LP EXERCISES

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author

Maximize $3X_1 + 2X_2$

subject to

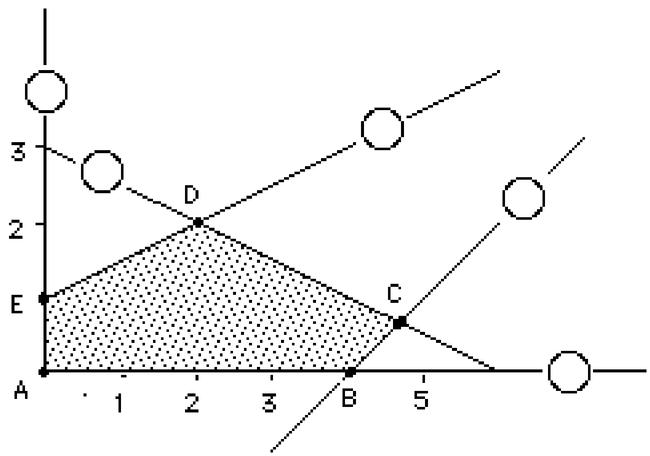
$$X_1 + 2X_2 \le 6 \quad \mathbf{1}$$

$$X_1 - X_2 \le 4 \quad \mathbf{2}$$

$$-X_1 + 2X_2 \le 2 \quad \mathbf{3}$$

$$X_1 \ge 0 \quad \mathbf{4}$$

$$X_2 \ge 0 \quad \mathbf{5}$$



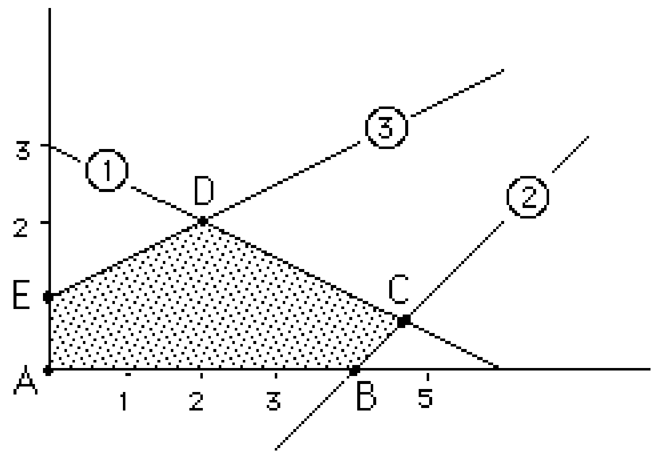
Match the 5 constraints with the 5 edges of the feasible region

Which variables are basic at each of the extreme points: A, B, C, D, & E?

$\text{Max } 3X_1 + 2X_2$

s.t.

$$X_1 + 2X_2 + X_3 = 6$$
$$X_1 - X_2 + X_4 = 4$$
$$-X_1 + 2X_2 + X_5 = 2$$
$$X_j \geq 0,\ j=1,2,...5$$



How many basic solutions does this LP have?
How many are feasible? ... infeasible?

$$\text{Max } 3X_1 + 2X_2$$

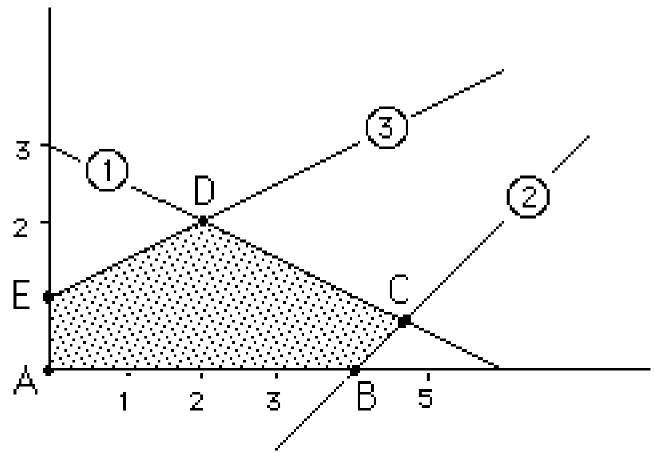
s.t.

$$X_1 + 2X_2 + X_3 = 6$$

$$X_1 - X_2 + X_4 = 4$$

$$-X_1 + 2X_2 + X_5 = 2$$

$$X_j \geq 0,\ j=1,2,..5$$



C is optimal... What can be inferred about the dual optimum, by Complementary Slackness Theorem?

