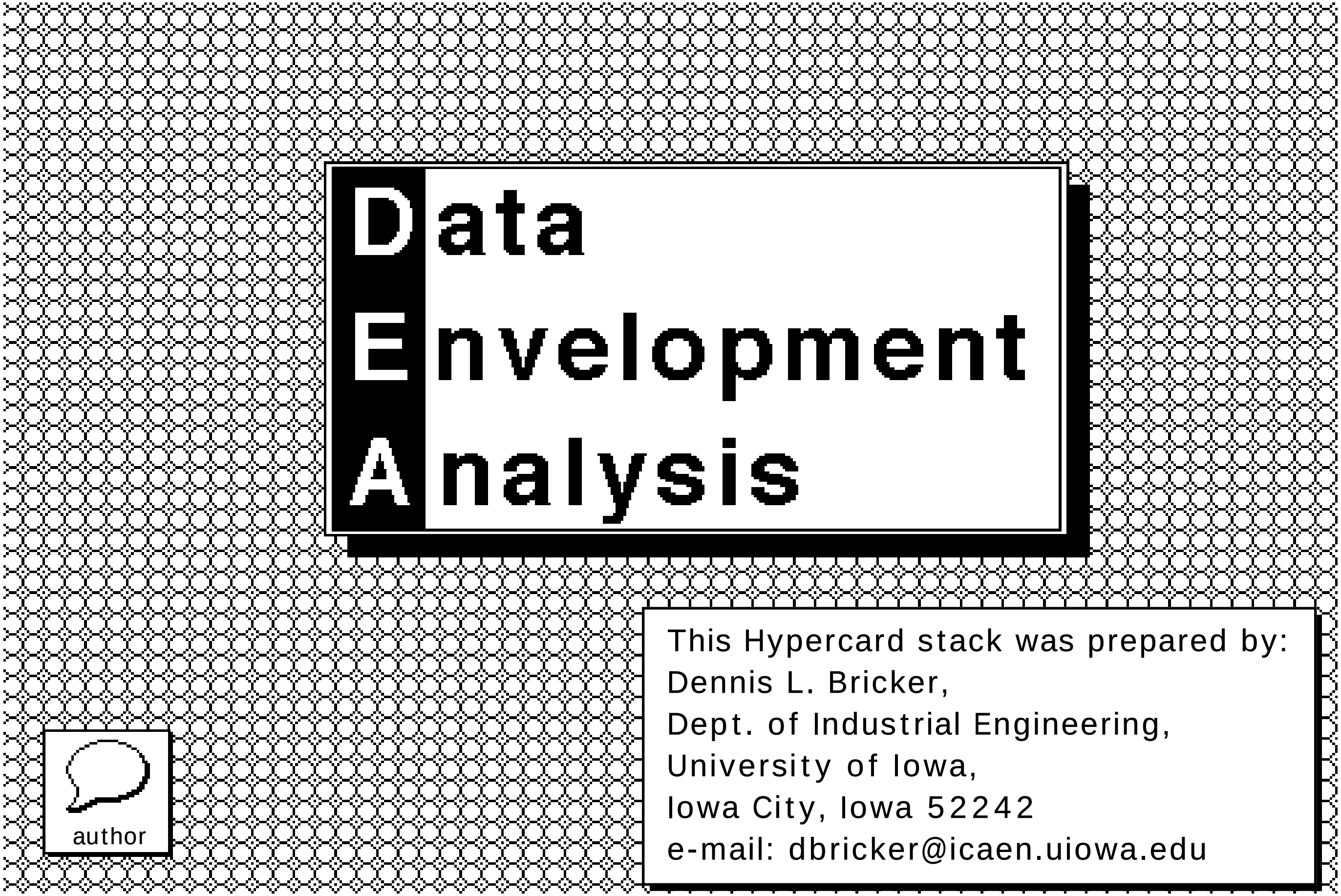

Data
Envelopment
Analysis

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- ☞ Definitions & Notation
- ☞ Nonlinear Programming Model
- ☞ DEA LP Model
- ☞ DEA Dual LP Model
- ☞ Examples
- ☞ Further References
- ☞ Summary of Strengths & Weaknesses

The Problem

Given:

- a set of **n** "decision-making units" (DMUs)
- for DMU #j, inputs X_{ij}, i=1,...m
  outputs Y_{rj}, r=1,.... s

Estimate:

- the relative efficiencies of the DMUs

If the DMUs are typical "firms" or "industries" and the inputs (resources) and outputs (goods & services) all have market values, then the efficiency of DMU #j could be computed by:

$$\boxed{E_j = \frac{\text{total market value of outputs}}{\text{total market value of inputs}}}$$

Suppose, however, that the DMUs are public entities or not-for-profit organizations which use resources and produce outputs which have no easily determined market values.

EXAMPLE The state wishes to evaluate different public schools, whose inputs might include not only funds provided by the state (which do have $ value) but also less tangible "inputs" such as

- teacher/student ratio
- socio-economic factors (median income of the population, ratio of single-parent families, etc.
- physical facilities
- parent participation in school activities

Likewise, the outputs may have intangible value, e.g.,

- standardized test scores of students
- psychological tests of student attitudes
- fraction of graduates seeking higher education

How can a state governing agency evaluate the performance of the schools in any kind of objective way?

DEA Suppose that each DMU was given the opportunity to evaluate its own "efficiency" (but must provide some justification for its computation).

That is, suppose that DMU #k may specify

a "value" u_i for each unit of input #i used by the DMU (i=1,2,... m), and

a "value" v_r for each unit of output #r which is produced by the DMU (r=1,2,... s)

Then, based upon these values, DMU #k will have an "efficiency" equal to

$$\frac{\sum_{r=1}^{s} v_r Y_{rk}}{\sum_{i=1}^{m} u_i X_{ik}}$$

total value of outputs

total value of inputs

Naturally, DMU #k wishes to choose the values or prices so as to maximize its perceived efficiency!

That is, DMU #k wishes to

$$\boxed{\text{Maximize} \quad \frac{\sum_{r=1}^{s} v_r Y_{rk}}{\sum_{i=1}^{m} u_i X_{ik}}}$$

subject to *??? Any other constraints to be imposed ???*

$$\boxed{u_i \geq 0 \ \forall\, i \ \& \ \ v_r \geq 0 \ \forall\, r}$$

The prices selected by DMU #k for the evaluation of its own efficiency cannot yield an efficiency greater than 100%,

either for itself or for any other DMU whose efficiency were to be computed based upon the same set of prices!

That is, the prices u & v must satisfy the inequalities

Efficiency of Facility j

$$\boxed{\frac{\sum_{r=1}^{s} v_r Y_{rj}}{\sum_{i=1}^{m} u_i X_{ij}} \leq 1,\ j=1, \cdots n}$$

Of course, we must give each of the other DMUs the same opportunity to choose the values of inputs and outputs by which their efficiencies are to evaluated!

$$\textit{Find} \quad E_k = \underset{u,v}{\text{Maximum}} \; \frac{\sum_{r=1}^{s} v_r Y_{rk}}{\sum_{i=1}^{m} u_i X_{ik}}$$

$$\text{subject to} \quad \frac{\sum_{r=1}^{s} v_r Y_{r\,j}}{\sum_{i=1}^{m} u_i X_{i\,j}} \leq 1, j=1, \cdots n$$

$$u_i \geq 0 \;\forall\; i \;\&\; v_r \geq 0 \;\forall\; r$$

Solving each of the n optimization problems (k=1,2,... n) would give us a set of values E_k, k=1,2,... n by which to compare the performances of the DMUs

Unfortunately, these optimization problems as stated have *nonlinear* objective function and *nonlinear* constraints...

