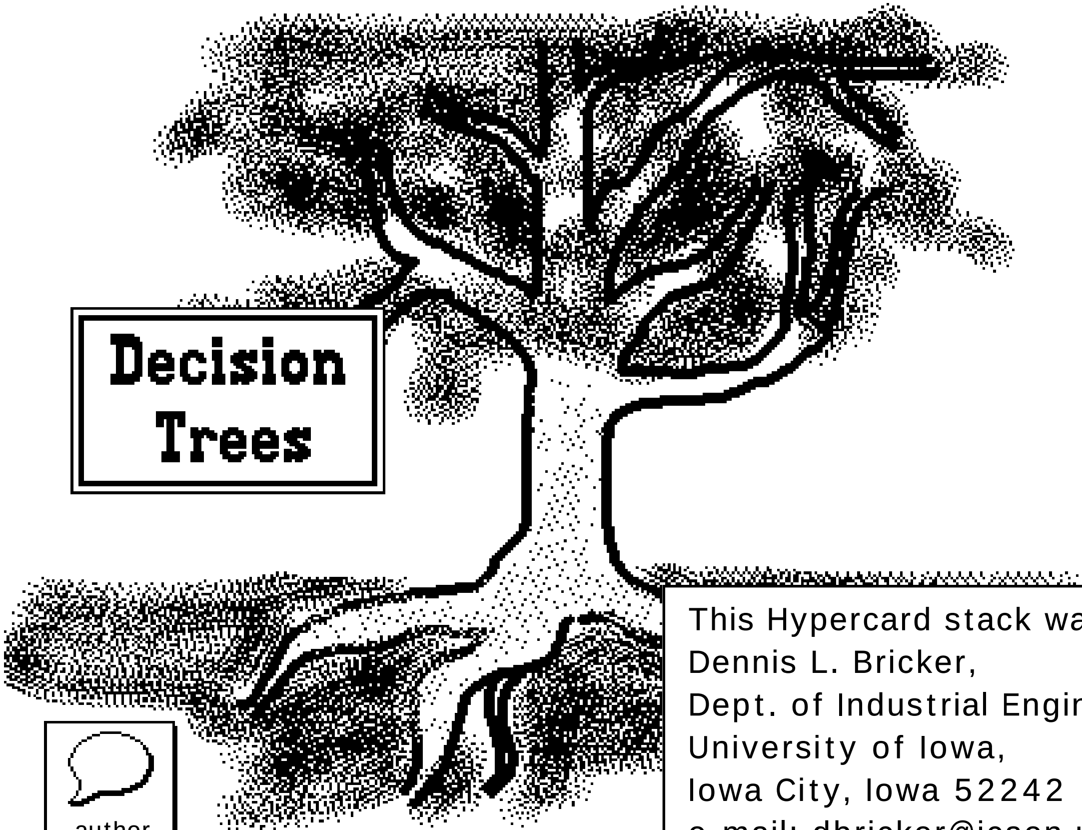


Decision Trees

author

This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dbricker@icaen.uiowa.edu

Decision Trees

- a diagram for analyzing decisions ***under risk***, i.e., when the probability dist'ns of the possible "states of nature" are known
- appropriate for a ***sequence*** of decisions, each of which could lead to one of several uncertain outcomes

EXAMPLES

- PROTRAC, Inc.
- AIRLINE TICKET PURCHASE
- OILCO

- INCORPORATING NEW INFORMATION

EXAMPLE PROTRAC, Inc., must decide on one of three marketing & prod'n strategies for a new line of home & garden tractors:

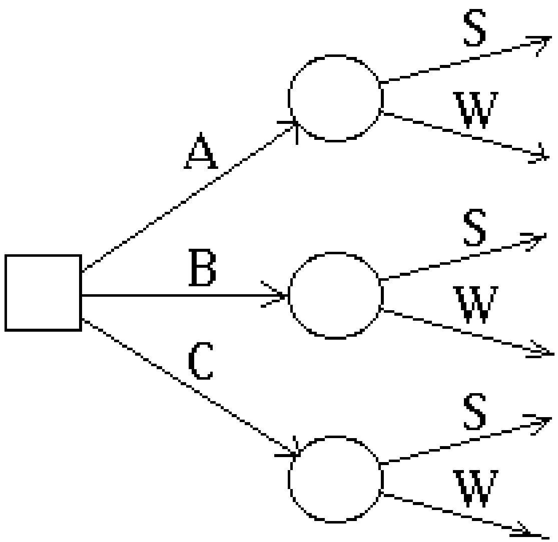
A: agressive
B: basic
C: cautious

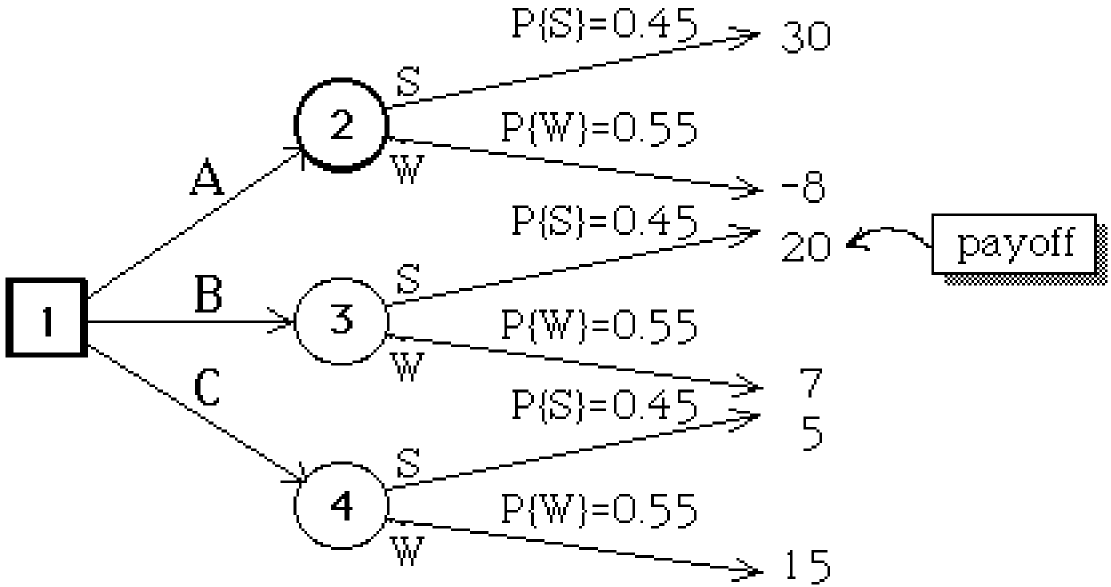
The condition of the market (as yet unknown) is categorized as "Strong" or "Weak", and determines the payoff:

The condition of the market (as yet unknown) is categorized as "Strong" or "Weak", and determines the payoff:

	State of "Nature"	
	S: strong	W: weak
Decision	0.45	0.55	Probability
A	30	-8
B	20	7
C	5	15

