56:171 Operations Research
Homework #1 - Due Wednesday, September 4, 2002

1. The Keyesport Quarry has two different pits from which it obtains rock. The rock is run through a crusher to produce two products: concrete grade stone and road surface chat. Each ton of rock from the South pit converts into 0.75 tons of stone and 0.25 tons of chat when crushed. Rock from the North pit is of different quality. When it is crushed it produces a "50-50" split of sone and chat. The Quarry has contracts for 60 tons of stone and 40 tons of chat this planning period. The cost per ton of extracting and crushing rock from the South pit is 1.6 times as costly as from the North pit.
   a. What are the decision variables in the problem? *Be sure to give their definitions, not just their names!*
   b. There are two constraints for this problem.
      - State them in words.
      - State them in equation or inequality form.
   c. State the objective function.
   d. Graph the feasible region (in 2 dimensions) for this problem.
   e. Draw an appropriate objective function line on the graph and indicate graphically and numerically the optimal solution.
   f. Use LINDO (or other appropriate LP solver) to compute the optimal solution.

2. a. Draw the feasible region of the following LP:

$$\begin{array}{ll} \text{Maximize} & 5X_1 + 2X_2 \\ \text{subject to} & 4X_1 + 3X_2 \le 24 \\ & X_1 + X_2 \le 8 \\ & 3X_1 + X_2 \le 9 \\ & X_1 \ge 0, X_2 \ge 0 \end{array}$$

   b. Indicate on the graph the optimal solution.

3. a. Compute the inverse of the matrix (showing your computational steps):

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -2 & -1 & 1 \end{bmatrix}$$

   b. Find a solution (if one exists) of the equations:

$$\begin{cases} X_1 + 2X_2 - X_3 = 4 \\ 2X_1 - X_2 + 2X_3 = 15 \\ 3X_2 - 2X_3 = -5 \end{cases}$$