56:171 Operations Research
Homework #5 - Due Friday, October 4, 2002

1. Consider the transportation tableau:

dstn→ ↓source	1	2	3	4	5	Supply
A	12	8	9	15	11	9
B	10	11	12	11	14	7
C	9	7	11	14	8	4
D	13	12	13	12	12	7
E	8	9	10	9	10	3
Demand=	4	7	5	5	9	

a. Use the initial basic solution: $X_{A3}=5$, $X_{A5}=4$, $X_{B1}=4$, $X_{B4}=3$, $X_{C4}=X_{C5}=2$, $X_{D2}=7$, $X_{E5}=3$ & _____ = 0. *(Choose one more variable to complete the basis. Any choice is valid except one that would create a "cycle" of basic cells in the tableau!)*

b. Compute two different sets of values for the dual variables U & V (*simplex multipliers*) for this basis.

c. Using each set of simplex multipliers, price all of the nonbasic cells. How do the reduced costs depend upon the choice of dual variables? Select the variable having the "most negative" reduced cost to enter the basis.

d. What variable will enter the basis as the new variable enters the basis?

e. Complete the computation of the optimal solution, using the transportation simplex method.

2. **Production scheduling** (adapted from O.R. text by Hillier & Lieberman, 7th edition, page 394) The MLK Manufacturing Company must produce two products in sufficient quantity to meet contracted sales in each of the next three months. The two products share the same production facilities, and each unit of both products requires the same amount of production capacity. The available production and storage facilities are changing month by month, so the production capacities, unit production costs, and unit storage costs vary by month. Therefore, it may be worthwhile to overproduce one or both products in some months and store them until needed.

   For each of the three months, the second column of the following table gives the maximum number of units of the two products combined that can be produced in Regular Time (RT) and in Overtime (OT). For each of the two products, the subsequent columns give (1) the number of units needed for the contracted sales, (2) the cost (in thousands of dollars) per unit produced in regular time, (3) the cost (in thousands of dollars) per unit produced in overtime, and (4) the cost (in thousands of dollars) of storing each extra unit that is held over into the next month.

In each case, the numbers for the two products are separated by a slash /, with the number for product 1 on the left and the number for product 2 on the right.

	Max combined production			Unit cost of production ($K)		
Month	RT	OT	Sales	RT	OT	Storage cost ($K)
1	10	3	5/3	15/16	18/20	1/2
2	8	2	3/5	17/15	20/18	2/1
3	10	3	4/4	19/17	22/22	

The production manager wants a schedule developed for the number of units of each of the two products to be produced in regular time and (if regular time production capacity is used up) in overtime in each of the three months. The objective is to minimize the total of the production and storage costs while meeting the contracted sales for each month. There is no initial inventory, and no final inventory is desired after the three months.

a. Formulate this problem as a balanced transportation problem by constructing the appropriate transportation tableau.

b. Use the Northwest Corner Method to find an initial basic feasible solution. Is it degenerate?

c. Use the transportation simplex algorithm to find the optimal solution. Is it degenerate? Are there multiple optima?

3. Assignment Problem. (adapted from O.R. text by Hillier & Lieberman, 7th edition, page 399.) Four cargo ships will be used for shipping goods from one port to four other ports (labeled 1, 2, 3, 4). Any ship can be used for making any one of these four trips. However, because of differences in the ships and cargoes, the total cost of loading, transporting, and unloading the goods for the different ship-port combinations varies considerably, as shown in the following table:

PORT→ ↓SHIP	1	2	3	4
1	$500	$400	$600	$700
2	$600	$600	$700	$500
3	$700	$500	$700	$600
4	$500	$400	$600	$600

The objective is to assign the four ships to four different ports in such a way as to minimize the total cost for all four shipments.

a. Use the Hungarian method to find an optimal solution.

b. Reformulate this as an equivalent transportation problem.

dstn→ ↓source	1	2	3	4	Supply=
1					
2					
3					
4					
Demand=					

c. Use the Northwest Corner Method to obtain an initial basic feasible solution. (This will be a *degenerate* solution. Be sure to specify which variables are basic!)

d. Use the transportation simplex method to find the optimal solution.

e. In how many iterations was the solution degenerate?

f. How many iterations produce a change in the *values* of the variables?

g. How many iterations leave the variables *unchanged in value* (although the basis changes)?

4. Return of Marky D. Sod Recall the LP model for this problem in HW#4:
Buster Sod's younger brother, Marky Dee, operates three ranches in Texas. the acreage and irrigation water available for the three farms are shown below:

FARM	ACREAGE	WATER AVAILABLE (ACRE-FT)
1	400	1500
2	600	2000
3	300	900

Three crops can be grown. However, the maximum acreage that can be grown of each crop is limited by the amount of appropriate harvesting equipment available. The three crops are described below. Any combination of crops may be grown on a farm.