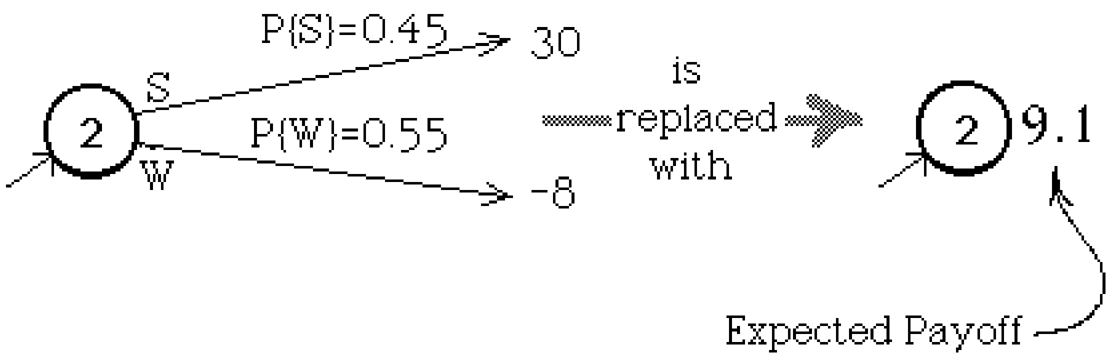
Represent the decision process as a "tree",
with a SQUARE representing a decision,
and a CIRCLE representing a random outcome:




P{S}=0.45
30
S
2
P{W}=0.55
W
A
-8
P{S}=0.45
20
payoff
1
B
S
3
P{W}=0.55
W
7
C
P{S}=0.45
5
S
4
P{W}=0.55
W
15

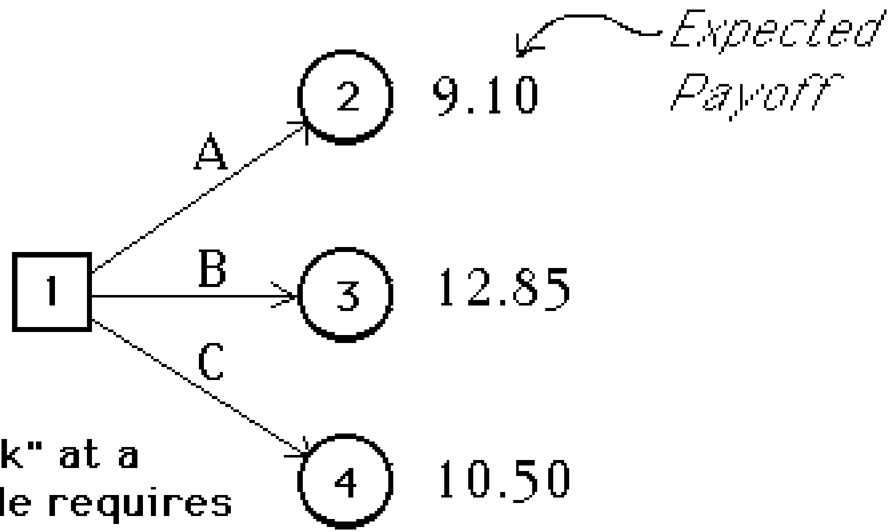
"Folding Back" Terminal Branches of the Tree

"Folding Back" at a random node requires computation of the expected payoff



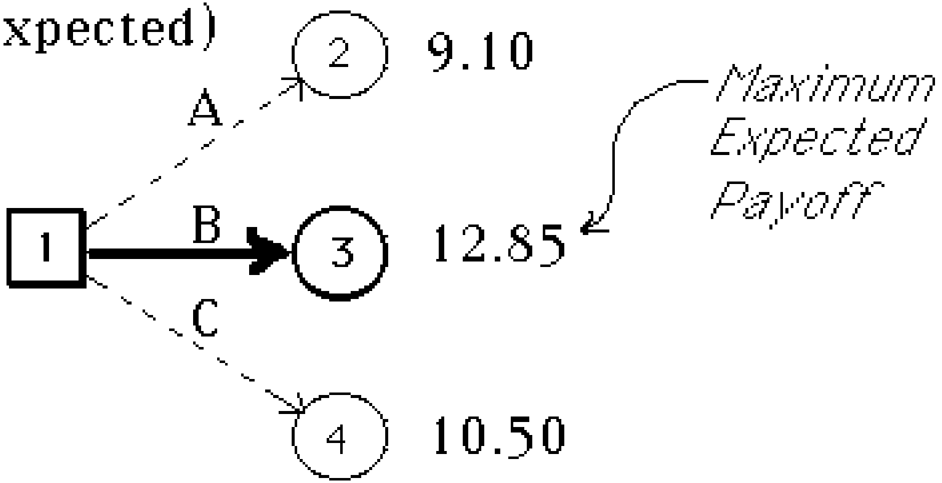
30(0.45)+(-8)(0.55)=9.10

Reduced Decision Tree



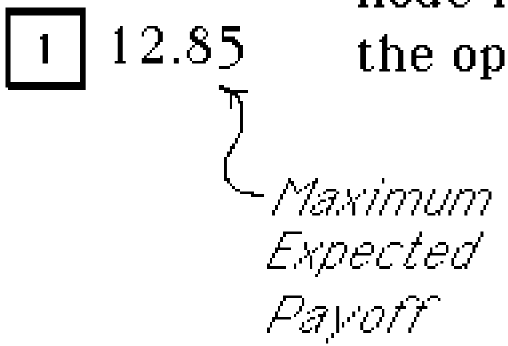
"Folding Back" at a decision node requires selection of branch with the maximum (expected) payoff (or minimum expected cost)

We now select the decision which leads to a greater (expected) payoff:



Reduced Decision Tree

When we "fold back" the tree at a decision node, the value of the node is the value of the optimal decision



EXAMPLE

Erica is going to fly to London on August 5 and return home on August 20. It is now July 1.

On July 1, she may buy a one-way ticket (for $350) or a round-trip ticket (for $660).

She may also wait until August 1 to buy a ticket.

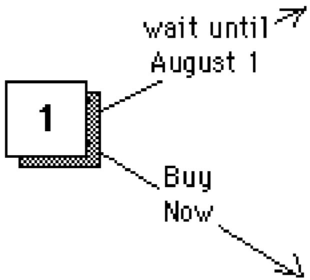
On August 1, a one-way ticket will cost $370 and a round-trip ticket will cost $730

It is possible (with probability 0.30) that between July 1 and August 1, her sister (who works for the airline) will be able to obtain a ***free*** one-way ticket for Erica.

If Erica has bought a round-trip ticket on July 1 and her sister has obtained a free ticket, she may return "half" of her round-trip ticket to the airline. In this case, her total cost will be $330 plus a $50 penalty.

Erica wishes to minimize the expected cost of obtaining round-trip transportation to London & return.

