

56:171 Operations Research
Homework #10 –due November 15, 2002

1. (#18.4-3, page 927, of *Intro to O.R.* by Hillier & Lieberman) The Garrett-Tompkins Company provides three copy machines in its copying room for the use of its employees. However, due to recent complaints about considerable time being wasted waiting for a copier to become free, management is considering adding one or more additional copy machines.

During the 2,000 working hours per year, employees arrive at the copying room according to a Poisson process at a mean rate of 30 per hour. The time each employee needs with a copy machine is believed to have an exponential distribution with a mean of 5 minutes. The lost productivity due to an employee spending time in the copying room is estimated to cost the company an average of \$25 per hour. Each copy machine is leased for \$3,000 per year. (Assume that the number of employees is so large that it could be assumed to be infinite.)

- a. According to this model, what is currently the average number of employees waiting to use a copier?
- b. What is currently the number of person-hours per year that is lost because of the waiting line?
- c. Determine how many copy machines the company should have in order to minimize its expected total cost per hour.



2. (#17.6-14, page 898, of *Intro to O.R.* by Hillier & Lieberman) Airplanes arrive for takeoff at the runway of an airport according to a Poisson process at a mean rate of 20/hour (i.e., time between arrivals has exponential distribution with mean $1/20$ hour = 3 minutes). The time required for an airplane to take off has an exponential distribution with a mean of 2 minutes, and this process must be completed before the next airplane can begin to take off.

Because a brief thunderstorm has just begun, all airplanes which have not commenced takeoff have just been grounded temporarily. However, airplanes continue to arrive at the runway during the thunderstorm to await its end.

Assuming steady-state operation before the thunderstorm, determine the expected number of airplanes that will be waiting to take off at the end of the thunderstorm if it lasts 30 minutes.

3. A production system consists of two machines which may operate simultaneously but independently. The machines fail randomly, with mean time to failure having exponential distribution with mean 15 hours. There is a single repairman whose responsibility is to return the machines to operating condition. Most of the failures require only an average of 1 hour to be made operational again, but 20% of the failures are severe and require an average of 8 hours. Assume the repair times have exponential distributions.
 - a. List the states of a continuous-time Markov chain model of this system.
 - b. Draw the transition diagram, with transition rates.
 - c. Write down Λ , the transition rate matrix.
 - d. Write down one of the balance equations which describe π , the steady state distribution.
 - e. Compute the steady state distribution. (You may use the APL workspace CTMC, or solve the system of linear equations using other software.)
 - f. What is the utilization of the machines, i.e., the fraction of the time that a machine is operable?
 - g. What is the utilization of the repairman, i.e., the fraction of the time that he is busy?