CROP	TOTAL HARVESTING CAPACITY (IN ACRES)	WATER REQMTS (ACRE-FT PER ACRE)	EXPECTED PROFIT ($/ACRE)
Milo	700	6	400
Cotton	800	4	300
Wheat	300	2	100

Decisionvariables: X_{ij} = # acreas of crop j planted on farm i.
The LINDO model (generated by LINGO) is:

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MAX     400 X1MILO + 300 X1COTTON + 100 X1WHEAT + 400 X2MILO
      + 300 X2COTTON + 100 X2WHEAT + 400 X3MILO + 300 X3COTTON + 100 X3WHEAT
SUBJECT TO
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        2)   X1MILO + X1COTTON + X1WHEAT <=   400
        3)   6 X1MILO + 4 X1COTTON + 2 X1WHEAT <=   1500
        4)   X2MILO + X2COTTON + X2WHEAT <=   600
        5)   6 X2MILO + 4 X2COTTON + 2 X2WHEAT <=   2000
        6)   X3MILO + X3COTTON + X3WHEAT <=   300
        7)   6 X3MILO + 4 X3COTTON + 2 X3WHEAT <=   900
        8)   X1MILO + X2MILO + X3MILO <=   700
        9)   X1COTTON + X2COTTON + X3COTTON <=   800
       10)   X1WHEAT + X2WHEAT + X3WHEAT <=   300
 END

       1)      320000.0

 VARIABLE        VALUE          REDUCED COST
   X1MILO         0.000000          0.000000
 X1COTTON       375.000000          0.000000
  X1WHEAT         0.000000         33.333332
   X2MILO        50.000000          0.000000
 X2COTTON       425.000000          0.000000
  X2WHEAT         0.000000         33.333332
   X3MILO       150.000000          0.000000
 X3COTTON         0.000000          0.000000
  X3WHEAT         0.000000         33.333332

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)        25.000000          0.000000
       3)         0.000000         66.666664
       4)       125.000000          0.000000
       5)         0.000000         66.666664
       6)       150.000000          0.000000
       7)         0.000000         66.666664
       8)       500.000000          0.000000
       9)         0.000000         33.333332
      10)       300.000000          0.000000

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
   X1MILO      400.000000         0.000000         INFINITY
 X1COTTON      300.000000         INFINITY         0.000000
  X1WHEAT      100.000000        33.333328         INFINITY
   X2MILO      400.000000         0.000000         0.000000
 X2COTTON      300.000000         0.000000         0.000000
  X2WHEAT      100.000000        33.333328         INFINITY
   X3MILO      400.000000         INFINITY         0.000000
 X3COTTON      300.000000         0.000000         INFINITY
  X3WHEAT      100.000000        33.333328         INFINITY

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2      400.000000         INFINITY        25.000000
        3     1500.000000       100.000000       300.000000
        4      600.000000         INFINITY       125.000000
        5     2000.000000       750.000000       300.000000
        6      300.000000         INFINITY       150.000000
        7      900.000000       900.000000       900.000000
        8      700.000000         INFINITY       500.000000
        9      800.000000        75.000000       425.000000
       10      300.000000         INFINITY       300.000000
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THE TABLEAU:

ROW	(BASIS)	X1MILO	X1COTTON	X1WHEAT	X2MILO	X2COTTON	X2WHEAT
1	ART	0.000	0.000	33.333	0.000	0.000	33.333
2	SLK 2	-0.500	0.000	0.500	0.000	0.000	0.000
3	X1COTTON	1.500	1.000	0.500	0.000	0.000	0.000
4	SLK 4	0.500	0.000	0.167	0.000	0.000	0.667
5	X2MILO	1.000	0.000	0.333	1.000	0.000	0.333
6	SLK 6	0.000	0.000	0.000	0.000	0.000	0.000
7	X3MILO	0.000	0.000	0.000	0.000	0.000	0.000
8	SLK 8	0.000	0.000	-0.333	0.000	0.000	-0.333
9	X2COTTON	-1.500	0.000	-0.500	0.000	1.000	0.000
10	SLK 10	0.000	0.000	1.000	0.000	0.000	1.000

ROW	X3MILO	X3COTTON	X3WHEAT	SLK 2	SLK 3	SLK 4	SLK 5
1	0.000	0.000	33.333	0.000	66.667	0.000	66.667
2	0.000	0.000	0.000	1.000	-0.250	0.000	0.000
3	0.000	0.000	0.000	0.000	0.250	0.000	0.000
4	0.000	-0.333	0.000	0.000	0.083	1.000	-0.167
5	0.000	-0.667	0.000	0.000	0.167	0.000	0.167
6	0.000	0.333	0.667	0.000	0.000	0.000	0.000
7	1.000	0.667	0.333	0.000	0.000	0.000	0.000
8	0.000	0.000	-0.333	0.000	-0.167	0.000	-0.167
9	0.000	1.000	0.000	0.000	-0.250	0.000	0.000
10	0.000	0.000	1.000	0.000	0.000	0.000	0.000

ROW	SLK 6	SLK 7	SLK 8	SLK 9	SLK 10	
1	0.00E+00	67.	0.00E+00	33.	0.00E+00	0.32E+06
2	0.000	0.000	0.000	0.000	0.000	25.000
3	0.000	0.000	0.000	0.000	0.000	375.000
4	0.000	0.000	0.000	-0.333	0.000	125.000
5	0.000	0.000	0.000	-0.667	0.000	50.000
6	1.000	-0.167	0.000	0.000	0.000	150.000
7	0.000	0.167	0.000	0.000	0.000	150.000
8	0.000	-0.167	1.000	0.667	0.000	500.000
9	0.000	0.000	0.000	1.000	0.000	425.000
10	0.000	0.000	0.000	0.000	1.000	300.000

a. Another farmer whose farm adjoins Sod Farm #3 might be willing to sell Marky a portion of his water rights. How much should Marky offer, and for how many acre-feet?

b. What increase in the profit per acre for wheat is required in order for it to be profitable for Marky to plant any?

c. If Marky were to plant 100 acres of wheat on Farm #1, how should he best adjust the optimal plan above?

d. Is there another optimal basic solution, besides the one given above? If so, how does it differ from that given above?