

Template for Posterior Probabilities

<table>
<tr><td>Data:</td><td></td><td colspan="2">P(Finding | State)</td></tr>
<tr><td>State of</td><td>Prior</td><td colspan="2">Finding</td></tr>
<tr><td>Nature</td><td>Probability</td><td>PR</td><td>NR</td></tr>
<tr><td>Survive</td><td>0.3</td><td>0.7</td><td>0.3</td></tr>
<tr><td>Fail</td><td>0.7</td><td>0.1</td><td>0.9</td></tr>
</table>

<table>
<tr><td>Posterior</td><td></td><td colspan="2">P(State | Finding)</td></tr>
<tr><td>Probabilities:</td><td></td><td colspan="2">State of Nature</td></tr>
<tr><td>Finding</td><td>P(Finding)</td><td>Survive</td><td>Fail</td></tr>
<tr><td>PR</td><td>0.28</td><td>0.75</td><td>0.25</td></tr>
<tr><td>NR</td><td>0.72</td><td>0.125</td><td>0.875</td></tr>
</table>



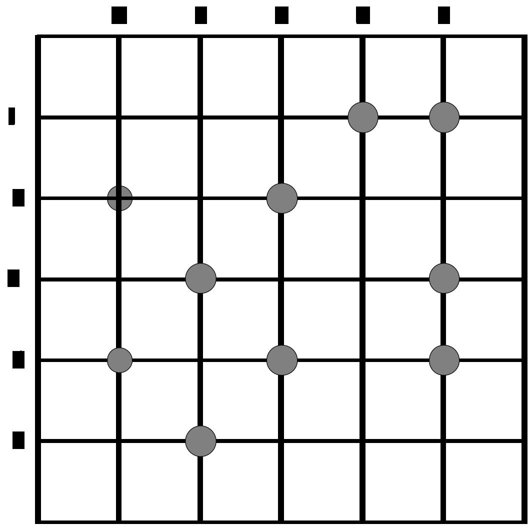
d. What is the expected value of the mechanic's advice?

Is it worth asking for the mechanic's opinion?

What is your optimal decision strategy?

Note: it is not necessary to ask for advice on car B because its problems could be repaired under the warranty!

2. **Integer Programming** A convenience store chain is planning to enter a growing market and must determine where to open several new stores. The map shows the major streets in the area being considered. (Adjacent streets are 1 mile apart. A Avenue, B Avenue, etc. are N-S streets (with A Ave. being the westernmost) while 1st Street, 2nd Street, etc. are E-W streets (with 1st Street being the furthest north.) . The symbol ● indicates possible store locations. All travel must follow the street network, so distance is determined with a rectilinear metric. For instance, the distance between corners A1 and C2 is 3 miles.



- The costs of purchasing property & constructing stores at the various locations are as follows:

Location	A2	A4	B3	B5	C2	C4	D1	E1	E3	E4
Cost	100	80	90	50	80	90	100	70	90	80

- No two stores can be on the same street (either north-south or east-west).
- Sttores must be <u>at least</u> 3 miles apart.
- Every grid point (A1, B2, etc.) must be <u>no more than</u> 3 miles from a store.

a. Set up an integer programming model that can be used to find the optimal store locations.
b. Find the optimal locations and the minimum cost..

