56:171 Operations Research
Homework #4 Solutions--Fall 2002

1. Ken & Larry's Ice Cream, *continued*. Refer to the problem description in last week's homework (HW#3). The optimal LP tableau provided by LINDO is as shown below.

```
THE TABLEAU

     ROW  (BASIS)         C          V          B      SLK 2      SLK 3
       1   ART         0.038      0.000      0.000      0.000      1.875
       2   SLK  2     -0.350      0.000      0.000      1.000     -2.000
       3         V     3.000      1.000      0.000      0.000     10.000
       4         B    -1.750      0.000      1.000      0.000     -7.500

     ROW      SLK 4
       1      1.000    341.250
       2      2.000     20.000
       3    -20.000    300.000
       4     20.000     75.000
```

a. Chocolate ice cream is not included in the optimal production plan. If one gallon of chocolate ice cream were to be produced, how would it change the quantity
   …of vanilla ice cream produced?
   …of banana ice cream produced?
   …of milk used?
   …of sugar used?
   …of cream used?

   ***Solution*:**

$$\begin{bmatrix} Profit \\ SLK2 \\ V \\ B \end{bmatrix} = \begin{bmatrix} 341.25 \\ 20 \\ 300 \\ 75 \end{bmatrix} - \begin{bmatrix} -0.038 \\ -0.35 \\ 3 \\ -1.75 \end{bmatrix} C$$

   The change of quantity of the vanilla ice cream produced: Decrease by 3 gallon $(-3\times 1 = -3)$.
   The change of quantity of the banana ice cream produced: Increase by 1.75 gallon $(-(-1.75)\times 1 = 1.75)$.
   The change of quantity of the milk used: Decrease by 0.35 (*increase of SLK2 by* 0.35).
   The quantities of sugar or cream used are not changed.

b. In last week's homework, you were asked about the effect on profit of a reduction in the quantity of available cream due to spoilage. That is, the effect of an increase in the unused cream (slack in the available cream constraint). According to the substitution rates in the tableau, what would be the effect of this spoilage on the quantity
   …of vanilla ice cream produced?
   …of banana ice cream produced?
   …of milk used?
   …of sugar used?

***Solution*:**

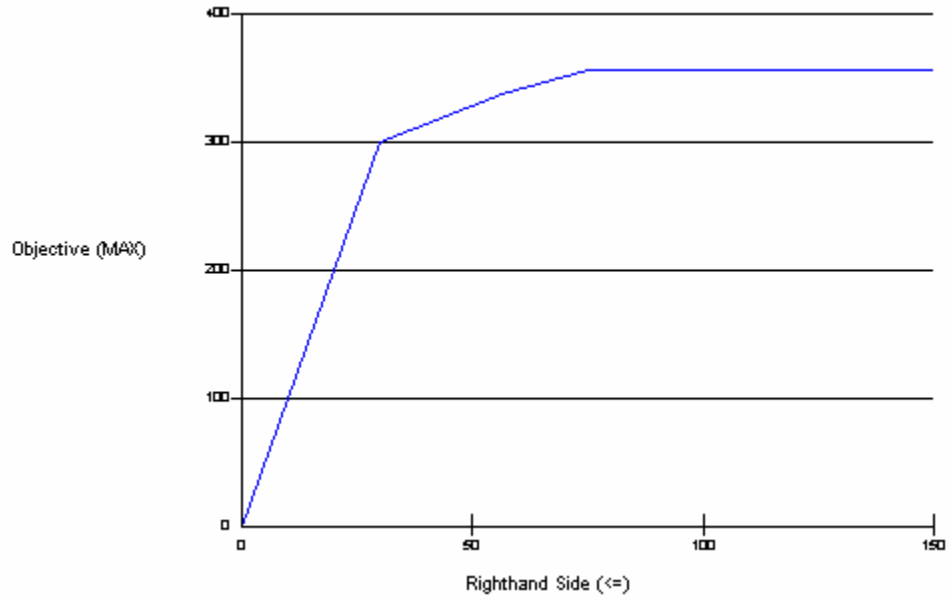
$$\begin{bmatrix} Profit \\ SLK\,2 \\ V \\ B \end{bmatrix} = \begin{bmatrix} 341.25 \\ 20 \\ 300 \\ 75 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ -20 \\ 20 \end{bmatrix} SLK\,4$$

The spoilage implies that *SLK4* is increased by 3 gallons.
The change of quantity of the vanilla ice cream produced: Increase by 60 gallons $(-(-20)\times 3 = 60)$.
The change of quantity of the banana ice cream produced: Decrease by 60 gallons $(-20\times 3 = -60)$.

