56:171 Operations Research
Homework #3 - Due Friday, September 20, 2002

1. Revised Simplex Method Consider the LP problem

$$\text{Maximize } z = 3x_1 - x_2 + 2x_3$$

subject to

$$\begin{cases} x_1 + x_2 + x_3 \le 15 \\ 2x_1 - x_2 + x_3 \le 2 \\ -x_1 + x_2 + x_3 \le 4 \end{cases}$$

$$\& \ x_j \ge 0, j = 1,2,3$$

a. Let x_4, x_5, & x_6 denote the slack variables for the three constraints, and write the LP with equality constraints.

After several iterations of the revised simplex method, the basis B={4,3,2} and the basis inverse matrix is

$$\left(A^B\right)^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$$

b. Proceed with one iteration of the revised simplex method, by
   i. computing the simplex multiplier vector π
   ii. “pricing”, i.e., computing the “relative profits”, of the nonbasic columns
   iii. selecting the column to enter the basis
   iv. computing the substitution rates of the entering column
   v. selecting the variable to leave the basis
   vi. updating the basis inverse matrix.

c. Write the dual of the above LP (i.e. with equality constraints & slack variables) in (a).

d. Substitute the vector π which you computed above in step (i) above to test whether it is feasible in the dual LP. Which constraint(s) if any are violated? How does this relate to the results in step (ii) above?

2. LP formulation: *Staffing a Call Center* (Case 3.3, pages 106-108, *Intro. to O.R.* by Hillier & Lieberman) Answer parts (a), (b), & (c) on page 108, using LINGO with sets to enter the model.

"California Children's Hospital has been receiving numerous customer complaints because of its confusing, decentralized appointment and registration process. When customers want to make appointments or register child patients, they must contact the clinic or department they plan to visit. Several problems exist with this current strategy. Parents do not always know the most appropriate clinic or department they must visit to address their children's ailments. They therefore spend a significant amount of time on the phone being transferred from clinic to clinic until they reach the most appropriate clinic for their needs. The hospital also does not publish the phone numbers of all clinics and departments, and parents must therefore invest a large amount of time in detective work to track down the correct phone number. Finally, the various clinics and departments do not communicate with each other. For example, when a doctor schedules a referral with a colleague located in another department or clinic that department or clinic almost never receives word of the referral. The parent must contact the correct department or clinic and provide the needed referral information.

In efforts to reengineer and improve its appointment and registration process, the children's hospital has decided to centralize the process by establishing one call center devoted exclusively to appointments and registration. The hospital is currently in the middle of the planning stages for the call center. Lenny Davis, the hospital manager, plans to operate the call center from 7 a.m. to 9 p.m. during the weekdays.

Several months ago, the hospital hired an ambitious management consulting firm, Creative Chaos Consultants, to forecast the number of calls the call center would receive each hour of the day. Since all appointment and registration- related calls would be received by the call center, the consultants decided that they could forecast the calls at the call center by totaling the number of appointment and registration-related calls received by all clinics and departments. The team members visited all the clinics and departments, where they diligently recorded every call relating to appointments and registration. they then totaled these calls and altered the totals to account for calls missed during data collection. They also altered totals to account for repeat calls that occurred when the same parent called the hospital many times because of the confusion surrounding the decentralized process. Creative Chaos Consultants determined the average number of calls the call center should expect during each hour of a weekday. The following table provides the forecasts:

Work Shift	*Average Number of Calls per hour*
7 a.m. — 9 a.m.	40
9 a.m. —11 a.m.	85
11 a.m. —1 p.m.	70
1 p.m.—3 p.m.	95
3 p.m. —5 p.m.	80
5 p.m.—7 p.m.	35
7 p.m.—9 p.m.	10

After the consultants submitted these forecasts, Lenny became interested in the percentage of calls from Spanish speakers since the hospital services many Spanish-speaking patients. Lenny knows that he has to hire some operators who speak Spanish to handle these calls. The consultants performed further data collection and determined that on average, 20 percent of the calls were from Spanish speakers.