$$E_k = \underset{u,v}{\text{Maximum}} \frac{\sum_{r=1}^{s} v_r Y_{rk}}{\sum_{i=1}^{m} u_i X_{ik}}$$

$$\text{subject to } \frac{\sum_{r=1}^{s} v_r Y_{rj}}{\sum_{i=1}^{m} u_i X_{ij}} \leq 1,\ j=1, \cdots n$$

$$u_i \geq 0\ \forall\, i \ \&\ \ v_r \geq 0\ \forall\, r$$

The constraints are easily linearized by multiplying both sides by the denominator, which is nonnegative

Linearizing the constraint

$$\frac{\sum_{r=1}^{s} v_r Y_{r\,j}}{\sum_{i=1}^{m} u_i X_{i\,j}} \leq 1$$

$$\sum_{r=1}^{s} v_r Y_{r\,j} \leq \sum_{i=1}^{m} u_i X_{i\,j}$$

$$\sum_{r=1}^{s} v_r Y_{r\,j} - \sum_{i=1}^{m} u_i X_{i\,j} \leq 0,$$

$$j=1, \cdots n$$

In order to linearize the objective function, we note that the objective & constraint functions are unchanged if the prices u and v were to be multiplied by any positive scalar, i.e.,

$$\frac{\sum_{r=1}^{s} (\alpha v_r) Y_{rj}}{\sum_{i=1}^{m} (\alpha u_i) X_{ij}} = \frac{\sum_{r=1}^{s} v_r Y_{rj}}{\sum_{i=1}^{m} u_i X_{ij}} \quad \text{for any } \alpha > 0$$

It should be valid, then, to arbitrarily restrict the total value of the inputs of DMU #k to be equal to 1.0.

$$\underset{u,v}{\text{Maximum}} \quad \frac{\sum_{r=1}^{s} v_r Y_{rk}}{\sum_{i=1}^{m} u_i X_{ik}} \quad \text{subject to} \quad \sum_{i=1}^{m} u_i X_{ik} = 1$$

is equivalent to

$$\underset{u,v}{\text{Maximum}} \quad \sum_{r=1}^{s} v_r Y_{rk} \quad \text{subject to} \quad \sum_{i=1}^{m} u_i X_{ik} = 1$$

which has a *linear* objective!

The DEA LP Model:

$$E_k = \underset{u,v}{\text{Maximum}} \sum_{r=1}^{s} v_r\, Y_{rk}$$

$$\textbf{subject to} \sum_{r=1}^{s} v_r Y_{rj} - \sum_{i=1}^{m} u_i X_{ij} \le 0,\ j=1, \cdots n$$

$$\sum_{i=1}^{m} u_i X_{ik} = 1$$

$$u_i \ge 0\ \forall\ i\ \&\ \ v_r \ge 0\ \forall\ r$$

\# of contraints = n+1 » # of variables = m+s

DEA dual LP

Computationally, it is more attractive to solve an LP with # rows ≪ # columns than # columns ≪ # rows.

dual variables		primal constraints
$\lambda_j, j=1,\cdots n$	⟷	$\sum_{r=1}^{s} v_r Y_{rj} - \sum_{i=1}^{m} u_i X_{ij} \leq 0, j=1,\cdots n$
z_0	⟷	$\sum_{i=1}^{m} u_i X_{ik} = 1$

DEA dual LP

Minimize z_0

subject to $\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{rk}$, r=1, ⋯ s

$X_{ik} z_0 - \sum_{j=1}^{n} X_{ij}\lambda_j \geq 0$, i=1, ⋯ m

$\lambda_j \geq 0$, j=1, ⋯ n

Interpretation of DEA Dual LP

We wish to form a linear combination of the hospitals in the group, i.e., a "hypothetical" hospital, whose outputs are at least as great as the target hospital,

$$\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{rk} \quad , r=1, \cdots s$$

and whose inputs are no more than the efficiency z_0 times the target hospital's inputs.

$$\sum_{j=1}^{n} X_{ij}\lambda_j \leq X_{ik}\, z_0 \quad , i=1, \cdots m$$

EXAMPLES

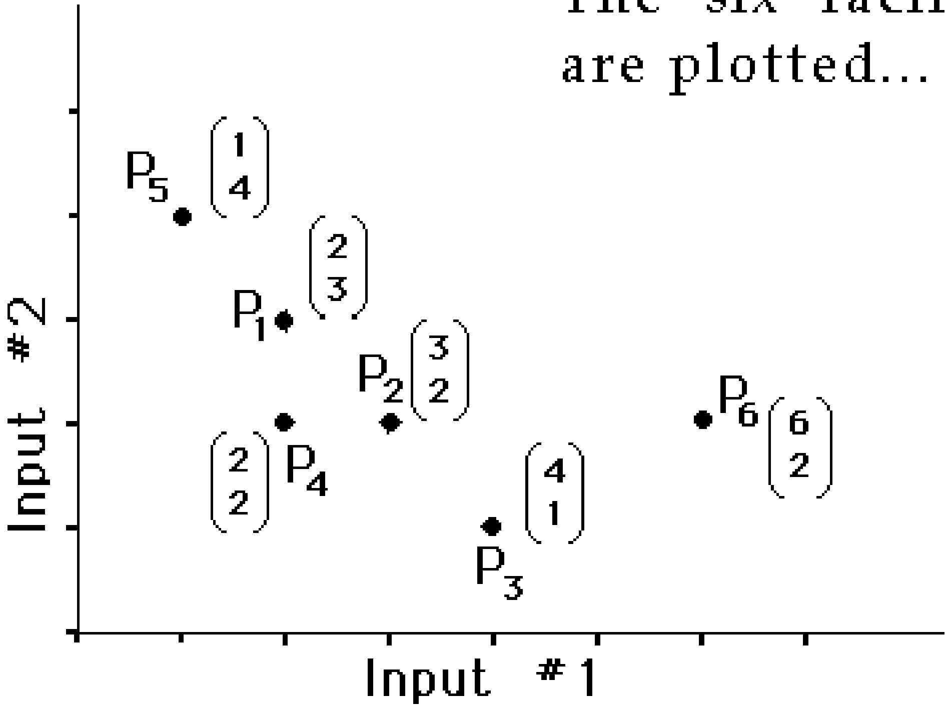
- ☞ Example A
- ☞ Example B
- ☞ Example C (Sherman)
- ☞ Nursing Home (Bricker & Nyman)
- ☞ University Departments (Beasley)

Consider six DMUs, with two inputs and one output. Normalize the inputs, i.e., define their units to be units of the input per unit of output which is produced.

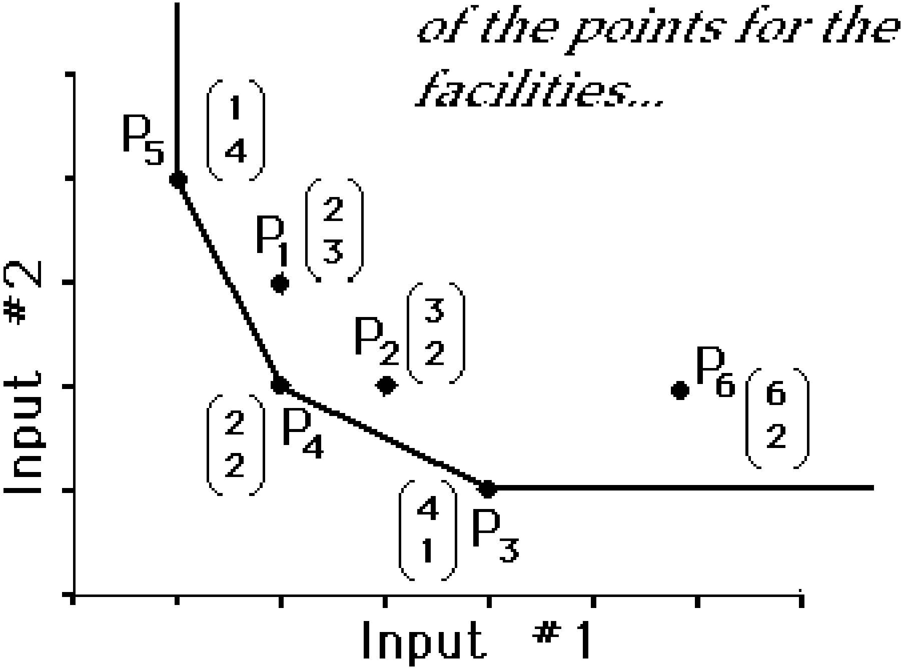
DMU	1	2	3	4	5	6
X_1	2	3	4	2	1	6
X_2	3	2	1	2	4	2