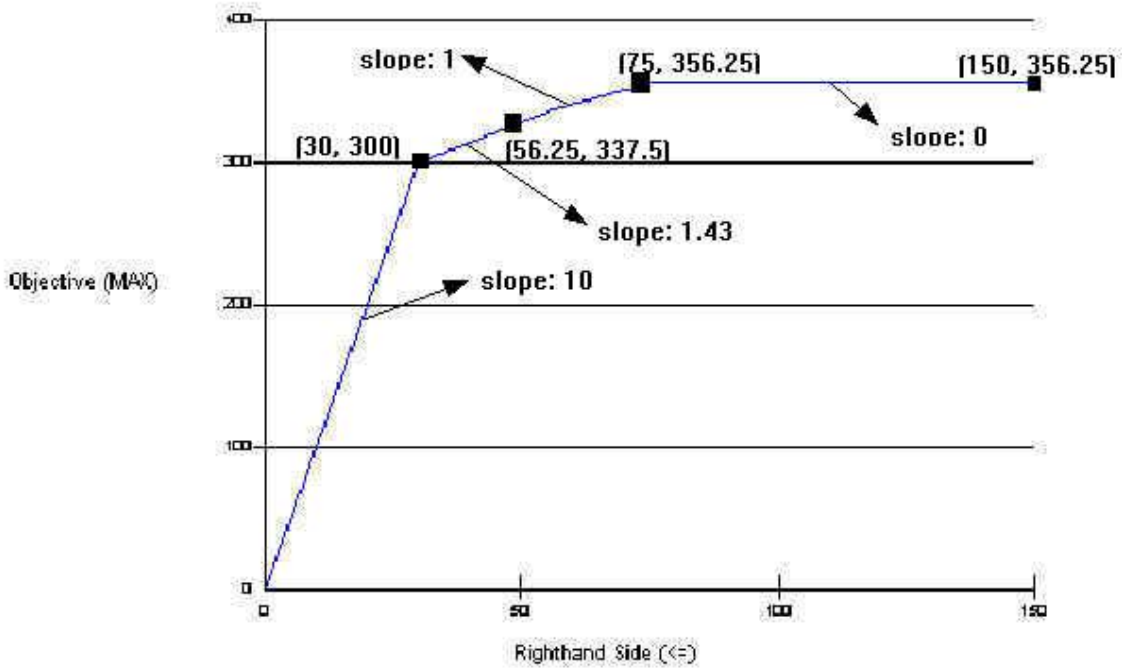
The right-hand-side of row #4 (available cream) was changed to zero, and then parametric analaysis performed with the right-hand-side increasing to 150 gallons, with the results below.

```
RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 4

   VAR     VAR   PIVOT    RHS    DUAL PRICE    OBJ
   OUT     IN     ROW     VAL   BEFORE PIVOT   VAL

                        0.0000    10.0000     0.000
 SLK 3       V    4    30.0000    10.0000   300.000
     C       B    3    56.2500     1.42857  337.500
     V   SLK 4    4    75.0000     1.00000  356.250
                     150.000      0.0000    356.250
```

The plot of optimal value vs gallons of cream available was also prepared by LINDO:



c. Using LINDO's report, indicate on the graph above the slope of each linear segment and the coordinates of each break-point (profit & gallons of cream).



2. LP model formulation. Buster Sod's younger brother, Marky Dee, operates three ranches in Texas. the acreage and irrigation water available for the three farms are shown below:

Farm	Acreage	Water available (acre-ft)
1	400	1500
2	600	2000
3	300	900

Three crops can be grown. However, the maximum acreage that can be grown of each crop is limited by the amount of appropriate harvesting equipment available. The three crops are described below. Any combination of crops may be grown on a farm.

Crop	Total harvesting capacity (in acres)	Water Reqmts (acre-ft per acre)	Expected profit ($/acre)
Milo	700	6	400
Cotton	800	4	300
Wheat	300	2	100

Using **LINGO**, the following sets were defined, with decision variables:

X_{ij} = # acreas of crop j planted on farm i.

```
MODEL:   ! MARKY DEE SOD'S RANCHES;

SETS:
      FARM/1..3/:ACREAGE, H20_AVAIL;
      CROP/MILO, COTTON, WHEAT/:CAPACITY, H20_RQMT, PROFIT;
      COMBO(FARM,CROP):X;
ENDSETS

DATA:
      ACREAGE   =  400  600 300;
      H20_AVAIL = 1500 2000 900;
      CAPACITY  =   700  800 300;
      H20_RQMT  =     6    4   2;
      PROFIT    =   400  300 100;
ENDDATA

!   INSERT OBJECTIVE & CONSTRAINTS HERE ;

END
```

a. Using LINGO, formulate the LP model to maximize the total expected profit of the three ranches.

Solution:

```
    MAX = @SUM(COMBO(I,J): PROFIT(J)*X(I,J) );
    @FOR(FARM(I):
          @SUM(COMBO(I,J): X(I,J)) <= ACREAGE(I) ;
          @SUM(COMBO(I,J): H20_RQMT(J)*X(I,J)) <= H20_AVAIL(I) ;
    );

    @FOR(CROP(J):
          @SUM(COMBO(I,J): X(I,J)) <= CAPACITY(J) ;
    );
```

b. Add the statements to the accompanying file (HW4_2.lg4) , and solve.

Solution: The primal solution:

```
                     Variable           Value        Reduced Cost
                      X( 1, MILO)       0.0000000           0.0000000
                    X( 1, COTTON)       375.0000            0.0000000
                     X( 1, WHEAT)       0.0000000           33.33333
                      X( 2, MILO)       50.00000            0.0000000
                    X( 2, COTTON)       425.0000            0.0000000
                     X( 2, WHEAT)       0.0000000           33.33333
                      X( 3, MILO)       150.0000            0.0000000
                    X( 3, COTTON)       0.0000000           0.0000000
                     X( 3, WHEAT)       0.0000000           33.33333
```

The dual solution:

Row	Slack or Surplus	Dual Price
2	25.00000	0.0000000
3	0.0000000	66.66667
4	125.0000	0.0000000
5	0.0000000	66.66667
6	150.0000	0.0000000
7	0.0000000	66.66667
8	500.0000	0.0000000
9	0.0000000	33.33333
10	300.0000	0.0000000