Solution



First, she must decide whether to

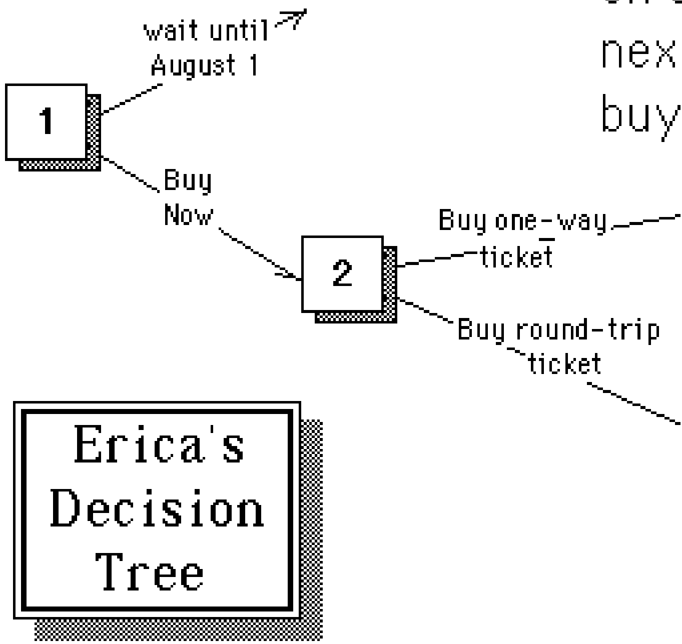
- buy a ticket now (July 1),

or

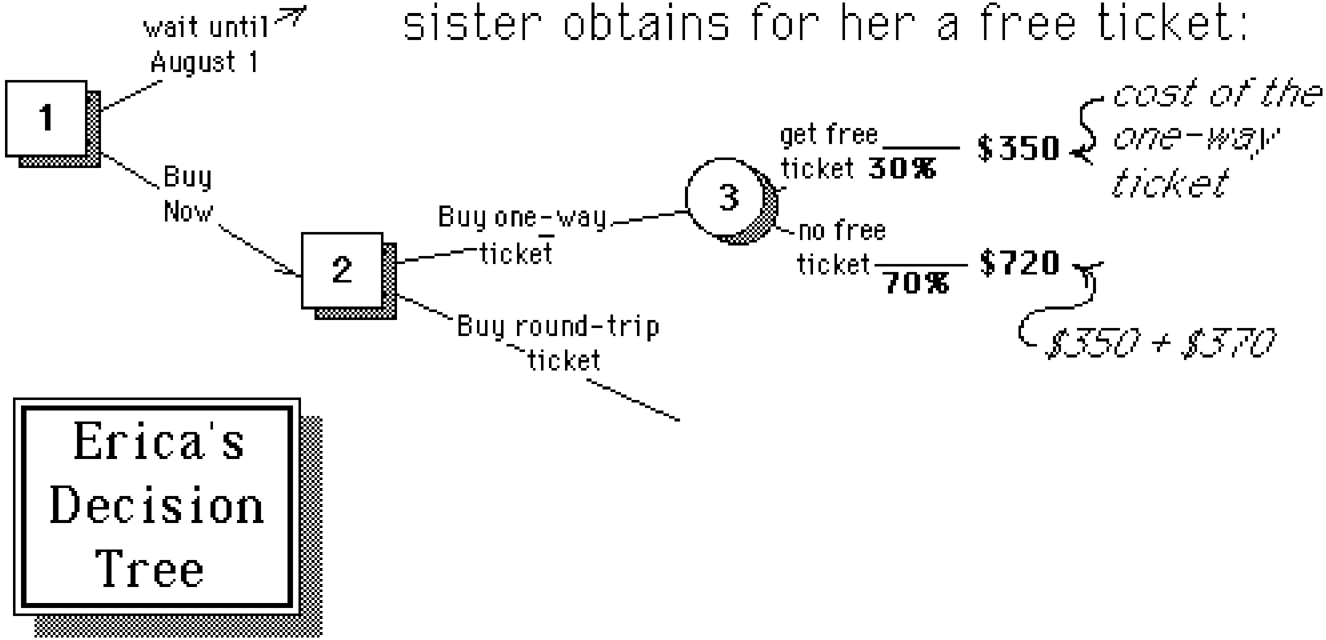
- wait until August 1

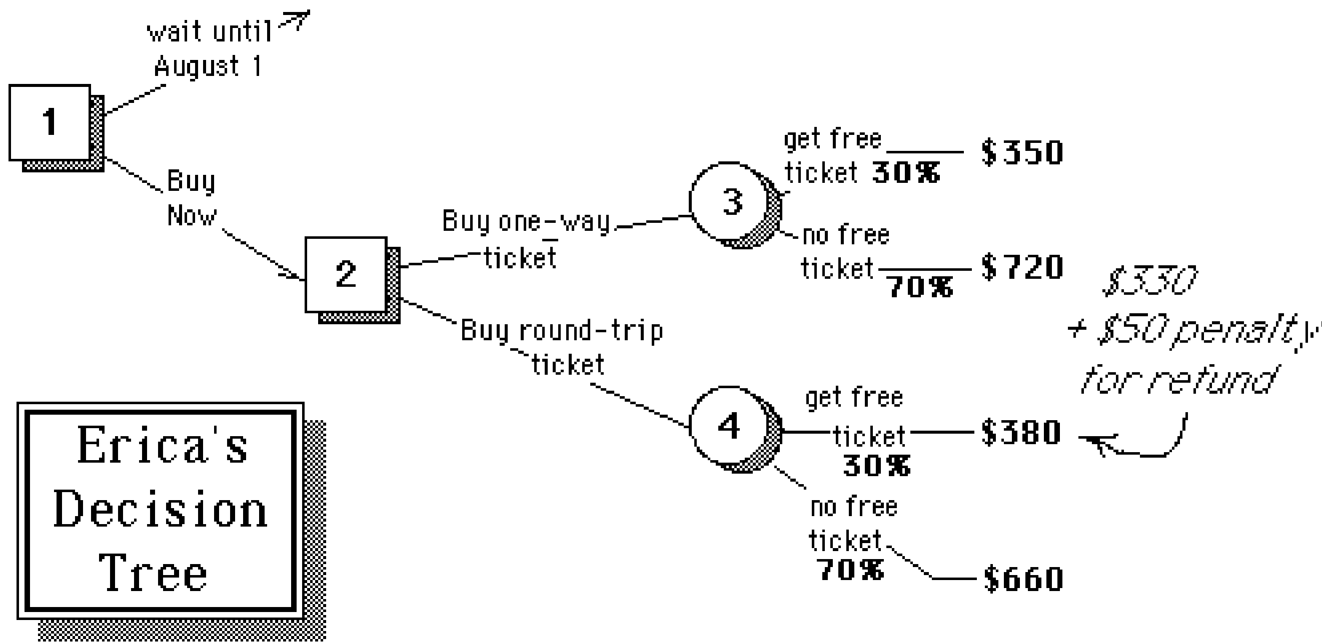
Erica's Decision Tree

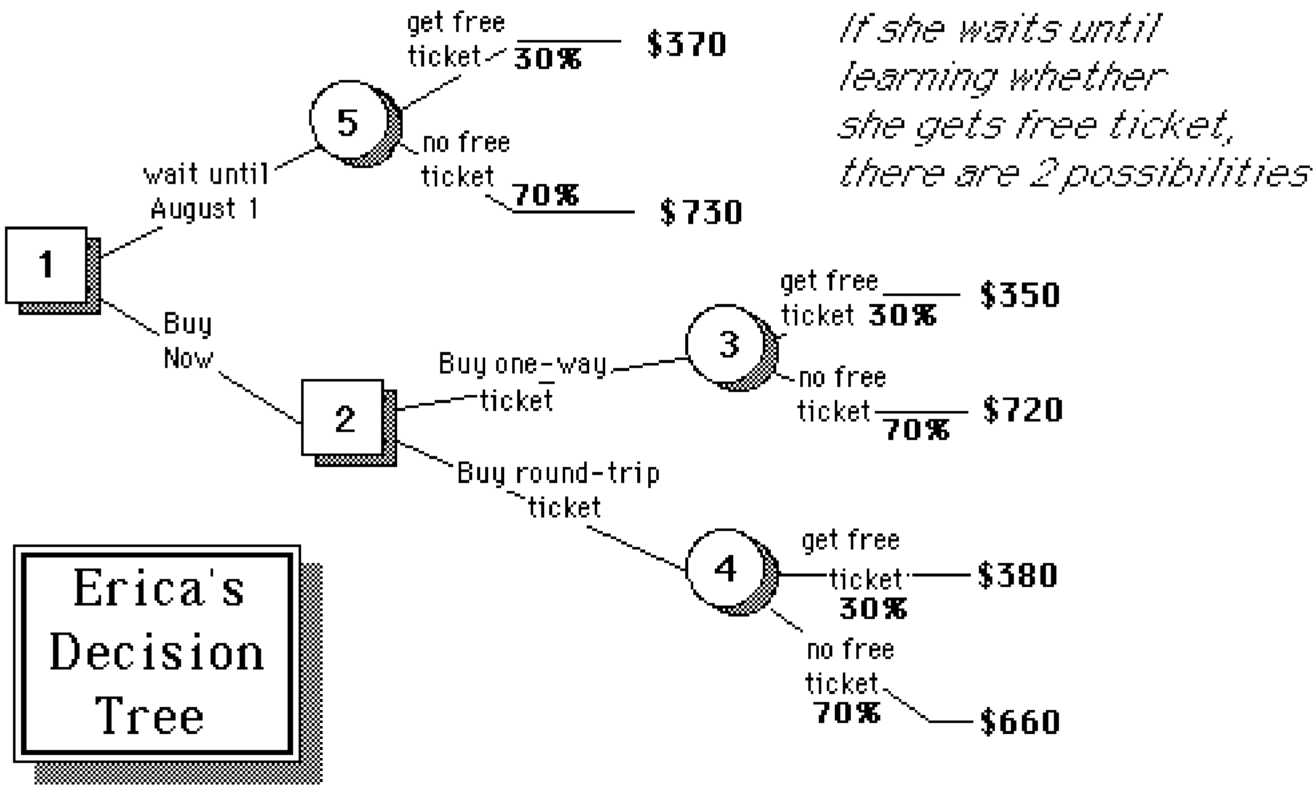
