56:171 Operations Research
Homework #1 Solutions –Fall 2002

1. The Keyesport Quarry has two different pits from which it obtains rock. The rock is run through a crusher to produce two products: concrete grade stone and road surface chat. Each ton of rock from the South pit converts into 0.75 tons of stone and 0.25 tons of chat when crushed. Rock from the North pit is of different quality. When it is crushed it produces a "50-50" split of stone and chat. The Quarry has contracts for 60 tons of stone and 40 tons of chat this planning period. The cost per ton of extracting and crushing rock from the South pit is 1.6 times as costly as from the North pit.

   a. What are the decision variables in the problem? *Be sure to give their definitions, not just their names!*
   Answer: S_ROCK = # of tons of rocks from the South pit.
   N_ROCK = # of tons of rocks from the North pit.

   b. There are two constraints for this problem.
   - State them in words.

   Answer:
   1. # of tons of concrete grade stone which is the sum of concrete grade stone from South pit and concrete grade stone from North pit is bigger than 60.
   2. # of tons of road surface chat which is the sum of road surface chat from South pit and road surface chat from North pit is bigger than 40.

   - State them in equation or inequality form.

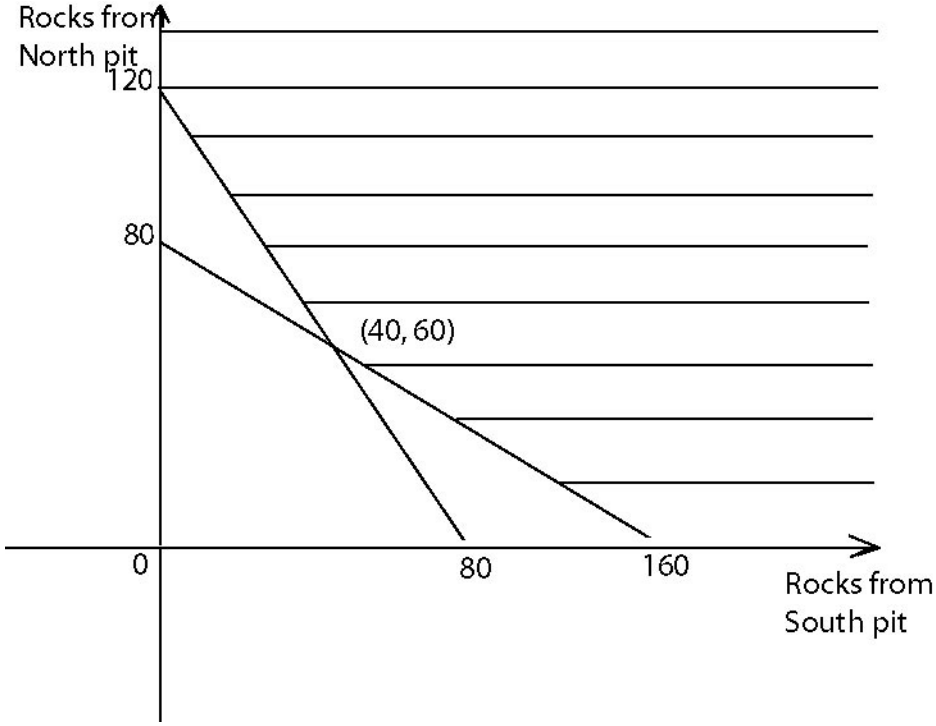
   Answer:
   $$0.75\ S\_ROCK + 0.5\ N\_ROCK \geq 60$$
   $$0.25\ S\_ROCK + 0.5\ N\_ROCK \geq 40$$

   c. State the objective function.
   Answer:
   Total cost of the processing rocks which is the sum of the cost of processing rocks from South pit and North pit in the unit of the cost of processing 1 ton's processing North pit (to be minimized):

