56:171 Operations Research
Homework #12 --Due Friday, 6 December 2002

1. ***Power Plant Construction.*** The construction of eight power plants is to be scheduled over an eight-year period, with a restriction that **no more than three** may be built during each one-year period. The cost per plant is 125M$. In addition, a cost of 100M$ is incurred in each year that construction is scheduled. (Assume for sake of simplicity that completion of the plant occurs

Year	1	2	3	4	5	6	7	8
# plants required at end of year	2	3	3	4	5	5	7	8

The table above gives the *cumulative* # of plants required by the end of each year, i.e., a total of 8 additional plants are required by the end of year #8.

The utility company wants to minimize the present value of the construction costs, using an internal rate-of-return of 20%.

A deterministic dynamic programming model was developed, with stages = years (stage 1 = year 1, etc.), state = cumulative # of plants built, and decision = # plants to be added in each stage. The tables below show the tables of computations of the optimal schedule:

```
                    ---Stage 8---
    s \ x:    0          1   | Minimum
    7   | 99999.999   225.000|   225.000
    8   |     0.000 99999.999|     0.000

                    ---Stage 7---
    s \ x:    0          1          2          3   | Minimum
    5   | 99999.999 99999.999   537.500   475.000|   475.000
    6   | 99999.999   412.500   350.000 99999.999|   350.000
    7   |   187.500   225.000 99999.999 99999.999|   187.500
    8   |     0.000 99999.999 99999.999 99999.999|     0.000

                    ---Stage 6---
    s \ x:    0          1          2          3   | Minimum
    5   |   395.833   516.667   506.250   475.000|   395.833
    6   |   291.667  |_______|  350.000 99999.999|   291.667
    7   |   156.250   225.000 99999.999 99999.999|   156.250
    8   |     0.000 99999.999 99999.999 99999.999|     0.000

                    ---Stage 5---
    s \ x:    0          1          2          3   | Minimum
    4   | 99999.999   554.861   593.056   605.208|   554.861
    5   |   329.861   468.056   480.208   475.000|   329.861
    6   |   243.056   355.208   350.000 99999.999|   243.056
    7   |   130.208   225.000 99999.999 99999.999|   130.208
    8   |     0.000 99999.999 99999.999 99999.999|     0.000

                    ---Stage 4---
    s \ x:    0          1          2          3   | Minimum
    3   | 99999.999   687.384   624.884   677.546|   624.884
    4   |   462.384   499.884   552.546   583.507|   462.384
    5   |   274.884   427.546   458.507   475.000|   274.884
    6   |   202.546   333.507   350.000 99999.999|   202.546
    7   |   108.507   225.000 99999.999 99999.999|   108.507
    8   |     0.000 99999.999 99999.999 99999.999|     0.000
```

---Stage 3---

s \ x:	0	1	2	3	Minimum
3	520.737	610.320	579.070	643.789	520.737
4	385.320	454.070	518.789	565.422	385.320
5	229.070	393.789	440.422	475.000	229.070
6	168.789	315.422	350.000	99999.999	168.789
7	90.422	225.000	99999.999	99999.999	90.422
8	0.000	99999.999	99999.999	99999.999	0.000

---Stage 2---

s \ x:	0	1	2	3	Minimum
2	99999.999	658.947	671.100	_______	658.947
3	433.947	546.100	540.892	615.657	433.947
4	321.100	415.892	490.657	550.352	321.100
5	190.892	365.657	425.352	475.000	190.892
6	140.657	300.352	350.000	99999.999	140.657
7	75.352	225.000	99999.999	99999.999	75.352
8	0.000	99999.999	99999.999	99999.999	0.000

---Stage 1---

s \ x:	0	1	2	3	Minimum
0	99999.999	99999.999	899.123	836.623	836.623
1	99999.999	774.123	711.623	742.583	711.623
2	549.123	586.623	617.583	634.077	549.123
3	361.623	492.583	509.077	592.214	361.623
4	267.583	384.077	467.214	537.793	267.583
5	159.077	342.214	412.793	475.000	159.077
6	117.214	287.793	350.000	99999.999	117.214
7	62.793	225.000	99999.999	99999.999	62.793
8	0.000	99999.999	99999.999	99999.999	0.000

a. What are the two values which are missing in the tables above, for stages 7 and 2?

b. What is the present value of the construction costs of the optimal schedule?

c. What is the optimal construction schedule?