If she decides to the ticket on July 1, then she must next decide whether to buy a one-way ticket or a round-trip ticket



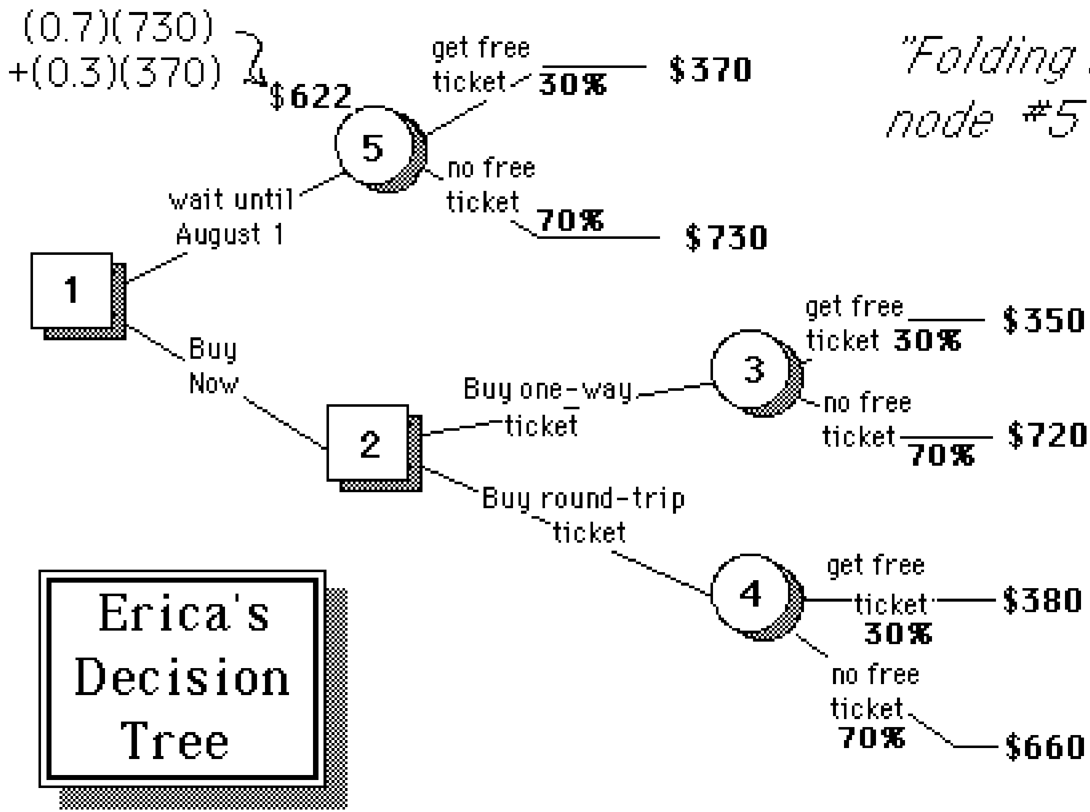
If she buys a one-way ticket, then her cost depends upon whether her sister obtains for her a free ticket:



