Consider the Geometric Programming problem

Minimize \( \frac{x_1}{x_2^2} + 4 \frac{x_2}{x_1} \)

subject to

\[ 0 < x_1 \leq \frac{1}{2} \]
\[ 0 < x_2 \]

a. Restate the first constraint in "standard" GP form.

b. Complete the GP dual of the above problem:

Maximize \( \frac{1}{\delta_1^a} \left( \frac{4}{\delta_2^b} \right)^{\beta_2} \left( \frac{2}{\delta_3^c} \right)^{\beta_3} \lambda_1^{\lambda_1} \)

subject to

\[ \delta_1 = \ldots, \delta_2 = \ldots, \delta_3 = \ldots, \lambda_1 = \ldots \]

c. Is the GP dual objective function above: (circle) convex? concave? both? neither?

d. What is the "degree of difficulty" of this problem? ______

e. What are the optimal values of the GP dual variables?

\( \delta_1 = \ldots, \delta_2 = \ldots, \delta_3 = \ldots, \lambda_1 = \ldots \)

f. What is the optimal value of the dual objective function? __________

g. In the optimal solution of the (primal) problem, what fraction of the optimal cost is due to the first term of the objective function, i.e., \( \frac{x_1}{x_2} \)?

h. What is the minimum value of the primal objective function? __________

i. What are the optimal values of \( x_1 \) and \( x_2 \)?

j. If the first constraint were \( 0 < x_1 \leq \frac{1}{4} \) instead, how would you answer part (g)?

Suppose the constraint \( x_2 - x_1 \geq \frac{1}{3} \) were to be added to the problem.

k. Can this new constraint be written in posynomial GP form? (If possible, do so.)

l. What would be the "degree of difficulty" if this new constraint were included? __________