1. Two products X and Y can be assembled from two components A & B with availabilities \( b_A \) & \( b_B \), respectively. The product mix LP model is

\[
\begin{align*}
\text{max} & \quad 250x + 185y \\
\text{s.t.} & \quad 2x + 3y \geq b_A \\
& \quad x + 2y \geq b_B \\
& \quad x, y \geq 0
\end{align*}
\]

For example, one unit of X requires 2 units of component A and one of component B. However, the availabilities \( b_A \) & \( b_B \) are random with cumulative distribution functions \( F_A \) & \( F_B \), respectively. For example, \( F_A(t) = P\{b_A \leq t\} \).

What (nonlinear) constraint would you use so as to be 80% confident that the solution \((x, y)\) will be feasible, i.e., so that sufficient components are available to assemble the specified X and Y?

-------------------------------------------------------------------------------

2. Consider again the GP model for designing the concentric cylinders:

\[
\begin{align*}
\text{Min} & \quad 4\pi r_1^2 h + 2\pi r_2 h + 2\pi r_2^2 \\
\text{s.t.} & \quad 1000r_2^{-2}h^{-1} + r_1^2 r_2^{-2} \leq 1 \\
& \quad 50r_1^{-1}h^{-1} \leq 1 \\
& \quad r_1, r_2, h > 0
\end{align*}
\]

where \( r_1 \) & \( r_2 \) are the radii of the outer & inner cylinders, respectively, and \( h \) their height.

This can be reformulated as a separable convex minimization problem

\[
\begin{align*}
\text{Min} & \quad 4\pi e^{z_1} + 2\pi e^{z_2} + 2\pi e^{z_3} \\
\text{s.t.} & \quad 1000e^{z_1} + e^{z_2} \leq 1 \\
& \quad 50e^{z_3} \leq 1
\end{align*}
\]

\[
\begin{align*}
z_1 &= u_1 + u_2 + u_3 \\
z_2 &= u_1 + u_2 + u_3 \\
z_3 &= u_1 + u_2 + u_3 \\
z_4 &= u_1 + u_2 + u_3 \\
z_5 &= u_1 + u_2 + u_3 \\
z_6 &= u_1 + u_2 + u_3
\end{align*}
\]

a. Complete the coefficients of the equations above.

b. What sign restrictions, if any, should be placed on \( z_i \)? ____________

c. What sign restrictions, if any, should be placed on \( u_i \)? ____________

d. What is the relationship between \( u_1 \) and \( r_1 \)? ____________