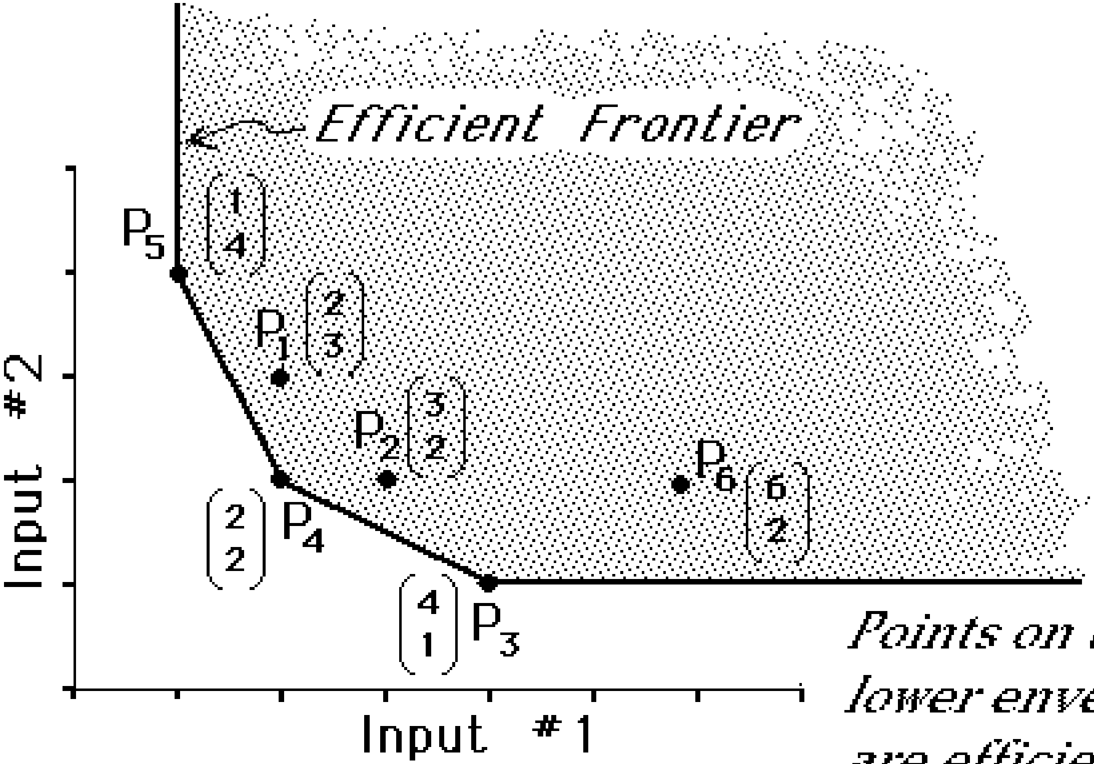
$Y_{1j} = 1$ for each j

The six facilities are plotted...

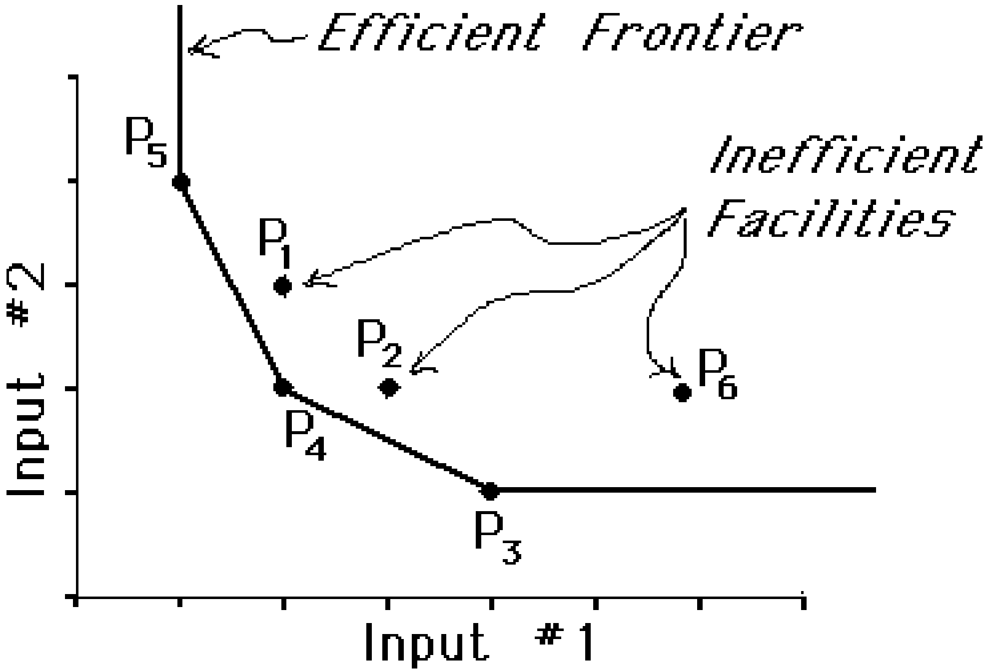


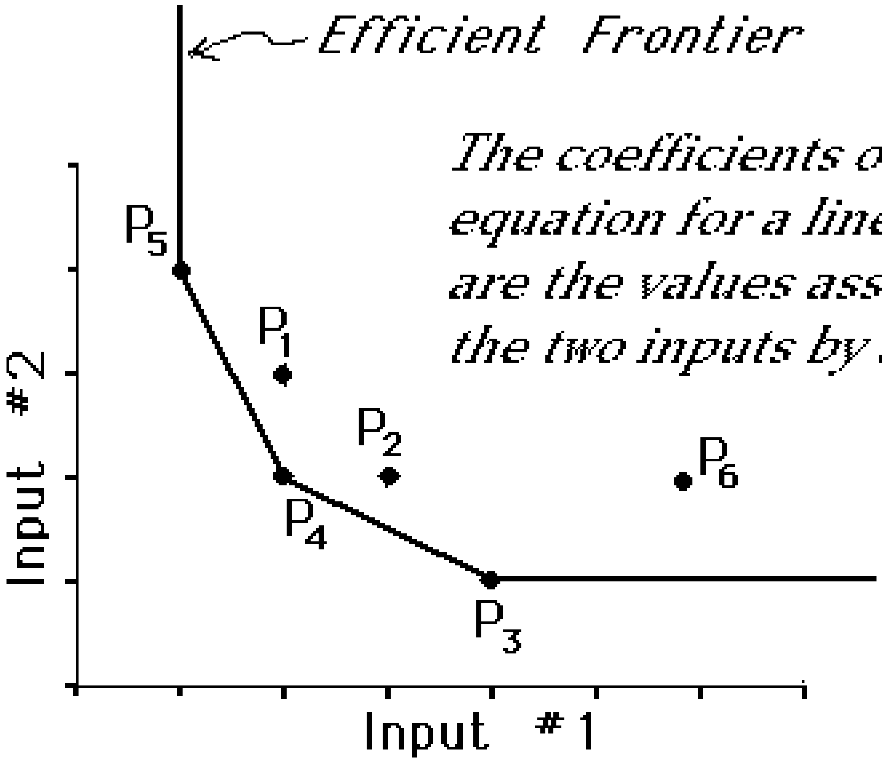
Find the lower envelope of the points for the facilities...





