56:171 Operations Research
Homework #6 - Due Friday, October 11, 2002

1. (From *O.R. Applications & Algorithms*, by Wayne Winston)
During the month of July, Pittsburgh resident B. Fly must make four round-trip flights between Pittsburgh and Chicago. The dates of the trips are as shown below.

Leave Pittsburgh	Leave Chicago
Monday, July 1	Friday, July 5
Tuesday, July 9	Thursday, July 11
Monday, July 15	Friday, July 19
Wednesday, July 24	Thursday, July 25

B. Fly must purchase four round-trip tickets. Without a discounted fare, a round-trip ticket between Pittsburgh and Chicago costs $500. If Fly's stay in a city includes a weekend, he gets a 20% discount on the round-trip fare. If his stay in a city is at least 21 days, he receives a 35% discount, and if his stay is more than 10 days, he receives a 30% discount. Of course, only one discount can be applied toward the purchase of any ticket.
Formulate and solve an assignment problem that minimizes the total cost of purchasing the four round-trip tickets. (Hint: Let $X_{ij} = 1$ if a round-trip ticket is purchased for use on the ith flight out of Pittsburgh and the jth flight out of Chicago. Also think about where Fly should buy a ticket if, for example, $X_{21} = 1$.)

2. (From *Practical Management Science*, by W. Winston & S. C. Albright)
The city of Spring View is taking bids from six bus companies on the eight routes that must be driven in the surrounding school district. Each company enters a bid on how much it will charge to drive selected routes, although not all companies bid on all routes. (Blank cells in the table indicates routes on chich a company does not bid.)

Company	1	2	3	4	5	6	7	8
A		8200	7800	5400		3900		
B	7800	8200		6300		3300	4900	
C		4800				4400	5600	3600
D			8000	5000	6800		6700	4200
E	7200	6400		3900	6400	2800		3000
F	7000	5800	7500	4500	5600		6000	4200

The city needs to select which companies to assign to which routes with the specifications that
(1) if a company does not bid on a route, it cannot be assigned to that route,
(2) exactly one company must be assigned to each route, and
(3) a company can be assigned to at most two routes.
The objective is to minimize the total cost of covering all routes. Formulate this as an assignment problem, and use the Hungarian method to solve it.

3. (Based on exercises 10.2-2, & 10.3-3, page 515f, *Introduction to O.R.,* Hillier & Lieberman (7th ed.)) Christine Phillips is in charge of planning and coordinating next spring's sales management training program for her company. Christine has listed the following activity information for this project. (Durations are given in weeks.)

Activity	Description	Immediate predecessors	Expected Duration	Estimated variance
A	Select location	--	2	1
B	Obtain keynote speaker	--	1	0.2
C	Obtain other speakers	B	2	1
D	Make travel plans for keynote speaker	A,B	2	1
E	Make travel plans for other speakers	A, C	3	1
F	Make food arrangements	A	2	1
G	Negotiate hotel rates	A	1	0.2
H	Prepare brochure	C, G	1	0.2
I	Mail brochure	H	1	0.2
J	Take reservations	I	3	1
K	Prepare handouts	C, F	4	4

a. Draw the AON (activity-on-node) project network.
b. Draw the AOA (activity-on-arrow) project network.
c. Label the nodes (events) of the AOA network so that if there is an arrow $i \rightarrow j$ then $i<j$.

In parts (d)-(h), assume that the expected durations are the actual durations.

d. Do a forward pass to compute the ET (earliest time) for each node.
e. What is the earliest completion time for this project?
f. Do a backward pass to compute the LT (latest time) for each node.
g. Complete the table below of ES (earliest start), EF (earliest finish), LS (latest start), LF (latest finish), and slack for each activity:

Activity	Description	ES	EF	LS	LF	Slack
A	Select location					
B	Obtain keynote speaker					
C	Obtain other speakers					
D	Make travel plans for keynote speaker					
E	Make travel plans for other speakers					
F	Make food arrangements					
G	Negotiate hotel rates					
H	Prepare brochure					
I	Mail brochure					
J	Take reservations					
K	Prepare handouts					

h. Which activities are critical?

Now assume that the durations are random with the variance shown above.

i. What is the expected completion time of the project?
j. What is the standard deviation of the completion time of the project?
k. Assuming that the project completion time has normal distribution, what is the probability that the project will be completed not later than two weeks after the expected completion time?