Introduction to QUEUEING: M/G/1

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M/G/1

- Arrival process is Memoryless, i.e., interarrival times have Exponential distribution with mean $\frac{1}{\lambda}$.
- Single server.
- Service times are independent, identically-distributed, but not necessarily exponential. Mean service time is $\frac{1}{\mu}$ with variance $\sigma^2$.
- Queue capacity is infinite.
A steady-state distribution exists if \( \rho = \frac{\lambda}{\mu} < 1 \)
i.e., if service rate exceeds the arrival rate.

\[
\pi_0 = 1 - \rho = \text{probability that server is idle}
\]

\[
1 - \pi_0 = \rho = \text{probability that server is busy}
\]
i.e., utilization of server

There is no convenient formula for the probability
of \( j \) customers in system when \( j > 0 \).
Steady-state Characteristics

\[ L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1 - \rho)} \]

average number of customers waiting

After calculating \( L_q \), Little's Formula allows us to compute:

\[ W_q = \frac{L_q}{\lambda}, \quad W = W_q + \frac{1}{\mu}, \]

\[ \& \quad L = \lambda W = L_q + \rho \]
For the M/M/1 queue, the standard deviation equals the mean service time, i.e., $\sigma = \frac{1}{\mu}$

Using these formulae for the M/G/1 queueing system with $\sigma^2 = \frac{1}{\mu^2}$ will give results consistent with the formulae for M/M/1.
Average number of customers waiting in the queue:

\[ L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)} \]

What is the effect of variability in the service time on \( L_q \)?

Illustration:
\[ \lambda = 7/\text{hr} \]
\[ \mu = 8/\text{hr} \]
\[ \sigma \in [0, 0.2] \]

(For exponential distribution, \( \sigma = \text{mean} = \frac{1}{\mu} = 0.125 \text{ hr.} \))
Average waiting time = $W_q = \frac{L_q}{\lambda}$

What is the effect of variability in the service time on $W_q$?

Illustration:

$\lambda = 7/\text{hr}$
$\mu = 8/\text{hr}$
$\sigma \in [0, 0.2]$