   $$\text{Min } 1.6\ S\_ROCK + N\_ROCK$$

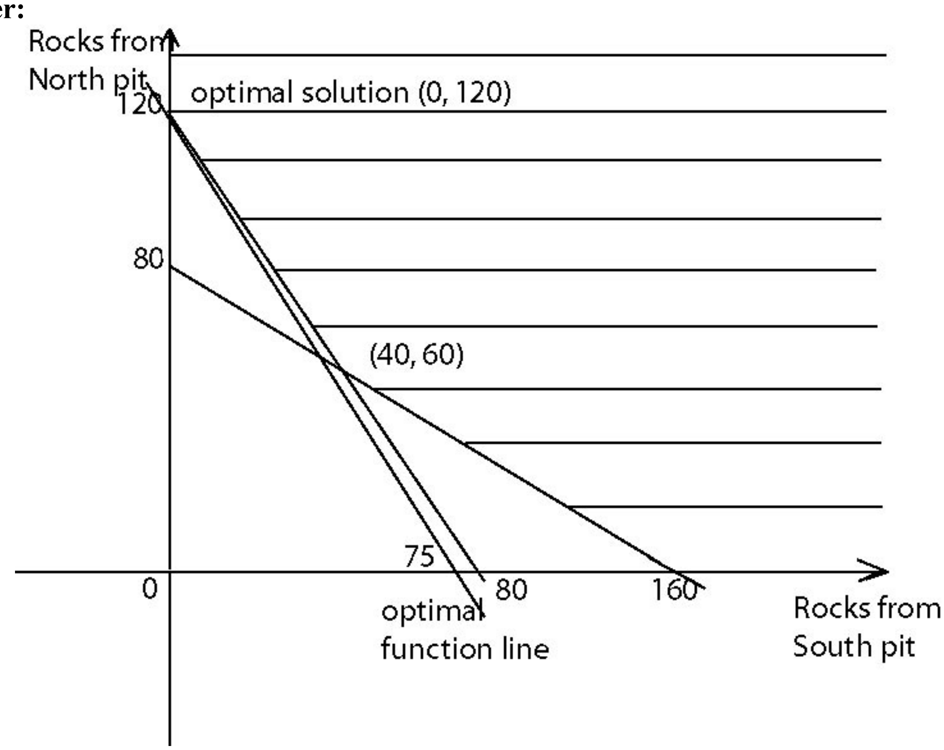
   d. Graph the feasible region (in 2 dimensions) for this problem.

Answer:



e. Draw an appropriate objective function line on the graph and indicate graphically and numerically the optimal solution.

Answer:



f. Use LINDO (or other appropriate LP solver) to compute the optimal solution.

Answer:

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 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      120.0000

  VARIABLE        VALUE          REDUCED COST
    S_ROCK         0.000000          0.100000
    N_ROCK       120.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -2.000000
        3)        20.000000          0.000000
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2. a. Draw the feasible region of the following LP:

