Introduction to QUEUEING : M/M/1/N

This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dennis-bricker@uiowa.edu

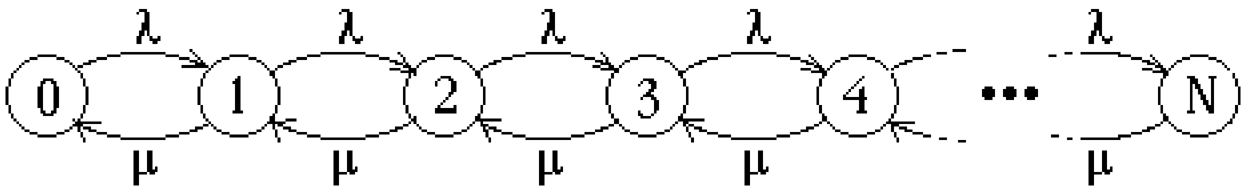
author

M/M/1/N

- *Arrival & Service processes are **M**emoryless, i.e.,*
  interarrival times have Exponential
  distribution with mean $1/\lambda$
  service times have Exponential
  distribution with mean $1/\mu$
- *Single server*
- ***Capacity** of queueing system is **finite:*** N
  (including customer currently being served)
- *Arriving customers **balk** when queue is full.*

M/M/1/N

Birth/Death Model



Steadystate distribution: $$\frac{1}{\pi_0} = 1 + \rho + \rho + \rho^2 + \cdots + \rho^N$$

finite geometric series, with sum: $$\frac{1 - \rho^{N+1}}{1 - \rho}$$

M/M/1/N

Steadystate Distribution

$$\pi_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

$$\pi_j = \rho^j \pi_0 = \rho^j \left(\frac{1 - \rho}{1 - \rho^{N+1}} \right)$$

where $\rho = \lambda / \mu \neq 1$

Note that ρ is not restricted to be less than 1 for steady state to exist!

Average Number of Customers in System

$$L = \sum_{j=0}^{N} j\,\pi_j$$

$$L = \frac{\rho\left[1 - (N+1)\rho^{N} + N\rho^{N+1}\right]}{(1 - \rho^{N+1})\,(1 - \rho)}$$

where $\rho = \lambda/\mu \neq 1$

M/M/1/N

Special Case: $\lambda = \mu$, i.e., $\rho = \lambda/\mu = 1$

Arrival rate = Service rate

$$\pi_j = \frac{1}{N+1}$$

$$L = \frac{N}{2}$$

All states are equally likely!

System is, on average, half-full!

Average Time in System per Customer

Little's Formula: $\mathbf{L} = \underline{\lambda}\mathbf{W}$

average arrival rate

$$\underline{\lambda} = \sum_{j=0}^{N-1} \lambda \, \pi_j = \lambda \sum_{j=0}^{N-1} \pi_j = \lambda \, (1 - \pi_N)$$

since arrival rate is zero when there are N in system

$$\mathbf{W} = \frac{\mathbf{L}}{\underline{\lambda}} = \frac{\mathbf{L}}{\lambda (1 - \pi_N)}$$

for M/M/1/N queue only!