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The decision process:

1) the decision-maker selects a decision from among the alternatives $d_k, k=1,...,n$

2) after the decision is selected, one of the possible "states of nature", $s_j$, occurs

3) the decision-maker receives a "payoff" $r_{kj}$ determined from a payoff table.
Three Classes of Decision Problems

- Decisions under *certainty*, i.e., a single state of nature is possible
- Decisions under *risk*, in which the probability distribution of the state of nature is known
- Decisions under *uncertainty*, in which the state of nature has an unknown probability distribution
Criteria for decision-making under...

- **risk**
  - maximize expected return
  - maximize expected utility
  - minimize expected regret

- **uncertainty**
  - maximize minimum return
  - maximize maximum return
  - minimize maximum regret
NEWSBOY EXAMPLE

MAKE or BUY EXAMPLE
NEWSBOY PROBLEM

- The newsboy buys newspapers from the delivery truck at the beginning of the day, at a cost of 10¢ per paper.
- During the day, he sells papers for 25¢ each.
- Demand is a random variable, but with a known probability distribution:
  \[ P_0 = 0.1, P_1 = 0.3, P_2 = 0.4, P_3 = 0.2 \]
- At the end of the day, any leftover papers are without any value.
**Newsboy Problem**

Let \( d \) = \# of papers ordered at beginning of the day (the "decision")
\( s \) = demand for papers ("state of nature")

\[
\text{Min}(s,d) = \# \text{ of papers sold}
\]

Payoff \( r_{ds} = 25(\# \text{ of papers sold}) - 10(\# \text{ of papers ordered}) \\
= 25 \min\{s,d\} - 10\times d \)
How many newspapers should the newsboy order from the delivery truck at the beginning of the day?

Because the probability distribution of the demand ("state of "nature") is known, this is decision-making under risk.
# Payoff Table

## State of Nature (demand)

<table>
<thead>
<tr>
<th>Decision</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>-20</td>
<td>5</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>-30</td>
<td>-5</td>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>
Calculation of Expected Payoff

\[
\sum_{j=1}^{4} r_{kj} \times P_j \quad \text{for } k=0,1,2,3
\]

Decision

0  \[0(0.1) + 0(0.3) + 0(0.4) + 0(0.2)\]  =  0
1  \[-10(0.1) + 15(0.3) + 15(0.4) + 15(0.2)\]  =  12.5
2  \[-20(0.1) + 5(0.3) + 30(0.4) + 30(0.2)\]  =  17.5
3  \[-30(0.1) - 5(0.3) + 20(0.4) + 45(0.2)\]  =  12.5

*To maximize the expected payoff, the newsboy should order 2 papers.*
NEWSBOY PROBLEM

Suppose that nothing is known about the probability distribution of the demand (although we still assume that possible demands are 0, 1, 2, & 3).

This is now an example of decision-making under uncertainty.
decision-making under uncertainty

Three commonly-used criteria:

- **maximin**, i.e., maximize the minimum payoff.
- **maximax**, i.e., maximize the maximum payoff.
- **minimax regret**, where "regret" is the opportunity cost of not making the best decision for a given state of nature.
MAXIMIN Criterion

\[
\text{Maximum} \left\{ \min_j r_{kj} \right\}
\]

- a very conservative or pessimistic approach
- each decision is evaluated by calculating the worst payoff that can be received if you make that decision
**MAXIMIN Criterion**  

\[ \text{Maximum} \left\{ \text{Minimum} \ r_{kj} \right\} \]

*State of Nature (demand)*

<table>
<thead>
<tr>
<th>Decision</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>-20</td>
<td>5</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>-30</td>
<td>-5</td>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>

The newsboy should order no papers from the delivery truck!
MAXIMAX Criterion

\[
\text{Maximum}_{k} \left( \text{maximum}_{j} r_{kj} \right)
\]

- a very optimistic approach

- each decision is evaluated by the best payoff that can be received if you make that decision
**MAXIMAX Criterion**

\[
\text{Maximum} \left\{ \max_j r_{kj} \right\}
\]

**State of Nature (demand)**

<table>
<thead>
<tr>
<th>Decision</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>-20</td>
<td>5</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>-30</td>
<td>-5</td>
<td>20</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>

The newsboy should order 3 papers from the delivery truck!
**MINIMAX REGRET**

\[
\text{Minimum } \left\{ \max_j \left( \max_i r_{ij} - r_{kj} \right) \right\}
\]

"Regret" is the opportunity cost of not making the best decision for a given state of nature.