Given these call forecasts, Lenny must now decide how to staff the call center during each 2 hour shift of a weekday. During the forecasting project, Creative Chaos Consultants closely observed the operators working at the individual clinics and departments and determined the number of calls that operators could process per hour. The consultants informed Lenny that an operator is able to process an average of six calls per hour. Lenny also knows that he has both full-time and part-time workers available to staff the call center. A full-time employee works 8 hours per day, but because of paperwork that must be completed, the employee spends only 4 hours per day on the phone. To balance the schedule, the employee alternates the 2-hour shifts between answering phones and completing paperwork. Full-time employees can start their day either by answering phones or by completing paperwork on the first shift. The full-time employees speak either Spanish or English, but none of them are bilingual. Both Spanish-speaking and English-speaking employees are paid $10 per hour for work before 5 p.m. and $12 per hour for work after 5 p.m. The full-time employees can begin work at the beginning of the 7 a.m. to 9 a.m. shift, 9 a.m. to 11 a.m. shift, 11 a.m. to 1 p.m. shift, or 1 p.m. to 3 p.m. shift. The part-time employees work for 4 hours, only answer calls, and only speak English. They can start work at the beginning of the 3 p.m.—5 p.m. shift or the 5 p.m.—7 p.m. shift, and like the full-time employees, they are paid $10 per hour for work before 5 p.m. and $12 per hour for work after 5 p.m.

For the following analysis, consider only the labor cost for the time employees spend answering phones. The cost for paperwork time is charged to other cost centers.

a. How many Spanish-speaking operators and how many English-speaking operators does the hospital need to staff the call center during each 21-hour shift of the day in order to answer all calls? Pleas provide an integer number since half a human operator makes no sense

b. Lenny needs to determine how many full-time employees who speak Spanish, full-time employees who speak English, and part-time employees he should hire to begin on each shift. Creative Chaos Consultants advise him that linear programming can be used to do this in such a way as to minimize operating costs while answering all calls. Formulate a linear programming model of this problem.

c. Obtain an optimal solution for the LP model formulated in part (b) to guide Lenny's decision.

3. Sensitivity Analysis (exercise 6.7-18, pages 296-297, *Intro. to O.R.* by Hillier & Lieberman) Answer parts (a) through (e), using the information shown on page 296.

Ken and Larry, Inc., supplies its ice cream parlors with three flavors of ice cream: chocolate, vanilla, and banana. Because of extremely hot weather and a high demand for its products, the company has run short of its supply of ingredients: milk, sugar, & cream. Hence, they will not be able to fill all the orders received from their retail outlets, the ice cream parlors. Owing to these circumstances, the company has decided to choose the amount of each product to produce that will maximize total profit, given the constraints on supply of the basic ingredients. The chocolate, vanilla, and banana flavors generate, respectively, $1.00, $0.90, and $0.95 per profit per gallon sold. The company has only 200 gallons of milk, 150 pounds of sugar, and 60 gallons of cream left in its inventory. The LP formulation for this problem has variables C, V, and B representing gallons of chocolate, vanilla, and banana ice cream produced, respectively.

LINDO model

```
!    Ken & Larry Ice Cream – from Intro to O.R.
!    Hillier &(7th ed) p. 296

MAXIMIZE  C+0.9V+0.95B
ST

0.45C + 0.50V + 0.40B <= 200  ! milk resource
0.50C + 0.40V + 0.40B <= 150  ! sugar resource
0.10C + 0.15V + 0.20B <= 60   ! cream resource

END
```

```
        OBJECTIVE FUNCTION VALUE
        1)      341.2500

  VARIABLE        VALUE          REDUCED COST
         C         0.000000         0.037500
         V       300.000000         0.000000
         B        75.000000         0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        20.000000         0.000000
        3)         0.000000         1.875000
        4)         0.000000         1.000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
C	1.000000	0.037500	INFINITY
V	0.900000	0.050000	0.012500
B	0.950000	0.021429	0.050000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	200.000000	INFINITY	20.000000
3	150.000000	10.000000	30.000000
4	60.000000	15.000000	3.750000

a. What is the optimal profit and the optimal solution?

b. Suppose the profit per gallon of banana changes to $1.00. Will the optimal solution change, and what can be said about the effect on total profit?

c. Suppose the profit per gallon of banana changes to 92 cents. Will the optimal solution change, and what can be said about the effect on total profit?

d. Suppose the company discovers that 3 gallons of cream have gone sour and so must be thrown out. Will the optimal solution change, and what can be said about the effect on total profit?

e. Suppose that the company has the opportuntiy to buy an additional 15 pounds of sugar at a total cost of $15. Should they buy it? Explain!