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
# plants required at end of year	**2**	**3**	**3**	**4**	**5**	**5**	**7**	**8**
# plants to be constructed	__	__	__	__	__	__	__	__
cumulative # of plants added								

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2. ***System Reliability*** A system is to be composed of five unreliable components with unit weights and reliabilities as shown in the table below.
The designer wishes to select the level of redundancy to maximize the system reliability, subject to a restriction that the total weight of the system cannot be more than 15 kg. (Note that the system requires *at least one* unit of each component for successful operation.)

Component	1	2	3	4	5
Weight (kg)	1	2	3	1	2
Reliability (%)	80	85	90	75	70

A deterministic dynamic programming model was developed, in which the components are assumed to be considered one at a time, beginning with component 5. The optimal value $f_n(s_n)$ is defined to be the maximum reliability which can be achieved for the subsystem of components n, n-1, …1 if s_n kilograms can be allocated to these n components. The tables below show the computation of the optimal design. (The value –99.9999 in the table indicates an infeasible combination of state and decision.)

---Stage 1---

s \ x:	1	2	3	Maximum
1	0.8000	-99.9999	-99.9999	0.8000
2	0.8000	0.9600	-99.9999	0.9600
3	0.8000	0.9600	0.9920	0.9920

---Stage 2---

s \ x:	1	2	3	Maximum
3	0.6800	-99.9999	-99.9999	0.6800
4	0.8160	-99.9999	-99.9999	0.8160
5	0.8432	0.7820	-99.9999	0.8432
6	0.8432	0.9384	-99.9999	0.9384
7	0.8432	0.9697	0.7973	0.9697
8	0.8432	0.9697	______	0.9697
9	0.8432	0.9697	0.9887	0.9887

---Stage 3---

s \ x:	1	2	3	Maximum
6	0.6120	-99.9999	-99.9999	0.6120
7	0.7344	-99.9999	-99.9999	0.7344
8	0.7589	-99.9999	-99.9999	0.7589
9	0.8446	0.6732	-99.9999	0.8446
10	0.8727	0.8078	-99.9999	0.8727
11	0.8727	0.8348	-99.9999	0.8727
12	0.8898	0.9290	0.6793	0.9290

---Stage 4---

s \ x:	1	2	3	Maximum
7	0.4590	-99.9999	-99.9999	0.4590
8	0.5508	0.5738	-99.9999	0.5738
9	0.5692	0.6885	0.6024	0.6885
10	0.6334	0.7115	0.7229	0.7229
11	0.6545	0.7918	0.7470	0.7918
12	0.6545	0.8182	0.8314	0.8314
13	0.6968	0.8182	0.8591	0.8591

---Stage 5---

s \ x:	1	2	3	Maximum
9	0.3213	-99.9999	-99.9999	0.3213
10	0.4016	-99.9999	-99.9999	0.4016
11	0.4820	0.4177	-99.9999	0.4820
12	0.5060	0.5221	-99.9999	0.5221
13	______	0.6265	0.4466	0.6265
14	0.5820	0.6579	0.5583	0.6579
15	0.6014	0.7205	0.6699	0.7205

a. What is the reliability of the system if only a single unit of each component were included?

b. Compute the two missing values in the tables above (in stages 2 & 5).

c. What is the maximum reliability of a system weighing 15 kg?

d. What is the optimal number of units of each component?

Component	1	2	3	4	5
# units					

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3. ***Stochastic Production Planning*** An optimal production policy is required for a product with random daily demand:

Demand d	0	1	2	3
Probability{d}	20%	30%	30%	20%

The state of the system is the "inventory position" at the end of the day, which if positive is the stock on hand, and if negative is the number of backorders which have accumulated. A maximum of 1 backorder is allowed, and a maximum of 4 units may be held in storage at the end of each day. (For simplicity, assume that any inventory in excess of 4 units is discarded.) There is a storage cost of $1 per unit, based upon the end-of-day inventory, and a shortage cost of $15 per unit, based upon any backorders at the end of the day.