LINDO model:

```
MIN 100XA2 + 80XA4 + 90XB3 + 50XB5 + 80XC2 + 90XC4 + 100XD1 + 70XE1 + 90XE3
    + 80XE4
ST

   !No two stores on same street(vertical)
   XA2 + XA4 <= 1
   XB3 + XB5 <= 1
   XC2 + XC4 <= 1
   XD1         <= 1
   XE1 + XE3 + XE4 <=1

   !No two stores on same street (horizontal)
   XD1 + XE1 <= 1
   XA2 + XC2 <= 1
   XB3 + XE3 <= 1
```

```
XA4 + XC4 + XE4 <= 1
XB5       <= 1

!Store A2 3 mile constraint
XA2 + XA4 <= 1
XA2 + XB3 <= 1
XA2 + XC2 <= 1

!Store A4 3 mile constraint
XA4 + XB3 <= 1
XA4 + XB5 <= 1
XA4 + XC4 <= 1

!Store B3 3 mile constraint
XB3 + XB5 <= 1
XB3 + XC2 <= 1
XB3 + XC4 <= 1

!Store B5 3 mile constraint
XB5 + XC4 <= 1

!Store C2 3 mile constraint
XC2 + XC4 <= 1
XC2 + XD1 <= 1
!XC2 + XE1 <= 1
!XC2 + XE3 <= 1

!Store C4 3 mile constraint
XC4 + XE3 <= 1
XC4 + XE4 <= 1

!Store D1 3 mile constraint
XD1 + XE1 <= 1
!XD1 + XE3 <= 1

!Store E1 3 mile constraint
XE1 + XE3 <= 1
!XE1 + XE4 <= 1

!Store E3 3 mile constraint
XE3 + XE4 <= 1

!Grid Point 3 mile constraint

!A
XA2 + XA4 + XB3 + XC2 >= 1
XA2 + XA4 + XB3 + XC2 >= 1
XA2 + XA4 + XB3 + XB5 + XC2 + XC4 >= 1
XA2 + XA4 + XB3 + XB5 + XC4 >= 1
XA2 + XA4 + XB3 + XB5 + XC4 >= 1

!B
XA2 + XB3 + XC2 + XD1 + XE1 >= 1
XA2 + XA4 + XB3 + XB5 + XC2 + XC4 + XD1 >= 1
XA2 + XA4 + XB3 + XB5 + XC2 + XC4 + XE3 >= 1
XA2 + XA4 + XB3 + XB5 + XC2 + XC4 + XE4 >= 1
XA4 + XB3 + XB5 + XC4 >= 1
```

```
  !C
  XA2 + XB3 + XC2 + XC4 + XD1 + XE1 >= 1
  XA2 + XB3 + XC2 + XC4 + XD1 + XE1 + XE3 >= 1
  XA2 + XA4 + XB3 + XB5 + XC2 + XC4 + XD1 + XE3 + XE4 >= 1
  XA4 + XB3 + XB5 + XC2 + XC4 + XE3 + XE4 >= 1
  XA4 + XB3 + XB5 + XC2 + XC4 + XE4 >= 1

  !D
  XC2 + XD1 + XE1 + XE3 >= 1
  XD2 + XA2 + XB3 + XC2 + XC4 + XD1 + XE1 + XE3 + XE4 >= 1
  XB3 + XC2 + XC4 + XD1 + XE1 + XE3 + XE5 >= 1
  XA4 + XB3 + XB5 + XC2 + XC4 + XD1 + XE3 + XE4 >= 1
  XB5 + XC4 + XE3 + XE4 >= 1

  !E
  XC2 + XD1 + XE1 + XE3 + XE4 >= 1
  XE2 + XC2 + XD1 + XE1 + XE3 + XE4 >= 1
  XB3 + XC2 + XC4 + XB1 + XE1 + XE3 + XE4 >= 1
  XC4 + XE1 + XE3 + XE4 >= 1
  XB5 + XC4 + XE3 + XE4 >= 1

END

INT 10
```

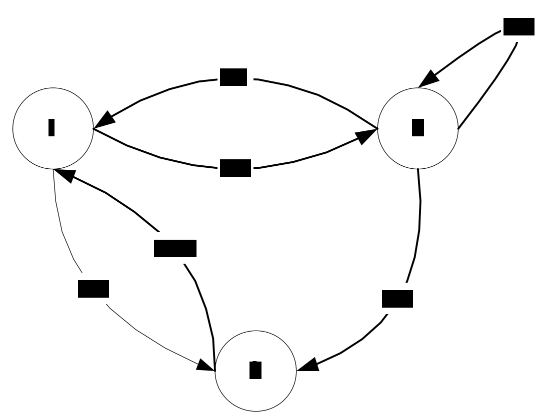
Solution:

```
        OBJECTIVE FUNCTION VALUE

         1)       200.0000

  VARIABLE         VALUE          REDUCED COST
       XB5          1.000000         50.000000
       XC2          1.000000         80.000000
       XE1          1.000000         70.000000
```

3. **Discrete-time Markov Chains** A stochastic process with three states has the transition probabilities shown below:



a. Write the transition probability matrix P.

Suppose that the system begins in state 1, and is in state 3 after two steps.

b. What are the possible sequences of two transitions that might have occurred?

c. What are the probabilities of each of these sequences?

d. What is the probability $p_{13}^{(2)}$?

c. Write the equations which determine π, the steadystate probability distribution.

d. Compute the steadystate probability distribution π.