Failure & Repair Characteristics of Production Systems

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cf. "Markovian Models for Investigating Failure and Repair Characteristics of Production Systems", by Joseph W. Foster, III and Alberto Garcia-Diaz, ***IIE Transactions***, Volume 15, No. 3 (Sept. 1983), pp. 202-209

A production system has **N** identical elements.

When an element fails, it cannot be repaired unless the entire system is shut down.

Thus, to avoid the greater loss of production capacity resulting from a shutdown, it may be better to continue production with some failed elements, until a sufficient number have failed.

Notation

f = # of failures causing shutdown to occur

P_b = probability that an element will fail during one time interval

R_f = probability that the system will be repaired during the next time interval, given that the system is now shut down.

(assumes that the repair process is memoryless, i.e., that the probability of completing system repairs is not dependent on the length of time already spent.)

We wish to evaluate the policy

> "Shut down the system for repairs only when the number of failures is at least **f**"

where the parameter **f** is specified.

For example,

- what average production rate can be expected?
- what will be the average length of a production run before a shutdown occurs?

Markov Chain Model

Define the state of the system to be

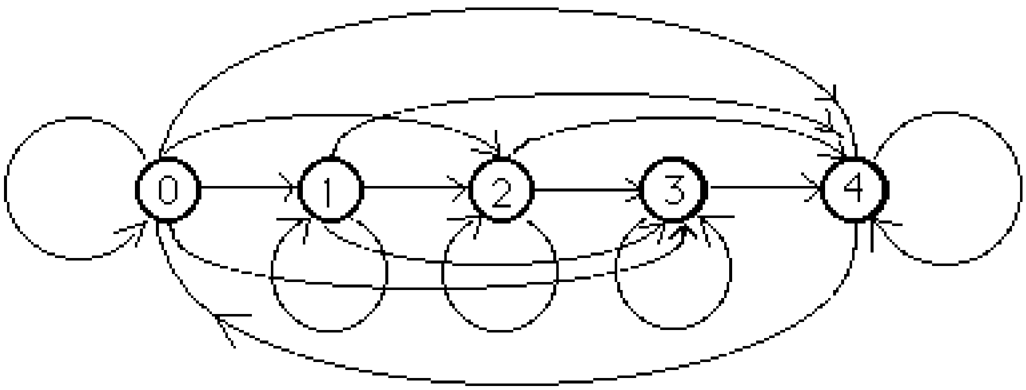
X_n = # of failed elements at stage n

Possible states are 0, 1, 2, ..., f

(where in state f, the number of failed elements is f *or more*.)

For example, suppose that the policy is f=4, i.e.,

"Shut down for repairs when 4 or more elements have failed."



Transition probabilities

If the system is in state i, then N−i elements are currently functioning.

If $i \le j \le f-1$,

$$p_{ij} = P\{\text{ j−i elements fail in a group of N−i }\}$$

$$= \binom{N-i}{j-i} P_b^{\,j-i} \left(1 - P_b\right)^{N-j}$$

(given by binomial distribution!)

If $i \le j = f$,

$$p \;\; = P\{\text{at least j−i out of N−i elements fail}\}$$

$$= 1 - \sum_{j=i}^{f-i} p_{ij}$$

Example

Suppose that we have N = 10 identical machines, and that the failure rate is P_b = 10% per hour, for each machine.

We will evaluate the policy f=4, i.e., shut down for repair of the system when at least 4 machines have failed.

The probability that the repair is completed after one hour is R_4 = 50%.