Points on this lower envelope are efficient

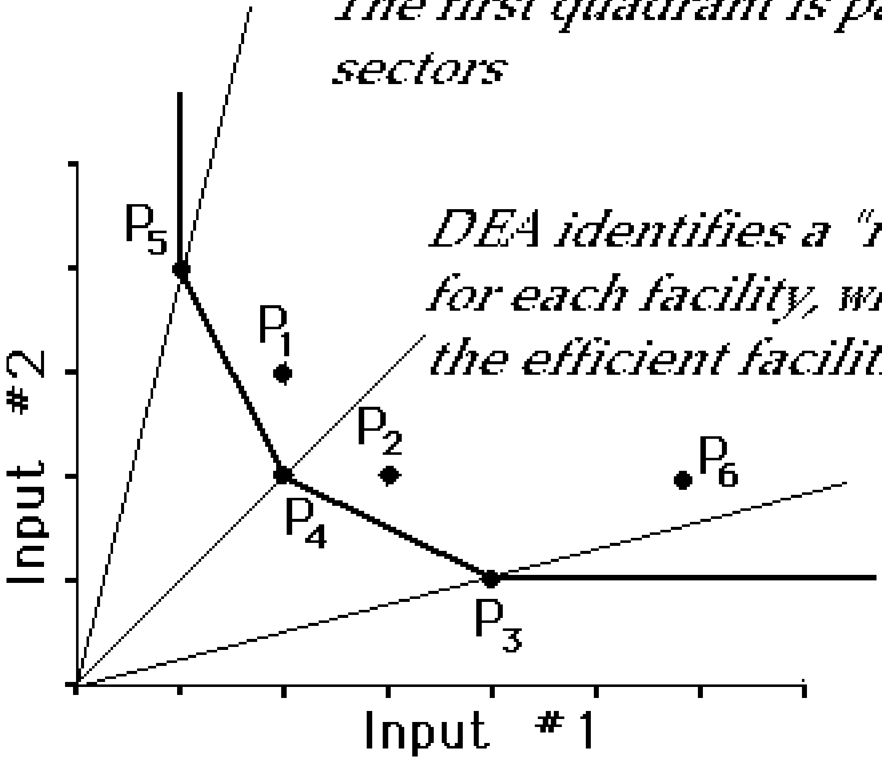

Efficient Frontier
Inefficient Facilities
P_5
P_1
P_2
P_6
P_4
P_3
Input #2
Input #1



The coefficients of the equation for a line segment are the values assigned to the two inputs by a facility

primal constraints

$$\sum_{r=1}^{s} v_r Y_{rj} - \sum_{i=1}^{m} u_i X_{ij} \le 0,\ j=1, \cdots n$$

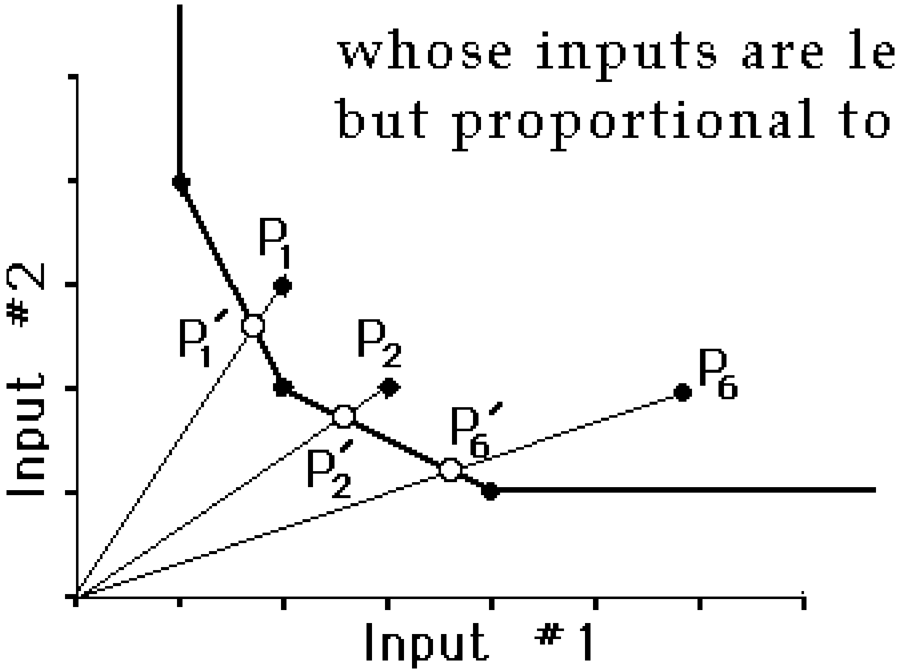


The first quadrant is partitioned into sectors

DEA identifies a "reference set" for each facility, which includes the efficient facilities in that sector

(Other facilities in the reference set are assigning the same values to inputs & outputs)

P_k' is a hypothetical hospital on the efficient frontier whose inputs are less than but proportional to those of P_k



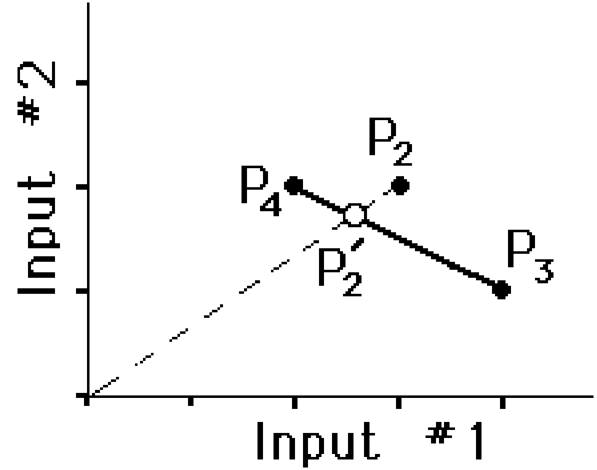
The "reference set" of facility P_2 is $\{P_3, P_4\}$

P_2 is a combination of P_3 and P_4:

$$\boxed{P_2 = \frac{5}{6} P_4 + \frac{1}{3} P_3}$$

i.e., $\lambda_4 = \frac{5}{6}$, $\lambda_3 = \frac{1}{3}$

The "hypothetical" efficient facility

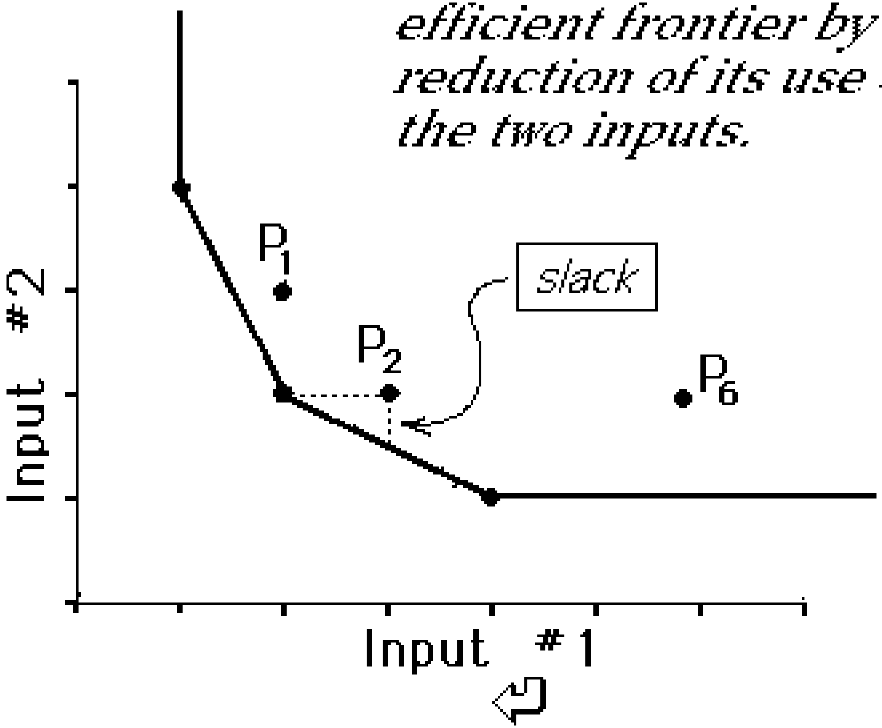


is

$$\boxed{P'_2 = \frac{6}{7} P_2 = \frac{5}{7} P_4 + \frac{2}{7} P_3}$$

To become efficient, P_2 should reduce each input by $\frac{1}{7}$.