For example, if the state of nature (i.e. demand) will be 2, the best decision that could have been made is of course 2, which earns a payoff of 30¢.

If we instead had ordered 3, our payoff will be 20¢, and our regret is 10¢.
Each payoff is subtracted from the maximum payoff in its column:

\[ \text{Regret}_{ij} = \left[ \text{Maximum}_{k} r_{kj} \right] - r_{ij} \]
MINIMAX REGRET

\[
\text{Minimum } \left\{ \max_k \left[ \max_i r_{ij} \right] - r_{kj} \right\}
\]

State of Nature (demand)

<table>
<thead>
<tr>
<th>Decision</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>max regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

The newsboy should order 2 newspapers from the delivery truck.
Imagine the current sequence of events:

- Mother Nature, using the probability distribution, generates a random demand
- The newsboy, not knowing what demand had been determined by Nature, orders his newspapers
- The demand is then revealed to the newsboy, and he then receives a payoff
EVPI (Expected Value of Perfect Information)

Consider a new scenario:
- The newsboy pays Mother Nature a fee
- Mother Nature determines the demand as before
- Mother Nature then tells the newsboy what the demand will be
- The newsboy orders his newspapers
- The newsboy receives his payoff

What is the largest fee which the newsboy should be willing to pay?
EVPI

\[ EVPI = \{\text{expected return with new scenario}\} - \{\text{expected return with current scenario}\} \]

Assuming that, after learning what the demand will be, the newsboy orders enough to exactly satisfy the demand,

Expected return with new scenario is

\[ \sum_{i=0}^{3} r_{ii} P_i \]

\[ = 0(0.1) + (15\varphi)(0.3) + (30\varphi)(0.4) + (45\varphi)(0.2) \]

\[ = 25.5\varphi \]
Since the newsboy’s expected return is currently 17.5¢
then
\[ \text{EVPI} = 25.5¢ - 17.5¢ = 8¢ \]

That is, possessing knowledge of the demand before he orders the newspapers will increase his expected return by 8¢.
Relationship between EVPI and "regret"

<table>
<thead>
<tr>
<th>Regret</th>
<th>demand</th>
<th>Expected regret</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>15 30 45</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10 0 15 30</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>20 10 0 15</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>30 20 10 0</td>
</tr>
</tbody>
</table>

Pj  0.1  0.3  0.4  0.2

EVPI = Minimum Expected Regret
EXAMPLE

A manufacturer has a choice of either

- buying 9000 of a certain part at $20 each,

or

- making them at a setup cost of $50,000 plus $12 each

\[
\text{Average cost} = \frac{50,000 + 9000 \times 12}{9000} = 17.56 \text{ per unit}
\]
Unfortunately, while the bought product is *always* satisfactory, the product he makes is often *defective*, having a distribution of the percent defective (p) as:

<table>
<thead>
<tr>
<th>p</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>P{p}</td>
<td>.1</td>
<td>.2</td>
<td>.3</td>
<td>.25</td>
<td>.15</td>
</tr>
</tbody>
</table>

If a defective part is installed and discovered on final test of the product, it must be corrected at a cost of $10 each.
Construct a Payoff table,
with the 5 "states of nature" being the % defective,
and the decisions being "make" and "buy".

<table>
<thead>
<tr>
<th>Decision</th>
<th>percent defective</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
</tr>
<tr>
<td>Make</td>
<td>-158000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
</tr>
</tbody>
</table>

(A cost is interpreted as a negative payoff, in order to be consistent with the criteria discussed earlier.)
### Payoff Table

<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>-158000</td>
<td>-167000</td>
<td>-176000</td>
<td>-185000</td>
<td>-194000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
</tr>
</tbody>
</table>

### Regret Table:

<table>
<thead>
<tr>
<th>Decision</th>
<th>percent defective</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
</tr>
<tr>
<td>Make</td>
<td>0</td>
</tr>
<tr>
<td>Buy</td>
<td>22000</td>
</tr>
</tbody>
</table>
What is the decision, using criterion...

