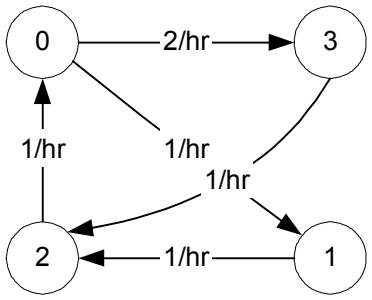
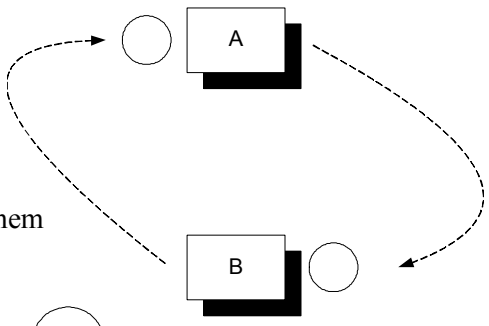
56:171 Operations Research
Homework #11 –due November 22, 2002

1. Consider the continuous-time Markov chain at the right.
   a. Write the matrix Λ of transition rates.
   b. Write the system of linear equations which determine the steady-state probabilities.
   c. Solve the equations to obtain the steady-state distribution.



2. Four customers circulate between two single-server systems, i.e., all customers leaving server A enter the queue of server B, and vice versa. Server A works can serve 2 customers per hour, while server B works at half the rate of server A.
   Define the states of a continuous-time Markov chain model of this system to be
   (i,j) where i=# customers in queue or being served at A, and
   j=# customers in queue or being served at B.



   a. Indicate the possible transitions from each state, and label them with the transition rates:

   b. Is this a birth-death process?

   c. Compute the steady-state probability distribution to find the expected number of customers in subsystem A and subsystem B.

3. At a taxi stand, taxis looking for customers and customers looking for taxis arrive according to Poisson processes with rates λ_t = 1/minute and λ_c = 2/minute, respectively.

A taxi will always wait if no customers are at the stand. However, an arriving customer waits only if there are 3 or fewer customers already waiting.

Define a continuous-time Markov chain model with states:

(–3): three customers waiting
(–2): two customers waiting
(–1): one customer waiting
(0): neither customer nor taxi waiting
(+1): one taxi waiting
(+2): two taxis waiting
(+3): three taxis waiting
etc.

Draw the transition diagram.

b. Compute the steady-state probability distribution.

c. What fraction of the time will customers be waiting for taxis?

4. Deterministic Dynamic Programming Production is to be scheduled for the next 8 weeks. You are given the quantity required during each of those weeks:

Week#	1	2	3	4	5	6	7	8
Demand	2	3	1	2	3	1	1	3

The production capacity limits the number produced during each week to no more than 4. There is a $10 setup cost which is incurred during each week that production is scheduled, and a marginal cost of $2 per unit produced for the first three units produced per week, and $3 for the fourth unit.

The state of the system is the inventory level at the end of the week, after any production has occurred and demand has been satisfied. The maximum quantity which may be stored is 6, and a holding cost is incurred of $1 per unit per week in storage for the first 4 units and $2 each for the fifth and sixth unit.. Shortages are not allowed. At the end of the 8 week planning period, the item is obsolete and has a salvage value of $2 per unit for 3 or fewer, and only $1 per unit for any units in excess of 3.

In the tables below, the stages are defined in the natural way, i.e., stage 1 is the first week and stage 8 is the final week. The value "9999.99" in the table indicates an infeasible combination of state & decision variables.

a. Two values are missing in the tables (stages 1 & 2). Compute their values and insert them into the tables.

Suppose that initially you have 2 units in inventory.

b. Determine the number of units to be produced during each of the 8 weeks *(where week #1 is the first week, i.e., stage 1, etc.)*

Week#	1	2	3	4	5	6	7	8
Beginning Inventory	**2**							
Production Quantity								

c. What is the minimum total cost (production cost + storage cost – salvage value) for the 8-week period?