Alternatively, P could move to the efficient frontier by an appropriate reduction of its use of either one of the two inputs.



EXAMPLE

Consider a group of 3 hospitals:
Each hospital is assumed to "convert" 2 inputs into 3 outputs:

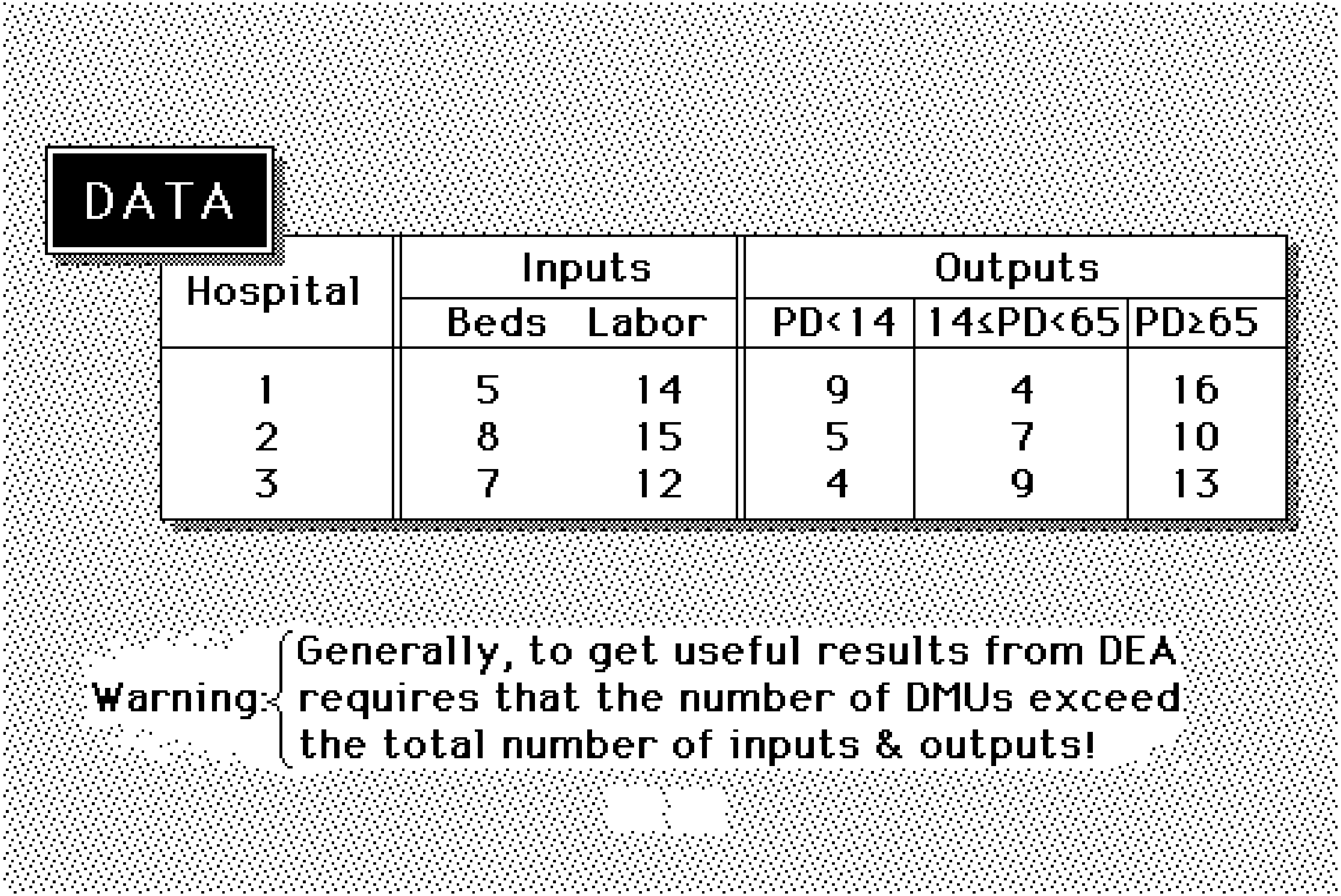
INPUTS

1. Capital (hospital beds)
2. Labor (10^3 hrs/month)

OUTPUTS (100s of days)

1. Patient days, age<14
2. Patient days, age between 14 & 65
3. Patient days, age≥65

↵



DATA

Hospital	Inputs		Outputs		
	Beds	Labor	PD<14	14≤PD<65	PD≥65
1	5	14	9	4	16
2	8	15	5	7	10
3	7	12	4	9	13

Warning: Generally, to get useful results from DEA requires that the number of DMUs exceed the total number of inputs & outputs!

The "efficiency" of Hospital #i is

$$E_i = \frac{\text{value of outputs of hospital \#i}}{\text{value of inputs of hospital \#i}}$$

To put Hospital #i in the "best light", we will assign values u & v to the inputs & outputs, respectively, in order to maximize E_i

For any choice of u_1, u_2, v_1, v_2, and v_3, the calculated efficiencies are:

$$E_1 = \frac{9\,v_1 + 4\,v_2 + 16\,v_3}{5\,u_1 + 14\,u_2}$$

$$E_2 = \frac{5\,v_1 + 7\,v_2 + 10\,v_3}{8\,u_1 + 15\,u_2}$$

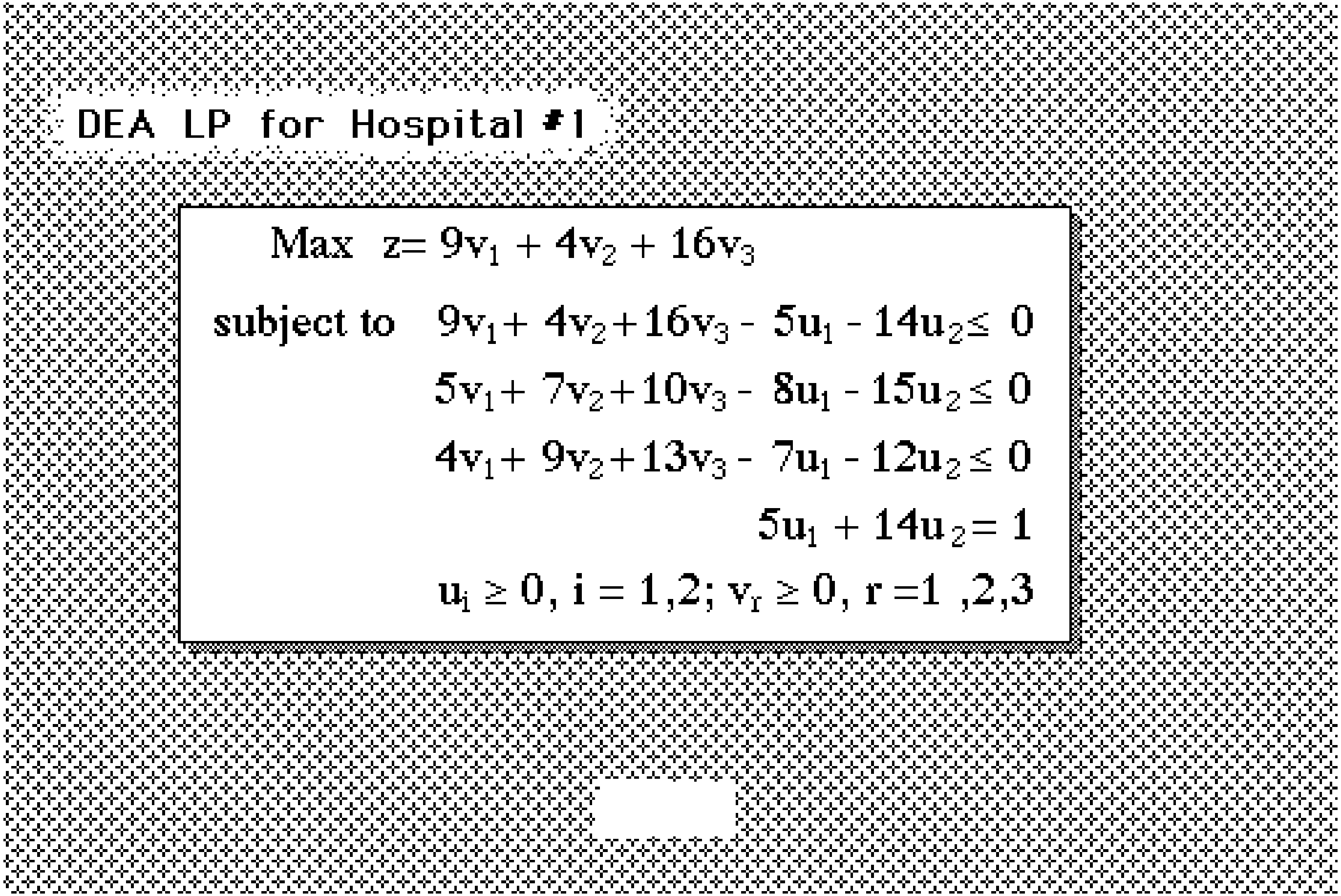
$$E_3 = \frac{4\,v_1 + 9\,v_2 + 13\,v_3}{7\,u_1 + 12\,u_2}$$