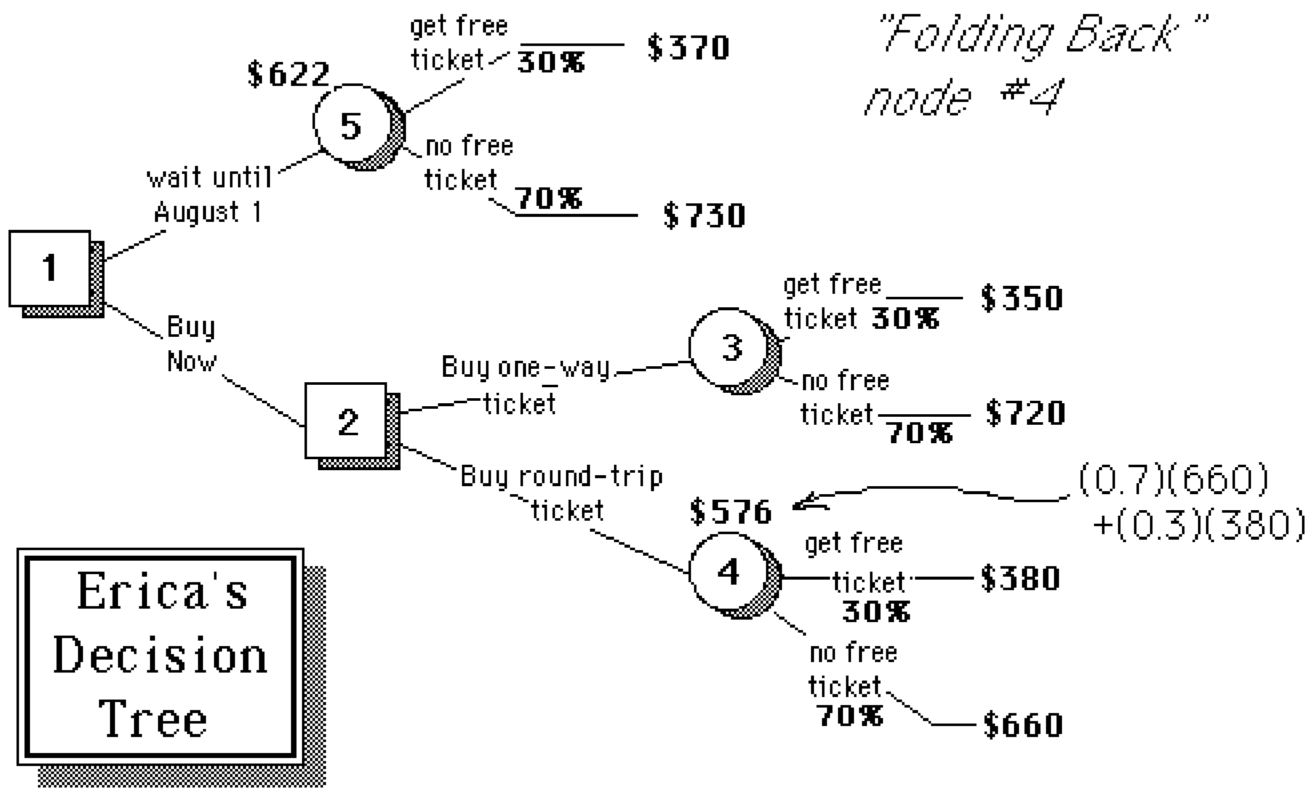

wait until August 1
1
Buy Now
2
Buy one-way ticket
Buy round-trip ticket
3
get free ticket 30%
$350
no free ticket 70%
$720
4
get free ticket 30%
$380
no free ticket 70%
$660
$330
+ $50 penalty for refund
Erica's Decision Tree

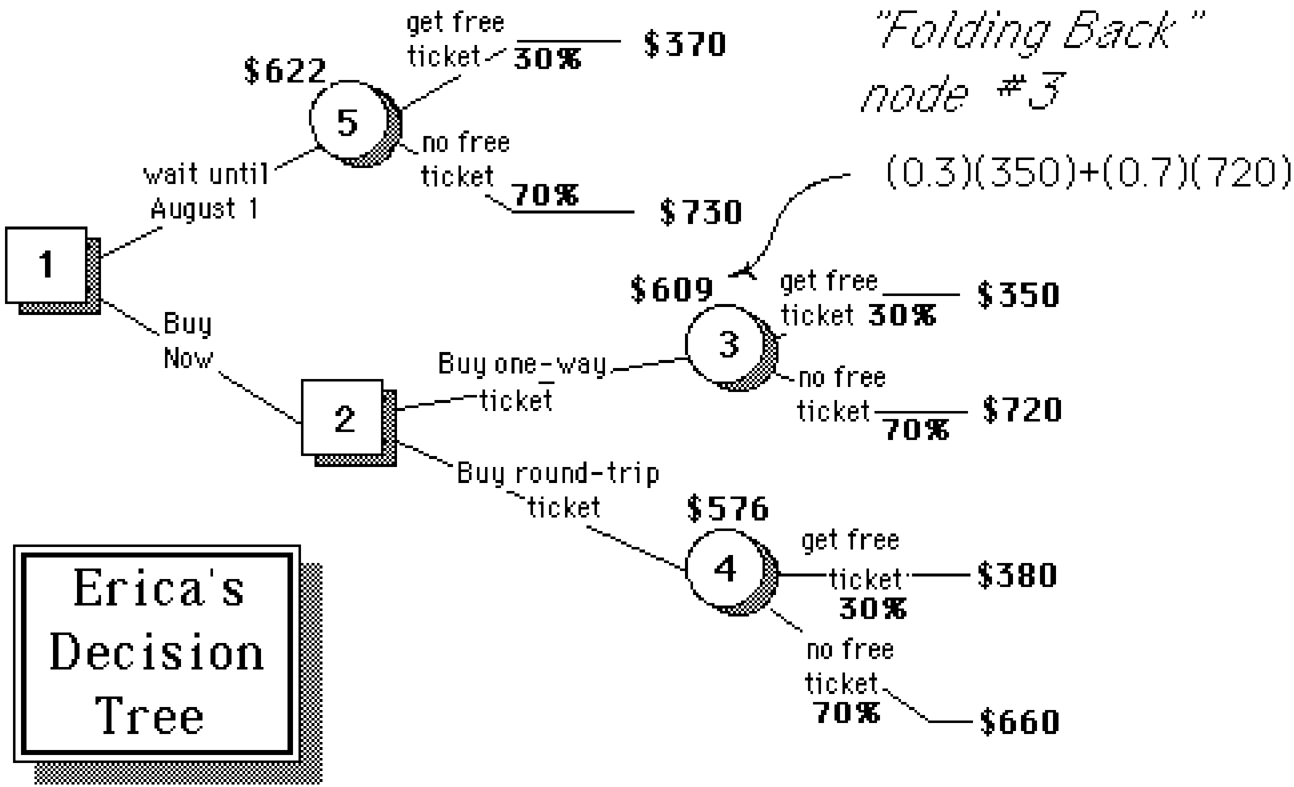


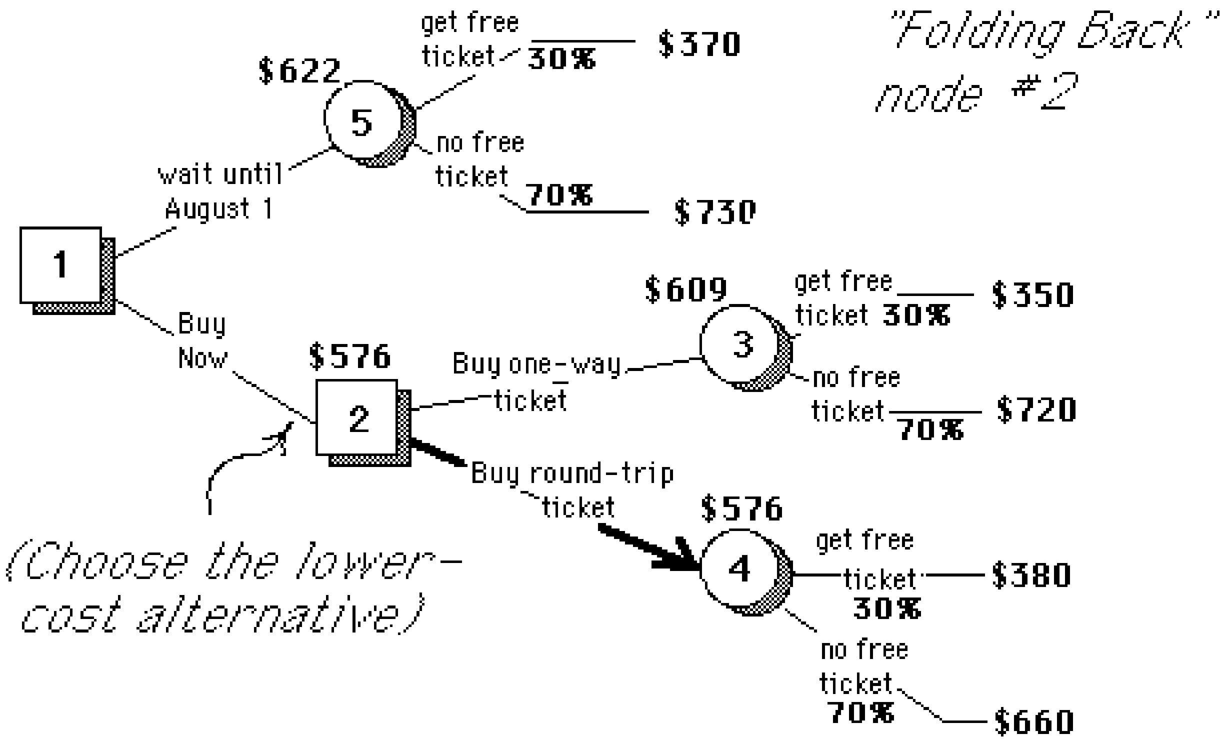
get free ticket 30%
$370
no free ticket 70%
$730
wait until August 1
5
1
Buy Now
2
Buy one-way ticket
3
get free ticket 30%
$350
no free ticket 70%
$720
Buy round-trip ticket
4
get free ticket 30%
$380
no free ticket 70%
$660
If she waits until learning whether she gets free ticket, there are 2 possibilities
Erica's Decision Tree