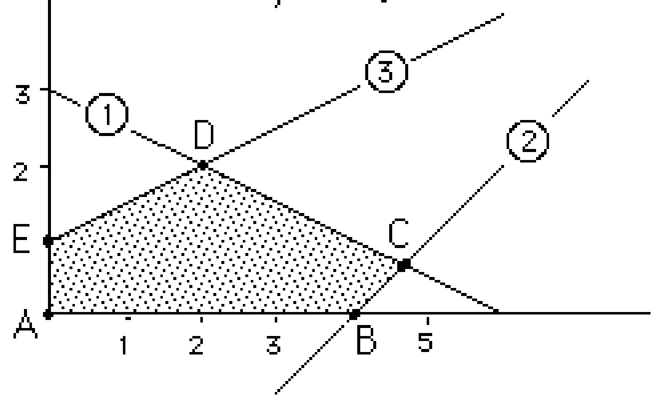
Primal

$$\text{Max } 3X_1 + 2X_2$$

s.t.

$$\begin{aligned} X_1 + 2X_2 + X_3 &= 6 \\ X_1 - X_2 + X_4 &= 4 \\ -X_1 + 2X_2 + X_5 &= 2 \end{aligned}$$

$$X_j \geq 0, \; j=1,2,..5$$



$$\text{Min } 6Y_1 + 4Y_2 + 2Y_3$$

s.t.

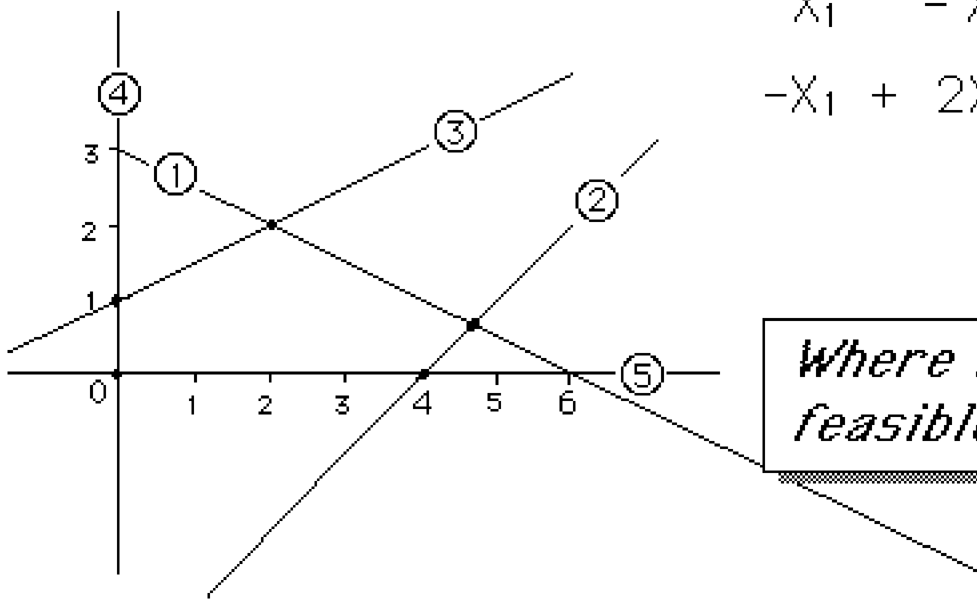
$$\begin{aligned} Y_1 + Y_2 - Y_3 &\geq 3 \\ 2Y_1 - Y_2 + 2Y_3 &\geq 2 \end{aligned}$$

$$Y_1 \geq 0, \; Y_2 \geq 0, \; Y_3 \geq 0$$

Dual

$$\text{Maximize } 3X_1 + 2X_2$$

$$\begin{aligned}
\text{subject to} \quad X_1 + 2X_2 &\le 6 && \textbf{1}\\
X_1 - X_2 &\ge 4 && \textbf{2}\\
-X_1 + 2X_2 &\le 2 && \textbf{3}\\
X_1 &\ge 0 && \textbf{4}\\
X_2 &\text{ urs} && \textbf{5}
\end{aligned}$$



Where is the feasible region?

Maximize $3X_1 + 2X_2$

subject to
$$X_1 + 2X_2 \le 6$$
$$X_1 - X_2 \ge 4$$
$$-X_1 + 2X_2 \le 2$$
$$X_1 \ge 0$$
$$X_2 \text{ urs}$$

Write the dual LP problem

Minimize $6Y_1 + 4Y_2 + 2Y_3$

s.t.
$$Y_1 + Y_2 - Y_3 \;\square\; 3$$
$$2Y_1 - Y_2 + 2Y_3 \;\square\; 2$$
$$Y_1 \;\square\; 0,\; Y_2 \;\square\; 0,\; Y_3 \;\square\; 0$$

$$\text{Maximize } 3X_1 + 2X_2$$

$$\text{subject to } X_1 + 2X_2 = 6$$

$$X_1 - X_2 \geq 4$$

$$-X_1 + 2X_2 \leq 2$$

$$X_1 \leq 0$$

$$X_2 \geq 0$$

Write the dual LP problem

$$\text{Minimize } 6Y_1 + 4Y_2 + 2Y_3$$

$$\text{s.t. } Y_1 + Y_2 - Y_3 \;\square\; 3$$

$$2Y_1 - Y_2 + 2Y_3 \;\square\; 2$$

$$Y_1 \;\square\; 0,\; Y_2 \;\square\; 0,\; Y_3 \;\square\; 0$$