For Hospital #1, for example, we should choose u & v so as to...

$$\text{Maximize } \frac{9\ v_1 + 4\ v_2 + 16\ v_3}{5\ u_1 + 14\ u_2}$$

$$\begin{cases} \dfrac{9\ v_1 + 4\ v_2 + 16\ v_3}{5\ u_1 + 14\ u_2} \le 1 \\ \dfrac{5\ v_1 + 7\ v_2 + 10\ v_3}{8\ u_1 + 15\ u_2} \le 1 \\ \dfrac{4\ v_1 + 9\ v_2 + 13\ v_3}{7\ u_1 + 12\ u_2} \le 1 \end{cases}$$

$$u_1 \ge 0, u_2 \ge 0, v_1 \ge 0, v_2 \ge 0, v_3 \ge 0$$



DEA LP for Hospital #1

$$\text{Max } z = 9v_1 + 4v_2 + 16v_3$$

subject to

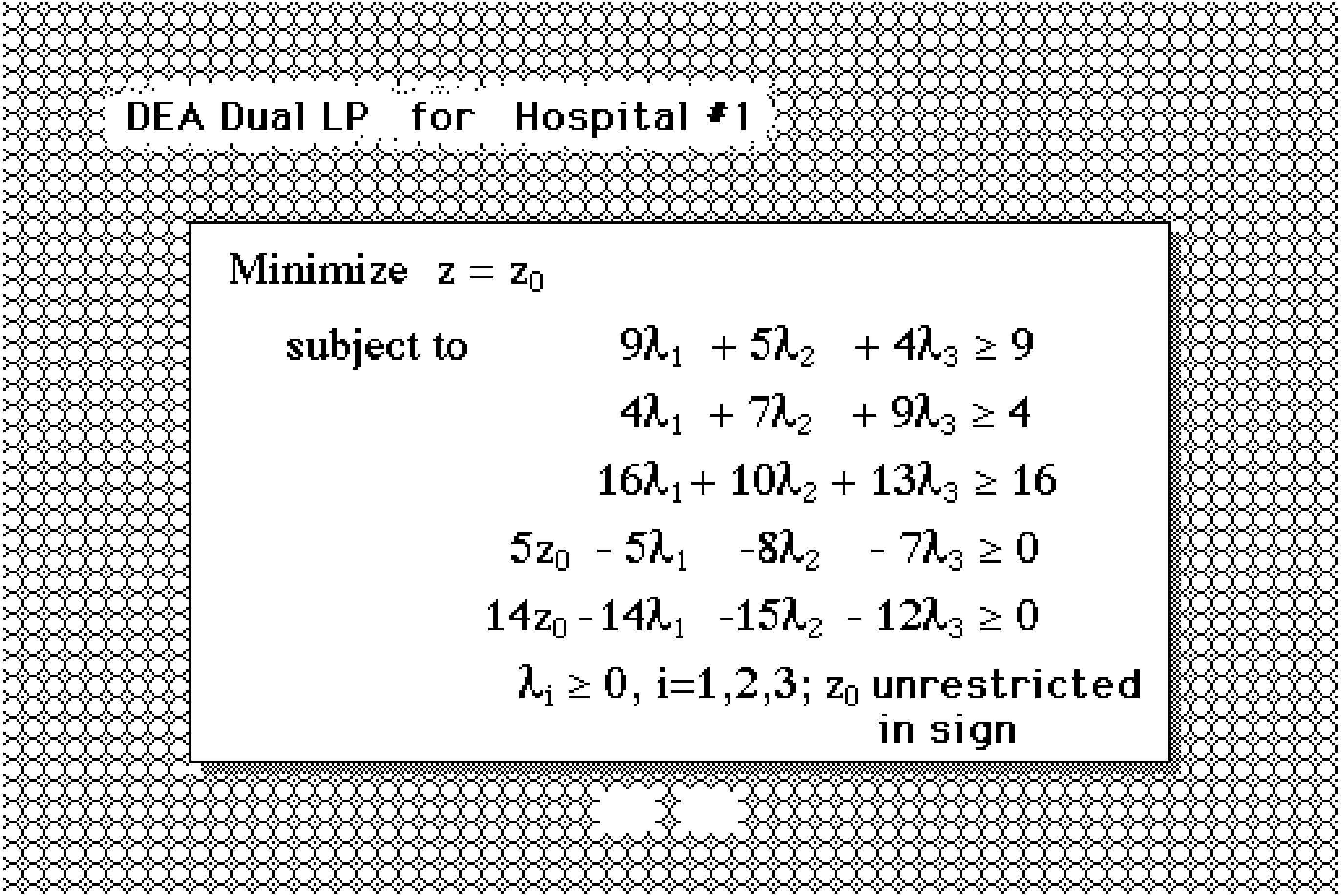
$$9v_1 + 4v_2 + 16v_3 - 5u_1 - 14u_2 \le 0$$

$$5v_1 + 7v_2 + 10v_3 - 8u_1 - 15u_2 \le 0$$

$$4v_1 + 9v_2 + 13v_3 - 7u_1 - 12u_2 \le 0$$

$$5u_1 + 14u_2 = 1$$

$$u_i \ge 0,\ i = 1,2;\ v_r \ge 0,\ r = 1,2,3$$



DEA Dual LP for Hospital #1

Minimize $z = z_0$

subject to

$$9\lambda_1 + 5\lambda_2 + 4\lambda_3 \geq 9$$

$$4\lambda_1 + 7\lambda_2 + 9\lambda_3 \geq 4$$

$$16\lambda_1 + 10\lambda_2 + 13\lambda_3 \geq 16$$

$$5z_0 - 5\lambda_1 - 8\lambda_2 - 7\lambda_3 \geq 0$$

$$14z_0 - 14\lambda_1 - 15\lambda_2 - 12\lambda_3 \geq 0$$

$\lambda_i \geq 0$, $i=1,2,3$; z_0 unrestricted in sign

Tableau for Hospital #1 Efficiency Computation

-z	surplus & slack var's					z_0	λ_1	λ_2	λ_3	
1	0	0	0	0	0	1	0	0	0	0
0	¯1	0	0	0	0	0	9	5	4	9
0	0	¯1	0	0	0	0	4	7	9	4
0	0	0	¯1	0	0	0	16	10	13	16
0	0	0	0	1	0	¯5	5	8	7	0
0	0	0	0	0	1	¯14	14	15	12	0

Efficiency of Facility 1

Prices

Efficiency = 1

Inputs		
i	var	u
1	Capital	0
2	Labor	0.0714286

Outputs		
i	var	v
1	P/Days Age<14	0.0857143
2	14≤P/Days Age<65	0.0571429
3	65≤P/Days Age	0

That is, to make Hospital #1 "look good", we place ***no*** value on "capital" and a value on "labor". Similarly, we place high values on inputs 1&2, and none on #3.