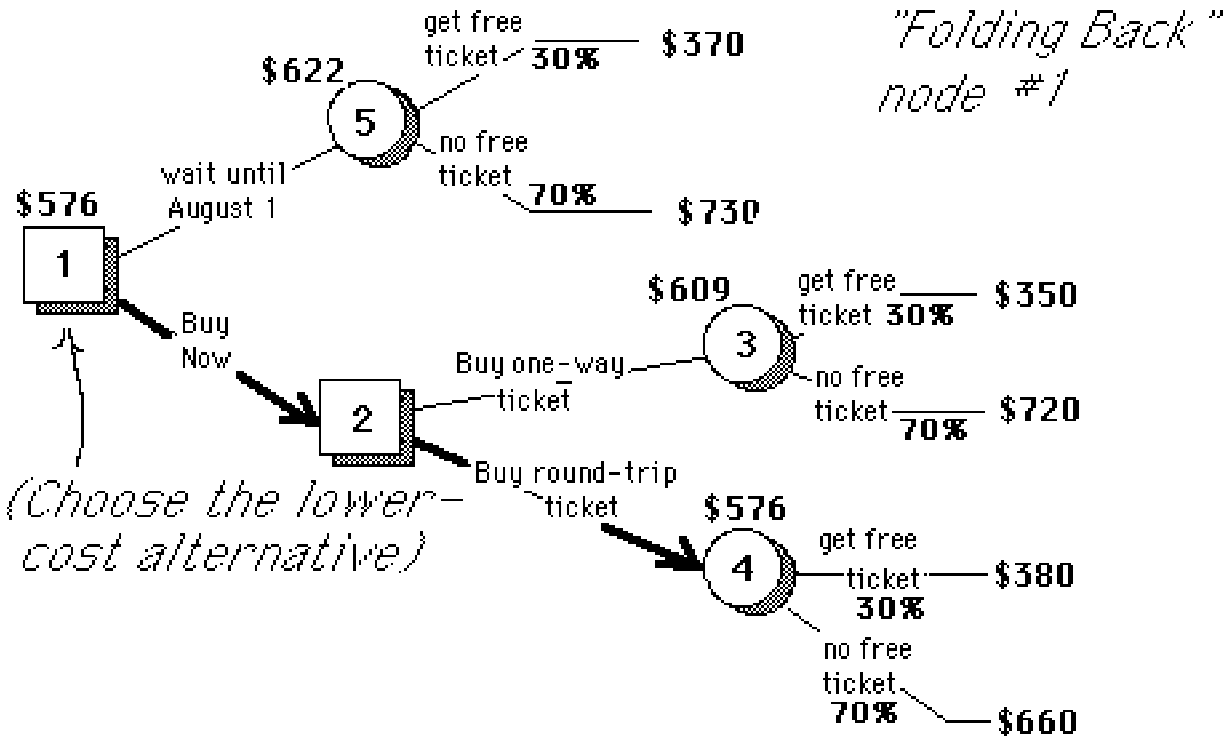


"Folding Back"
node #5


"Folding Back"
node #4
1
wait until August 1
$622
5
get free ticket
30%
$370
no free ticket
70%
$730
Buy Now
2
Buy one-way ticket
3
get free ticket
30%
$350
no free ticket
70%
$720
Buy round-trip ticket
$576
(0.7)(660)
+(0.3)(380)
4
get free ticket
30%
$380
no free ticket
70%
$660
Erica's Decision Tree


"Folding Back"
node #3
(0.3)(350)+(0.7)(720)
$622
5
get free ticket
30%
$370
no free ticket
70%
$730
wait until August 1
1
Buy Now
2
Buy one-way ticket
$609
3
get free ticket 30%
$350
no free ticket
70%
$720
Buy round-trip ticket
$576
4
get free ticket
30%
$380
no free ticket
70%
$660
Erica's Decision Tree


"Folding Back" node #2
$622
5
get free ticket 30%
$370
no free ticket 70%
$730
wait until August 1
1
Buy Now
$576
2
Buy one-way ticket
$609
3
get free ticket 30%
$350
no free ticket 70%
$720
Buy round-trip ticket
$576
4
get free ticket 30%
$380
no free ticket 70%
$660
(Choose the lower-cost alternative)


"Folding Back"
node #1
$576
1
wait until
August 1
$622
5
get free
ticket
30%
$370
no free
ticket
70%
$730
Buy
Now
2
Buy one-way
ticket
$609
3
get free
ticket
30%
$350
no free
ticket
70%
$720
Buy round-trip
ticket
$576
4
get free
ticket
30%
$380
no free
ticket
70%
$660
(Choose the lower-
cost alternative)

The optimal strategy is not to wait until August 1,
but to buy a round-trip ticket now.
Then, if she gets the free ticket,
she should cancel half of the round-trip ticket
which she purchased.

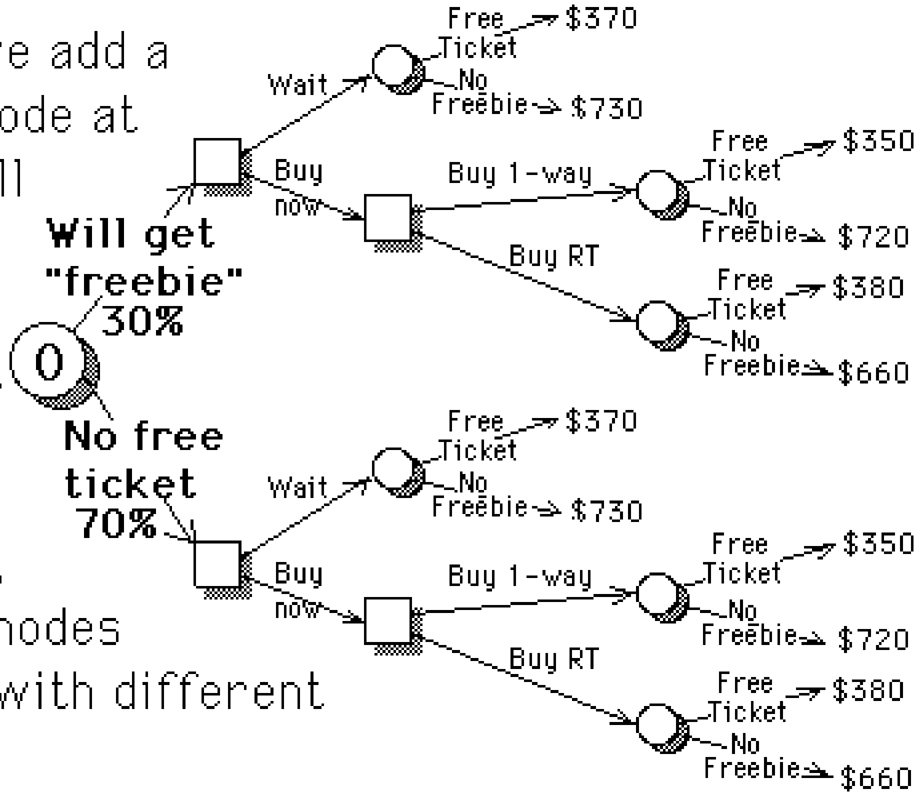
EVPI Expected Value of Perfect Information