Maximize $5X_1 + 2X_2$

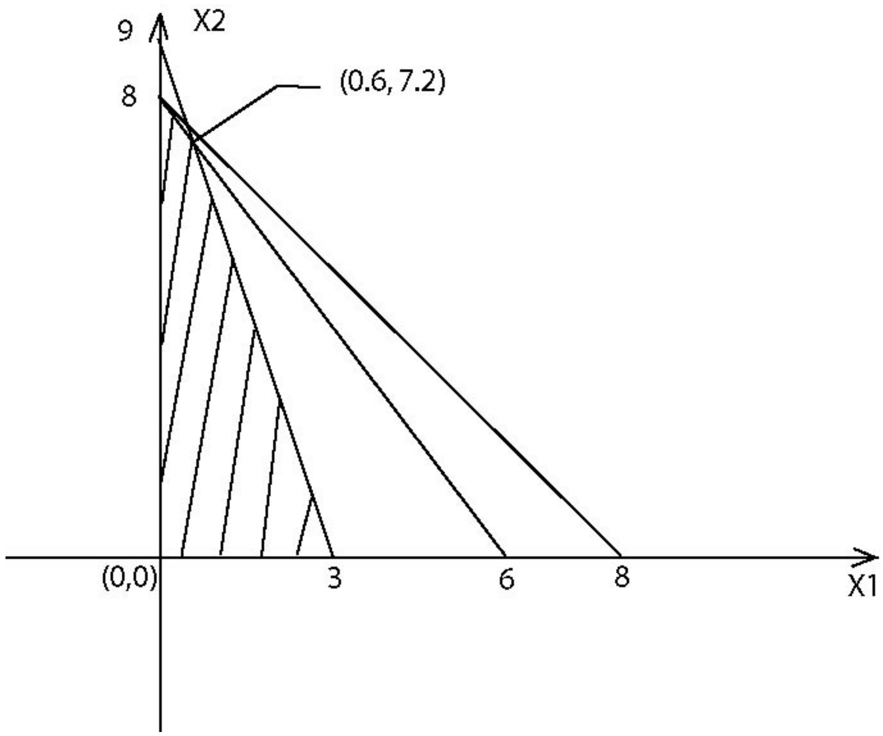
subject to $4X_1 + 3X_2 \leq 24$

$X_1 + X_2 \leq 8$

$3X_1 + X_2 \leq 9$

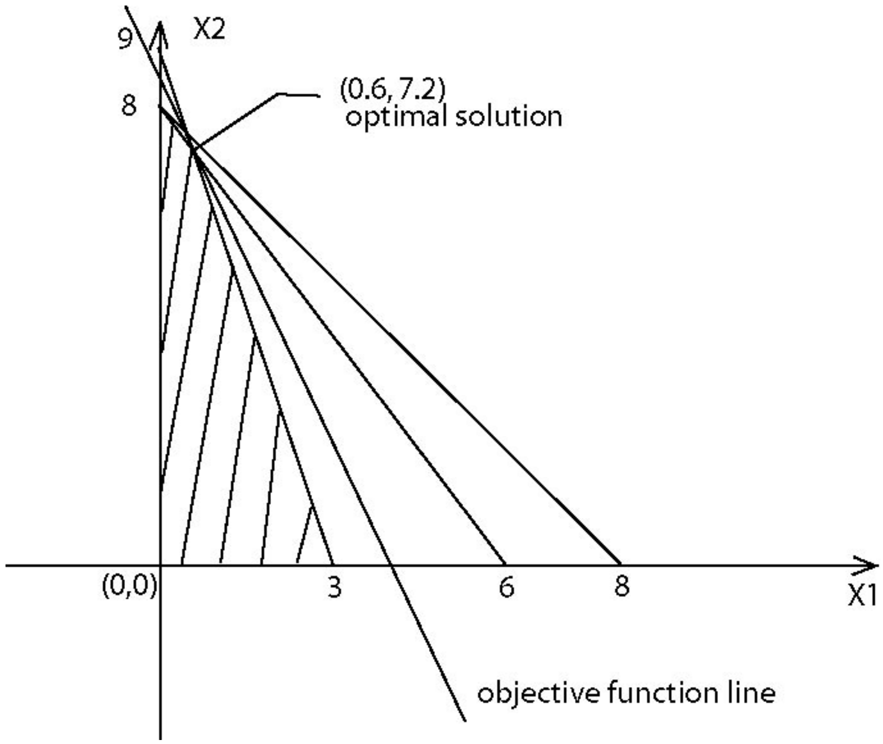
$X_1 \geq 0, X_2 \geq 0$

Answer:



Note that the point (0,8) is the intersection of three boundary lines, indicating a degeneracy!

b. Indicate on the graph the optimal solution.
Answer:



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3. a. Compute the inverse of the matrix (showing your computational steps):

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -2 & -1 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ -2 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 + 2R_1 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & -1 & -1 & 2 & 0 & 1 \end{bmatrix} \begin{matrix} R_2 = R_2 / 2 \\ R_3 = R_3 + R_2 / 2 \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & -1/2 & 1/2 & 0 \\ 0 & 0 & -1/2 & 3/2 & 1/2 & 1 \end{bmatrix} \begin{matrix} R_1 = R_1 - 2R_3 \\ R_2 = R_2 + R_3 \\ R_3 = -2R_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -2 & -1 & -2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 & -1 & -2 \end{bmatrix}$$

Hence $A^{-1} = \begin{bmatrix} -2 & -1 & -2 \\ 1 & 1 & 1 \\ -3 & -1 & -2 \end{bmatrix}$

b. Find a solution (if one exists) of the equations:

$$\begin{cases} X_1 + 2X_2 - X_3 = 4 \\ 2X_1 - X_2 + 2X_3 = 15 \\ 3X_2 - 2X_3 = -5 \end{cases}$$

Answer:

Using Gauss elimination, we reduce the augmented coefficient matrix to echelon form:

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & -1 & 2 & 15 \\ 0 & 3 & -2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & -5 & 4 & 7 \\ 0 & 3 & -2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & -4/5 & -7/5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

By back substitution $\begin{cases} X_1 = 4 - 2X_2 + X_3 = 8 \\ X_2 = -7/5 + (4/5)X_3 = -3 \\ X_3 = -2 \end{cases}$

Note that this could also have been done by Gauss-Jordan elimination.