Computing the efficiencies based upon these values of inputs & outputs, we find:

$$E_1 = \frac{1.00}{1.00} = 1.00$$

$$E_2 = \frac{0.828572}{1.07143} = 0.773333$$

$$E_3 = \frac{0.857143}{0.857143} = 1.00$$

That is, if we apply the values selected by facility 1, we would find that #2 is inefficient while #1 and #3 are efficient.

DEA LP for Hospital #2

$$\text{Max} \quad z = 5v_1 + 7v_2 + 10v_3$$

$$\begin{aligned} \text{subject to} \quad & 9v_1 + 4v_2 + 16v_3 - 5u_1 - 14u_2 \leq 0 \\ & 5v_1 + 7v_2 + 10v_3 - 8u_1 - 15u_2 \leq 0 \\ & 4v_1 + 9v_2 + 13v_3 - 7u_1 - 12u_2 \leq 0 \\ & 8u_1 + 15u_2 = 1 \\ & u_i \geq 0,\ i = 1,2;\ v_r \geq 0,\ r = 1,2,3 \end{aligned}$$

DEA Dual LP for Hospital #2

Minimize $z = z_0$

subject to

$$\begin{aligned}
9\lambda_1 + 5\lambda_2 + 4\lambda_3 &\geq 5 \\
4\lambda_1 + 7\lambda_2 + 9\lambda_3 &\geq 7 \\
16\lambda_1 + 10\lambda_2 + 13\lambda_3 &\geq 10 \\
8z_0 - 5\lambda_1 - 8\lambda_2 - 7\lambda_3 &\geq 0 \\
15z_0 - 14\lambda_1 - 15\lambda_2 - 12\lambda_3 &\geq 0
\end{aligned}$$

$\lambda_i \geq 0$, $i=1,2,3$; z_0 unrestricted in sign

Tableau for Hospital #2 Efficiency Computation

$-z$	surplus & slack var's					z_0	λ_1	λ_2	λ_3	
1	0	0	0	0	0	1	0	0	0	0
0	¯1	0	0	0	0	0	9	5	4	5
0	0	¯1	0	0	0	0	4	7	9	7
0	0	0	¯1	0	0	0	16	10	13	10
0	0	0	0	1	0	¯8	5	8	7	0
0	0	0	0	0	1	¯15	14	15	12	0

Efficiency of Facility 2

Efficiency = 0.773333

Reference set: 1 3

Facility expressed in terms of reference set:

Combination		
i	wt	wt*
1	0.261538	0.283333
3	0.661538	0.716667

(wt* are weights convexified by dividing by sum of weights, 0.923077)

Interpretation of DEA Dual LP

We wish to form a linear combination of the hospitals in the group, i.e., a "hypothetical" hospital, whose outputs are at least as great as the target hospital,

$$\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{rk} \quad , r=1,\cdots s$$

and whose inputs are no more than the efficiency z_0 times the target hospital's inputs.

$$\sum_{j=1}^{n} X_{ij}\lambda_j \leq X_{ik}\, z_0 \quad , i=1,\cdots m$$

$$\overset{\text{efficiency of Hospital \#2}}{0.7733}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}6.186\\11.6\end{bmatrix}\geq \underset{\lambda_1}{0.261538}\overset{\text{inputs of Hosp.\#1}}{\begin{bmatrix}5\\14\end{bmatrix}}+\underset{\lambda_3}{0.661538}\overset{\text{inputs of Hosp.\#3}}{\begin{bmatrix}7\\12\end{bmatrix}}=\begin{bmatrix}5.93846\\11.6\end{bmatrix}$$

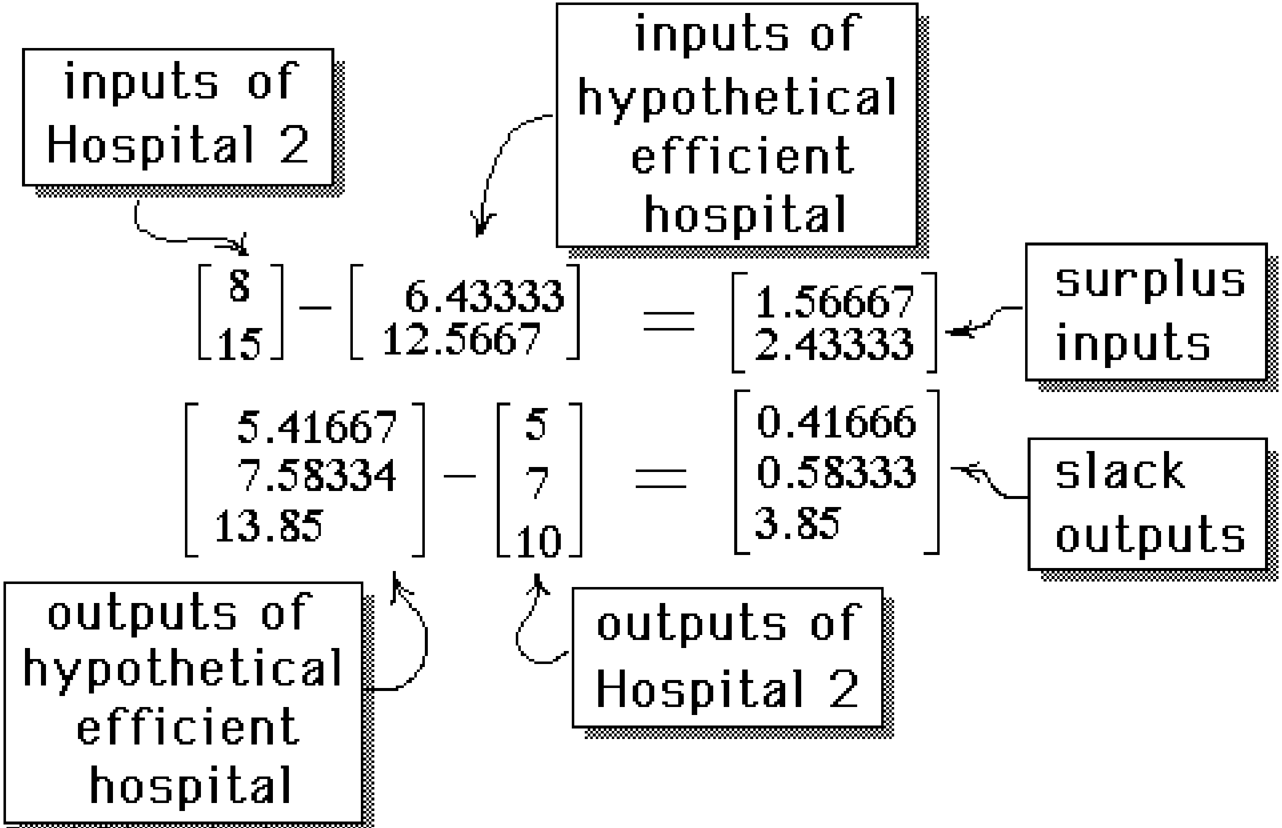
$$\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}5\\7\\10\end{bmatrix}\leq \underset{\lambda_1}{0.261538}\underset{\text{outputs of Hosp. \#1}}{\begin{bmatrix}9\\4\\16\end{bmatrix}}+\underset{\lambda_3}{0.661538}\underset{\text{outputs of Hosp. \#3}}{\begin{bmatrix}4\\9\\13\end{bmatrix}}=\begin{bmatrix}5\\7\\12.7846\end{bmatrix}$$