EVWOI = Expected Value Without Information

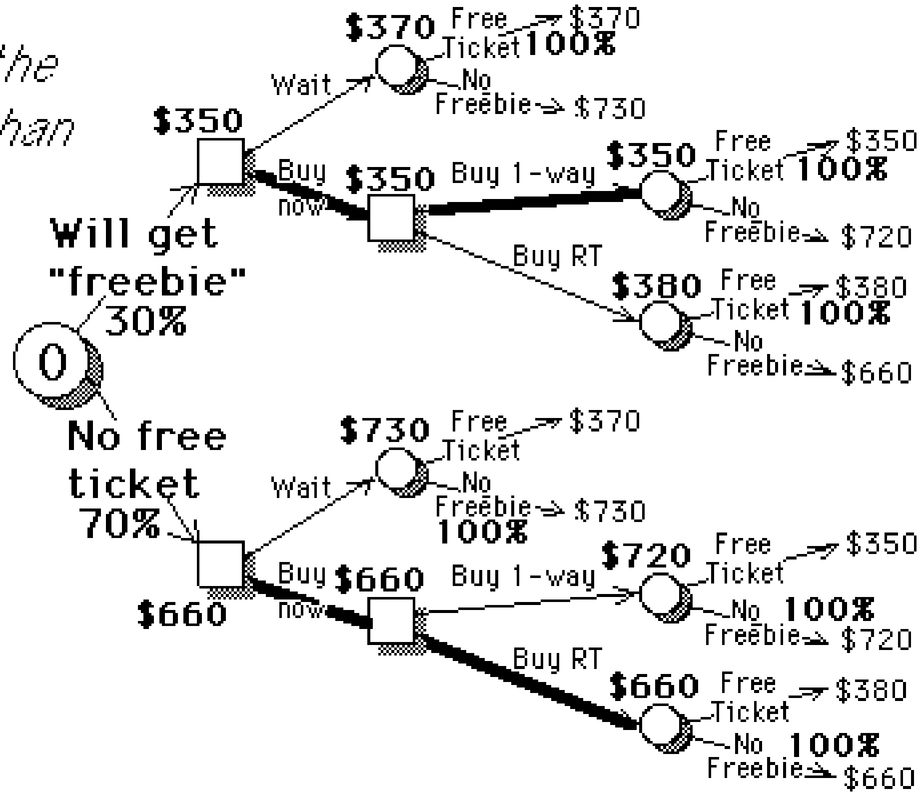
= $576 (cost)

What is EVWPI = Expected Value With Perfect Information?

Suppose that we add a random-type node at which Erica will learn whether she will get the free ticket. At the end of each branch from this node, we have same nodes as before, but with different probabilities.

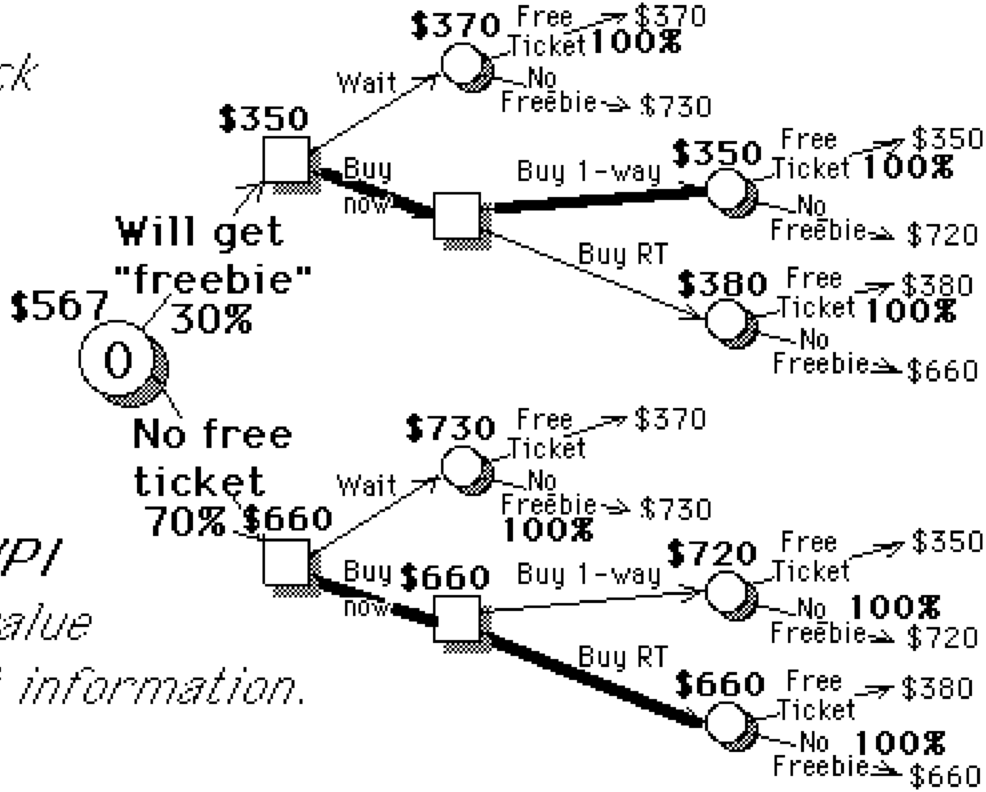


Folding back the nodes other than node 0 is quite trivial:



Folding back node #0

350(0.3)
+660(0.7)
= 567



This is ***EVWPI***
= expected value with perfect information.