Transition Probability Matrix

from \ to	1	2	3	4	5
1	0.34868	0.38742	0.19371	0.057396	0.012795
2	0	0.38742	0.38742	0.17219	0.052972
3	0	0	0.43047	0.38264	0.1869
4	0	0	0	0.4783	0.5217
5	0.5	0	0	0	0.5

states

i	name
1	# Functioning= 10
2	# Functioning= 9
3	# Functioning= 8
4	# Functioning= 7
5	Shut down

$N=10$, $f=4$,
$P_b = 0.1$,
$R_4 = 0.5$

Steady State Distribution

i	name	P{i}
1	# Functioning= 10	0.2179
2	# Functioning= 9	0.13781
3	# Functioning= 8	0.16786
4	# Functioning= 7	0.19257
5	Shut down	0.28385

What is the utilization of the machines?

i	State	Pi	C	Pi×C
1	# Functioning= 10	0.2179	10	2.179
2	# Functioning= 9	0.13781	9	1.2403
3	# Functioning= 8	0.16786	8	1.3429
4	# Functioning= 7	0.19257	7	1.348
5	Shut down	0.28385	0	0

The average cost/period in steady state is 6.1102

The average number of machines functioning at any time is 6.1102,

i.e., the system will operate at 61.1% of capacity.

What will be the average length of a production run?

Mean First Passage Times

from \ to	1	2	3	4	5
1	4.5892	5.7493	5.4165	6.0369	5.0459
2	6.0961	7.2563	4.8972	5.1058	4.0961
3	5.0436	10.793	5.9574	4.3932	3.0436
4	3.9168	9.6661	9.3333	5.1929	1.9168
5	2	7.7493	7.4165	8.0369	3.523

states

i	name
1	# Functioning= 10
2	# Functioning= 9
3	# Functioning= 8
4	# Functioning= 7
5	Shut down

If the system begins with all machines functioning, the expected time until shutdown occurs will be 5.0459 hours.

Transition Probabilities:

From \ To	0	1	2	3	4
0	0.348678	0.38742	0.19371	0.057396	0.012795
1	0	0.38742	0.38742	0.172187	0.052972
2	0	0	0.43047	0.382638	0.186895
3	0	0	0	0.478297	0.521703
4	0.5	0	0	0	0.5

Steadystate Distribution:

i	name	P{i}
1	#failed= 0	0.217905
2	#failed= 1	0.137812
3	#failed= 2	0.167859
4	#failed= 3	0.192572
5	#failed= 4	0.283852

Average number of functioning machines:

6.11023

Mean First Passage Times

From \ To	0	1	2	3	4
0	4.58916	5.7493	5.41647	6.03687	5.04593
1	6.09613	7.2563	4.89721	5.10584	4.09613
2	5.04362	10.7929	5.95736	4.39317	3.04362
3	3.91680	9.6661	9.33327	5.19286	1.91680
4	2	7.7493	7.41647	8.03687	3.52296

That is, the average length of a production run (m_{04}) is 5.046 hours.

Consider now the case f=5, but with $P_5 = 45\%$:

Transition Probabilities

From \ To	0	1	2	3	4	5
0	0.34868	0.38742	0.19371	0.057396	0.01116	0.0016349
1	0	0.38742	0.38742	0.17219	0.044641	0.0083311
2	0	0	0.43047	0.38264	0.1488	0.038092
3	0	0	0	0.4783	0.37201	0.14969
4	0	0	0	0	0.53144	0.46856
5	0.55	0	0	0	0	0.45

Steadystate Distribution

i	name	P{i}
1	#failed= 0	0.18178
2	#failed= 1	0.11497
3	#failed= 2	0.14003
4	#failed= 3	0.16065
5	#failed= 4	0.1873
6	#failed= 5	0.21527

Average number of functioning machines:

6.2211

Mean First Passage Times

From \ To	0	1	2	3	4	5
0	5.5011	6.7024	6.0968	6.6748	7.4422	6.6279
1	7.4969	8.6982	5.6637	5.7481	6.4897	5.6787
2	6.4418	13.144	7.1412	5.1005	5.4481	4.6237
3	5.2568	11.959	11.354	6.2248	4.5739	3.4386
4	3.9524	10.655	10.049	10.627	5.339	2.1342
5	1.8182	8.5206	7.915	8.493	9.2604	4.6453

That is, the average length of a production run (m_{04}) is 6.628 hours.