Name ________________________

56:271 Nonlinear Programming
Quiz #5 – Fall 2003

Part ONE: Separable Programming

Consider the nonlinear programming problem:

$$\begin{aligned} &\textit{Minimize} \quad x_1 \ln x_1 + x_2^2 \\ &\textit{subject to} \quad x_1 + 2x_2 = 1 \\ &0 \le x_1 \le 1, 0 \le x_2 \le 1 \end{aligned}$$

Both terms in the objective function are convex. We wish to use separable programming to solve a piecewise-linear approximation of this problem, using (for each variable) grid points at 0, $^1/_2$, and 1.

x_1	0	0.5	1		x_2	0	0.5	1
$f_1(x_1)$	0	–0.3466	1		$f_2(x_2)$	0	0.25	1
Weight	λ_{11}	λ_{12}	λ_{13}		Weight	λ_{21}	λ_{22}	λ_{23}

Insert the coefficients in the tableau below, where the first row is the objective row:

λ_{11}	λ_{12}	λ_{13}	λ_{21}	λ_{22}	λ_{23}		RHS
						=	
						=	
						=	

The optimal solution of this LP is λ_{11}=0,λ_{12}=1, λ_{13}=0, λ_{21}=0.5, λ_{22}=0.5, λ_{23}=0.
What are the corresponding values of x_1 & x_2? x_1= ____ , x2= ____.

Part TWO: Quadratic Programming

Consider the problem

Minimize $3x_1^2 + 2x_1x_2 + x_2^2 - 30x_1 - 14x_2$

subject to

$x_1 + x_2 \le 3$

$2x_1 - x_2 \le 4$

$0 \le x_1, \quad 0 \le x_2 \le 2$

1. What is the Hessian matrix of the objective function?

Solution using a complementary pivoting algorithm:

<u>Tableau (before adding artificial variable)</u>

1	2	3	4	5	6	7	8	9	0	b
1	1	0	0	0	1	0	0	0	0	3
2	¯1	0	0	0	0	1	0	0	0	4
0	1	0	0	0	0	0	1	0	0	2
3	1	1	2	0	0	0	0	¯1	0	30
1	1	1	¯1	1	0	0	0	0	¯1	14

These represent the K.T. conditions:

$AX \le B$, i.e. $AX + Y = B$

$HX+(A^t)U+C\ge 0$, i.e. $HX + (A^t)U - V = -C$

(In addition, we must impose:
Complementary Slackness: $XV=0$, $YU=0$
Nonnegativity: $X\ge 0$, $Y\ge 0$, $U\ge 0$, $V\ge 0$)

Variable numbers:
X: 1 2
Y: 6 7 8 (slack variables for primal constraints)
U: 3 4 5 (Lagrangian multipliers for $Ax\le b$ constraints)
V: 9 10 (Lagrangian multipliers for $x\ge 0$ constraints)

Tableau, after pivoting slack and surplus variables into basis:

1	2	3	4	5	6	7	8	9	0	b
1	1	0	0	0	1	0	0	0	0	3
2	¯1	0	0	0	0	1	0	0	0	4
0	1	0	0	0	0	0	1	0	0	2
¯3	¯1	¯1	¯2	0	0	0	0	1	0	¯30
¯1	¯1	¯1	1	¯1	0	0	0	0	1	¯14

2. Are the <u>complementaryslacknessconditions</u> satisfied by the basic solution shown? _____

TABLEAU with artificial variable (a) included:

1	2	3	4	5	6	7	8	9	0	a	b
1	1	0	0	0	1	0	0	0	0	0	3
2	¯1	0	0	0	0	1	0	0	0	0	4
0	1	0	0	0	0	0	1	0	0	0	2
¯3	¯1	¯1	¯2	0	0	0	0	1	0	¯1	¯30
¯1	¯1	¯1	1	¯1	0	0	0	0	1	¯1	¯14

Artificial variable (a) enters the basis, replacing variable 9 whose complement is 1

<u>Resulting tableau</u>:

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1	2	3	4	5	6	7	8	9	0	a	b
1	1	0	0	0	1	0	0	0	0	0	3
2	¯1	0	0	0	0	1	0	0	0	0	4
0	1	0	0	0	0	0	1	0	0	0	2
3	1	1	2	0	0	0	0	¯1	0	1	30
2	0	0	3	¯1	0	0	0	¯1	1	0	16

3. Are the complementary slackness conditions satisfied by the basic solution of this tableau? _____

4. Which will be the next pivot column? column # _____

5. Which is the next pivot row? row# _____

6. What column leaves the basis as a result of this pivot? column #______

After several iterations, we obtain the tableau:

1	2	3	4	5	6	7	8	9	0	a	b
0	1	0	0	0	0.666667	¯0.333333	0	0	0	0	0.666667
1	0	0	0	0	0.333333	0.333333	0	0	0	0	2.33333
0	0	0	0	0	¯0.666667	0.333333	1	0	0	0	1.33333
0	0	1	0	0.666667	¯1.22222	¯0.222222	0	¯0.333333	¯0.666667	1	14.7778
0	0	0	1	¯0.333333	¯0.222222	¯0.222222	0	¯0.333333	0.333333	0	3.77778

7. Are the complementary slackness conditions satisfied by the basic solution of this tableau? _____

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Entering: 3, Leaving: 11 (Pivot in row 4)
                                   TABLEAU
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1	2	3	4	5	6	7	8	9	0	a	b
0	1	0	0	0	0.666667	¯0.333333	0	0	0	0	0.666667
1	0	0	0	0	0.333333	0.333333	0	0	0	0	2.33333
0	0	0	0	0	¯0.666667	0.333333	1	0	0	0	1.33333
0	0	1	0	0.666667	¯1.22222	¯0.222222	0	¯0.333333	¯0.666667	1	14.7778
0	0	0	1	¯0.333333	¯0.222222	¯0.222222	0	¯0.333333	0.333333	0	3.77778

8. Why does the algorithm terminate with this tableau?

9. What are the optimal values of X_1 and X_2? X_1= __________ , X_2= ____________