---Stage 8---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	9999.99	9999.99	16.00	17.00	16.00
1	9999.99	9999.99	15.00	15.00	16.00	15.00
2	9999.99	14.00	14.00	14.00	15.00	14.00
3	3.00	13.00	13.00	13.00	15.00	3.00
4	2.00	12.00	12.00	13.00	15.00	2.00
5	2.00	12.00	13.00	14.00	16.00	2.00
6	2.00	13.00	14.00	15.00	9999.99	2.00

---Stage 7---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	28.00	29.00	30.00	22.00	22.00
1	17.00	28.00	29.00	20.00	22.00	17.00
2	17.00	28.00	19.00	20.00	23.00	17.00
3	17.00	18.00	19.00	21.00	24.00	17.00
4	7.00	18.00	20.00	22.00	9999.99	7.00
5	8.00	20.00	22.00	9999.99	9999.99	8.00
6	10.00	22.00	9999.99	9999.99	9999.99	10.00

---Stage 6---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	34.00	31.00	33.00	36.00	31.00
1	23.00	30.00	32.00	34.00	27.00	23.00
2	19.00	31.00	33.00	25.00	29.00	19.00
3	20.00	32.00	24.00	27.00	32.00	20.00
4	21.00	23.00	26.00	30.00	9999.99	21.00
5	13.00	26.00	30.00	9999.99	9999.99	13.00
6	16.00	30.00	9999.99	9999.99	9999.99	16.00

---Stage 5---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	9999.99	9999.99	47.00	42.00	42.00
1	9999.99	9999.99	46.00	40.00	39.00	39.00
2	9999.99	45.00	39.00	37.00	41.00	37.00
3	34.00	38.00	36.00	39.00	43.00	34.00
4	27.00	35.00	38.00	41.00	36.00	27.00
5	25.00	38.00	41.00	35.00	41.00	25.00
6	28.00	41.00	35.00	40.00	9999.99	28.00

---Stage 4---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	9999.99	56.00	55.00	56.00	55.00
1	9999.99	55.00	54.00	54.00	54.00	54.00
2	44.00	53.00	53.00	52.00	48.00	44.00
3	42.00	52.00	51.00	46.00	47.00	42.00
4	41.00	50.00	45.00	45.00	51.00	41.00
5	40.00	45.00	45.00	50.00	9999.99	40.00
6	35.00	45.00	50.00	9999.99	9999.99	35.00

---Stage 3---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	67.00	68.00	60.00	61.00	60.00
1	56.00	67.00	59.00	59.00	61.00	56.00
2	56.00	58.00	58.00	59.00	61.00	56.00
3	47.00	57.00	58.00	59.00	57.00	47.00
4	46.00	57.00	58.00	55.00	9999.99	46.00
5	47.00	58.00	55.00	9999.99	9999.99	47.00
6	48.00	55.00	9999.99	9999.99	9999.99	48.00

---Stage 2---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	9999.99	9999.99	76.00	75.00	75.00
1	9999.99	9999.99	75.00	73.00	76.00	73.00
2	9999.99	74.00	72.00	74.00	68.00	68.00
3	63.00	71.00	73.00	66.00	68.00	63.00
4	60.00	72.00	65.00	66.00	70.00	60.00
5	62.00	65.00	66.00	69.00	73.00	
6	55.00	66.00	69.00	72.00	9999.99	55.00

---Stage 1---

s \ x:	0	1	2	3	4	Minimum
0	9999.99	9999.99	89.00	89.00	87.00	87.00
1	9999.99	88.00	88.00	85.00	83.00	83.00
2	77.00	87.00		81.00	81.00	77.00
3	76.00	83.00	80.00	79.00	84.00	76.00
4	72.00	79.00	78.00	82.00	78.00	72.00
5	69.00	78.00	82.00	77.00	9999.99	69.00
6	68.00	82.00	77.00	9999.99	9999.99	68.00