56:271 Nonlinear Programming
Quiz #1

Indicate whether true (+) or false (o). If false, briefly explain why, or give a counterexample.

___ 1. Cholesky factorization of a matrix $A$ requires that $A$ be symmetric.
___ 2. A pivot matrix is a product of elementary matrices.
___ 3. Given an LU factorization of the matrix $A$, the equation $Ax=b$ (for any given vector $b$) may be solved by first backward substitution followed by forward substitution.
___ 4. If $E_1$, $E_2$, $E_k$ are the elementary matrices corresponding to the $k$ elementary row operations required for Gaussian elimination of matrix $A$, then the matrix product $(E_1 E_2 \ldots E_k)A$ is an echelon matrix.
___ 5. If $E$ is an elementary matrix and $A$ is a matrix of the same dimensions, then the product $EA$ yields the same result as performing the corresponding elementary row operation on the matrix $A$.
___ 6. The inverse of an elementary matrix is also an elementary matrix.
___ 7. An echelon matrix must be square.
___ 8. Gauss-Jordan elimination is a more efficient method for solving $Ax=b$ (for a specific $b$) than is Gauss elimination with backsubstitution.
___ 9. The inverse of a lower triangular elementary matrix is upper triangular.
___ 10. Given an LU factorization of the matrix $A$, the equation $Ax=b$ (for any given vector $b$) may be solved by first solving $Ly=b$ for vector $y$ and then $Ux=y$ for vector $x$.
___ 11. If $E$ is an elementary matrix and $A$ is a matrix of the same dimensions, then $EA = AE$.

12. Use forward & backward substitution (in the proper order) to solve the equation $LUx=b$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 6 \\ 5 \end{bmatrix}$$

$x_1 = \text{____}$, $x_2 = \text{____}$, $x_3 = \text{____}$

___ 13. Which of the following is the result of a pivot in the upper left corner of the matrix

$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$?

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -3 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 5/2 & -1 \\ 0 & -3 & -3 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -5 & -1 \end{bmatrix}$$

(Note: All three can be obtained by row transformations, but only one is the result of a pivot!)