The next day's production must be scheduled after the inventory position is determined. Assume that production on the next day is completed in time to meet any demand that occurs that same day. The maximum number of units which can be produced each day is 3. If production is scheduled, there is a setup cost of $10, plus a $3 marginal cost per unit.

Finally, at the end of the planning period (6 days), a salvage value of $2 per unit is received for any remaining inventory.

A backward recursion is used, where the stage **n** is the number of days remaining in the planning period, the state s_n is the inventory position (–1 = "1 backorder", +3 = "three units in stock", etc.), and optimal value function $f_n(s_n)$ = minimum expected cost for stages n, n-1, ...2, 1 if the inventory position at stage n is s_n.

---Stage 1---

s \ x:	0	1	2	3	Minimum
¯1	99999.999	50.400	44.600	38.200	38.200
0	22.400	26.600	20.200	16.000	16.000
1	14.600	18.200	14.000	15.000	14.000
2	6.200	12.000	13.000	14.400	6.200
3	0.000	11.000	12.400	14.400	0.000
4	¯1.000	10.400	12.400	15.000	¯1.000

---Stage 2---

s \ x:	0	1	2	3	Minimum
¯1	99999.999	61.760	57.700	51.880	51.880
0	33.760	39.700	33.880	28.260	28.260
1	27.700	31.880	26.260	24.460	24.460
2	19.880	24.260	22.460	21.740	19.880
3	12.260	20.460	19.740	21.200	12.260
4	8.460	17.740	19.200	22.000	8.460

---Stage 3---

s \ x:	0	1	2	3	Minimum
¯1	99999.999	75.156	70.310	64.168	64.168
0	47.156	52.310	46.168	40.406	40.406
1	40.310	44.168	38.406	36.226	36.226
2	32.168	36.406	34.226	32.884	32.168
3	24.406	32.226	30.884	31.220	24.406
4	20.226	28.884	29.220	31.460	20.226

---Stage 4---

s \ x:	0	1	2	3	Minimum
¯1	99999.999	87.416	82.451	76.257	76.257
0	59.416	64.451	58.257	52.481	52.481
1	52.451	56.257	50.481	48.263	48.263
2	44.257	48.481	46.263	44.868	44.257
3	36.481	44.263	42.868	43.062	36.481
4	32.263	40.868	41.062	43.226	32.263

---Stage 5---

s \ x:	0	1	2	3	Minimum
¯1	99999.999	99.502	94.525	88.326	88.326

0	71.502	76.525	70.326	64.548	64.548
1	64.525	68.326	62.548	60.326	60.326
2	56.326	60.548	58.326	56.927	56.326
3	48.548	56.326	54.927	55.106	48.548
4	44.326	52.927	53.106	55.263	44.326

---Stage 6---

s \ x:	0	1	2	3	Minimum
-1	99999.999	111.570	106.593	100.393	100.393
0	83.570	88.593	82.393	76.615	76.615
1	76.593	80.393	|______|	72.393	72.393
2	68.393	72.615	70.393	68.993	68.393
3	60.615	68.393	66.993	67.171	60.615
4	56.393	64.993	65.171	67.326	56.393

a. Compute the missing value in the table for stage 6 (the first day of the planning period).

Suppose that initially there is one unit in stock.

b. What is the expected total cost of production, storage, & shortages during the six days, minus salvage value received for any final inventory?

c. What is the optimal production quantity for the first day (stage 6)?

d. What is the optimal production policy for each of the first five days?

If inventory position is…	Produce…
-1	
0	
1	
2	
3	
4	

e. What is the optimal production policy for the final day (stage 1)?

If inventory position is…	Produce…
-1	
0	
1	
2	
3	
4	

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4. ***Equipment Replacement*** A machine initially costs $5000. The annual operating cost increases with the age of the machine, while the trade-in value decreases, according to the following table (assume that management policy is to keep no machine more than four years) :

Year of operation	**1**	**2**	**3**	**4**
Operating cost($)	1000	1200	1500	1800
Trade-in value at end of year ($)	3500	3000	2200	1500

Assume an interest rate of 12%, that initially you have a new machine, and that the current machine at the end of eight years will be traded in, but not replaced.

Determine the replacement strategy for the next 8 years which will minimize the present value of replacement & operating costs, minus trade-in value.

What is the present value of the costs for the eight-year period