- [ ] **MAXIMIN** ?
- [ ] **MAXIMAX** ?
- [ ] **MINIMAX REGRET** ?
- [ ] **MAXIMUM EXPECTED PAYOFF** ?

What is...

- [ ] **EVPI** ? Expected Value of Perfect Information
<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>-158000</td>
<td>-167000</td>
<td>-176000</td>
<td>-185000</td>
<td>-194000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
</tr>
</tbody>
</table>

Minimum payoff for the decision "Make" is -194000,

Minimum payoff for the decision "Buy" is -180000.

Therefore, the decision selected by the maximin criterion will be "Buy", since

-180000 > -194000.
### MAXIMAX

<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>-158000</td>
<td>-167000</td>
<td>-176000</td>
<td>-185000</td>
<td>-194000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
</tr>
</tbody>
</table>

Maximum payoff for decision "Make" is -158000, maximum payoff for decision "Buy" is -180000. Therefore, the decision selected by the maximax criterion is "Make", since \(-158000 > -180000\).
The maximum regret for decision "Make" is 14000, and for "Buy" is 22000. Therefore, the decision selected by the "minimax regret" criterion is "Make".

### MINIMAX REGRET

<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5000</td>
<td>14000</td>
</tr>
<tr>
<td>Buy</td>
<td>22000</td>
<td>13000</td>
<td>4000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### Maximum Expected Payoff

<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>-158000</td>
<td>-167000</td>
<td>-176000</td>
<td>-185000</td>
<td>-194000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
</tr>
</tbody>
</table>

The expected payoff for decision "Make" is
\[
0.1x(-158000) + 0.2x(167000) + 0.3x(-176000) + 0.25x(-185000) + 0.15x(-194000) = -177350
\]
while for the decision "Buy" it is -180000. Therefore, the decision selected by this criterion is "Make".

↑
**EVPI** Expected Value of Perfect Information:

If the manufacturer had a prediction of the defective rate in advance (*possessed perfect information*), he would choose

and  
"Make" if $p = 0, 10, \text{ or } 20\%$, 
"Buy" if $p = 30 \text{ or } 40\%$:

<table>
<thead>
<tr>
<th>Decision</th>
<th>p = 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>-158000</td>
<td>-167000</td>
<td>-176000</td>
<td>-185000</td>
<td>-194000</td>
</tr>
<tr>
<td>Buy</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
<td>-180000</td>
</tr>
</tbody>
</table>

probability: 0.10 0.20 0.30 0.25 0.15
<table>
<thead>
<tr>
<th>defec rate:</th>
<th>0</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff:</td>
<td>-158</td>
<td>-167</td>
<td>-176</td>
<td>-180</td>
<td>-180</td>
</tr>
<tr>
<td>probability:</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.25</td>
<td>0.15</td>
</tr>
</tbody>
</table>

\[
\text{EVWPI} = \text{Expected Value With Perfect Information} \\
= 0.1 \times (-158000) + 0.2 \times (167000) + 0.3 \times (-176000) \\
\quad + 0.25 \times (-180000) + 0.15 \times (-180000) \\
= -174000. \\
\text{EVWOI} = \text{Expected Value Without Information} \\
= -177350.
\]
\textbf{EVPI}

\[ \text{EVWPI} = -174000 \]

\[ \text{EVWOI} = -177350 \]

\[ \text{EVPI} = \text{EVWPI} - \text{EVWOI} = 3350 \]

i.e., with perfect information, the manufacturer's payoff is 3350 more than without.
### Regret Table:

<table>
<thead>
<tr>
<th>Decision</th>
<th>p= 0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>Expected regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5000</td>
<td>14000</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>22000</td>
<td>13000</td>
<td>4000</td>
<td>0</td>
<td>0</td>
<td>$3350</td>
</tr>
</tbody>
</table>

The decision which maximizes expected payoff is "Make".

The expected regret of this decision is

\[
0 + 0 + 0 + 0.25 \times 5000 + 0.15 \times 14000 = 3350
\]

EVPI = Minimum Expected Regret