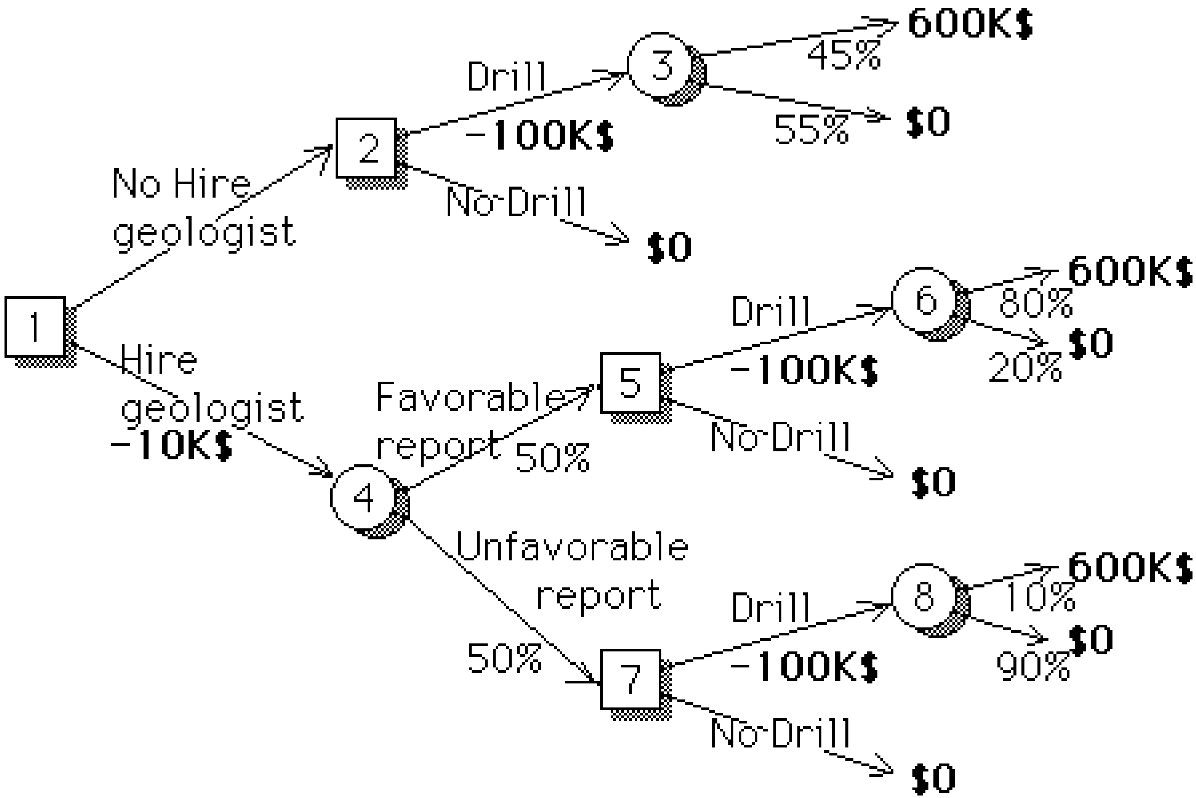
EVPI = EVWPI - EVWOI

= (-$567) - (-$576)

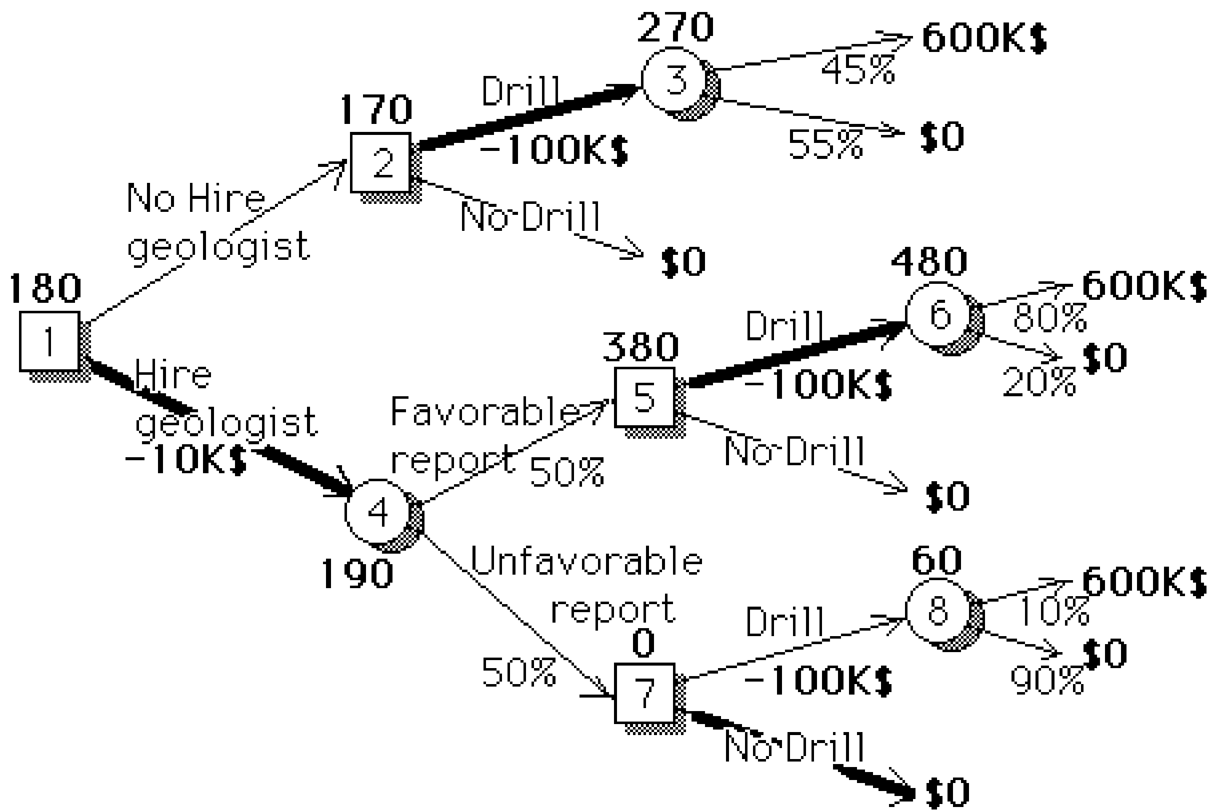
= $9

If Erica had foreknowledge whether she would receive the free ticket, her expected cost would be reduced by $9.

- Oilco must decide whether to drill for oil in the South China Sea.
- Cost of drilling is $100,000.
- If oil is found, its value is estimated at $600,000.
- Current estimate of P{oil} is 45%.
- Before drilling, the company can hire a geologist for $10,000.
- There is 50% probability he will issue a favorable report, in which case P{oil} is 80%
- If unfavorable, P{oil} is 10%.



1
No Hire geologist
Hire geologist
-10K$
2
Drill
-100K$
No Drill
$0
3
45%
600K$
55%
$0
4
Favorable report
50%
5
Drill
-100K$
6
80%
600K$
20%
$0
No Drill
$0
Unfavorable report
50%
7
Drill
-100K$
8
10%
600K$
90%
$0
No Drill
$0



270
600K$
45%
3
Drill
170
2
-100K$
55%
$0
No Hire geologist
No Drill
$0
480
180
600K$
1
6
Drill
80%
380
$0
Hire geologist
-100K$
20%
5
Favorable report
-10K$
50%
No Drill
$0
4
190
Unfavorable report
60
600K$
8
10%
Drill
0
$0
50%
-100K$
90%
7
No Drill
$0

Solution

Oilco should hire the geologist;
if his report is favorable, they should drill,
but if not favorable, they should not drill.

What is the expected value of

- *sample information, i.e., the report of the geologist?*
- *perfect information?*

$$EVSI = EVWSI - EVWOI = \$20{,}000$$

$$EVWSI = 190\ K\$$$

$$EVWOI = 170\ K\$$$

$$EVPI = EVWPI - EVWOI = \$55{,}000$$

$$EVWPI = 500(0.45) = 225\ K\$$$

$$EVWOI = 170\ K\$$$