Hypothetical Efficient Hospital

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}8\\15\end{bmatrix}\geq 0.283333\overset{\text{inputs of Hosp.\#1}}{\begin{bmatrix}5\\14\end{bmatrix}}+0.716667\overset{\text{inputs of Hosp.\#3}}{\begin{bmatrix}7\\12\end{bmatrix}}=\begin{bmatrix}6.43333\\12.5667\end{bmatrix}$$

$$\frac{\lambda_1}{\lambda_1+\lambda_3} \qquad\qquad \frac{\lambda_3}{\lambda_1+\lambda_3}$$

$$\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}5\\7\\10\end{bmatrix}\leq 0.283333\underset{\text{outputs of Hosp.\#1}}{\begin{bmatrix}9\\4\\16\end{bmatrix}}+0.716667\underset{\text{outputs of Hosp. \#3}}{\begin{bmatrix}4\\9\\13\end{bmatrix}}=\begin{bmatrix}5.41667\\7.58334\\13.85\end{bmatrix}$$



inputs of Hospital 2

inputs of hypothetical efficient hospital

$$\begin{bmatrix}8\\15\end{bmatrix} - \begin{bmatrix}6.43333\\12.5667\end{bmatrix} = \begin{bmatrix}1.56667\\2.43333\end{bmatrix}$$

surplus inputs

$$\begin{bmatrix}5.41667\\7.58334\\13.85\end{bmatrix} - \begin{bmatrix}5\\7\\10\end{bmatrix} = \begin{bmatrix}0.41666\\0.58333\\3.85\end{bmatrix}$$

slack outputs

outputs of hypothetical efficient hospital

outputs of Hospital 2

Slack inputs/outputs

Inputs	Hospital 2
Capital	1.566666667
Labor	2.433333333

Outputs	Hospital 2
PD-Age>14	0.4166666667
PD-14≤Age<65	0.5833333333
PD-65≤Age	3.85

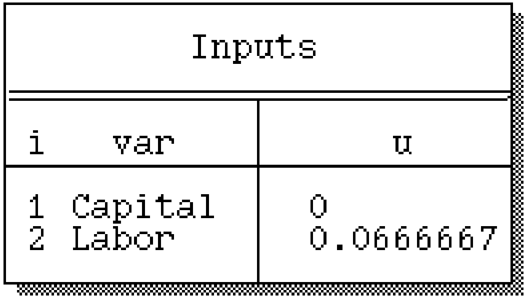
To become efficient, Hospital #2 should

reduce its inputs by the surplus amounts

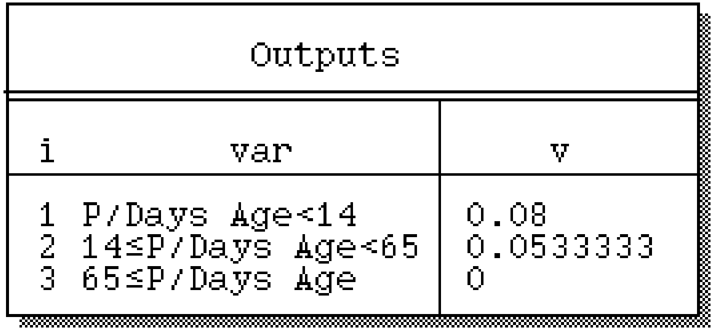
and

increase its outputs by the slack amounts

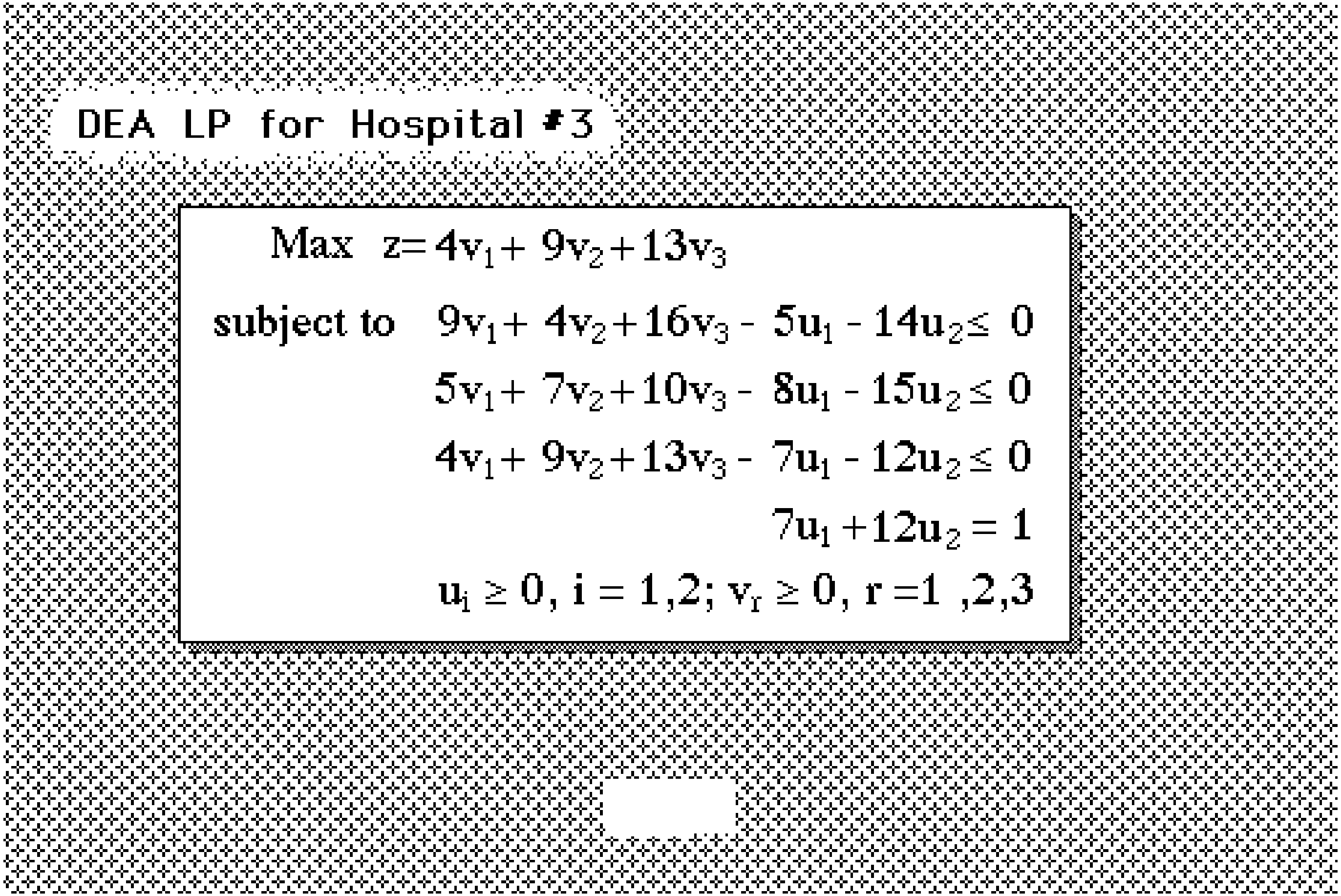
Prices



Inputs	
i var	u
1 Capital	0
2 Labor	0.0666667



Outputs	
i var	v
1 P/Days Age<14	0.08
2 14≤P/Days Age<65	0.0533333
3 65≤P/Days Age	0



DEA LP for Hospital #3

$$\text{Max } z = 4v_1 + 9v_2 + 13v_3$$

subject to

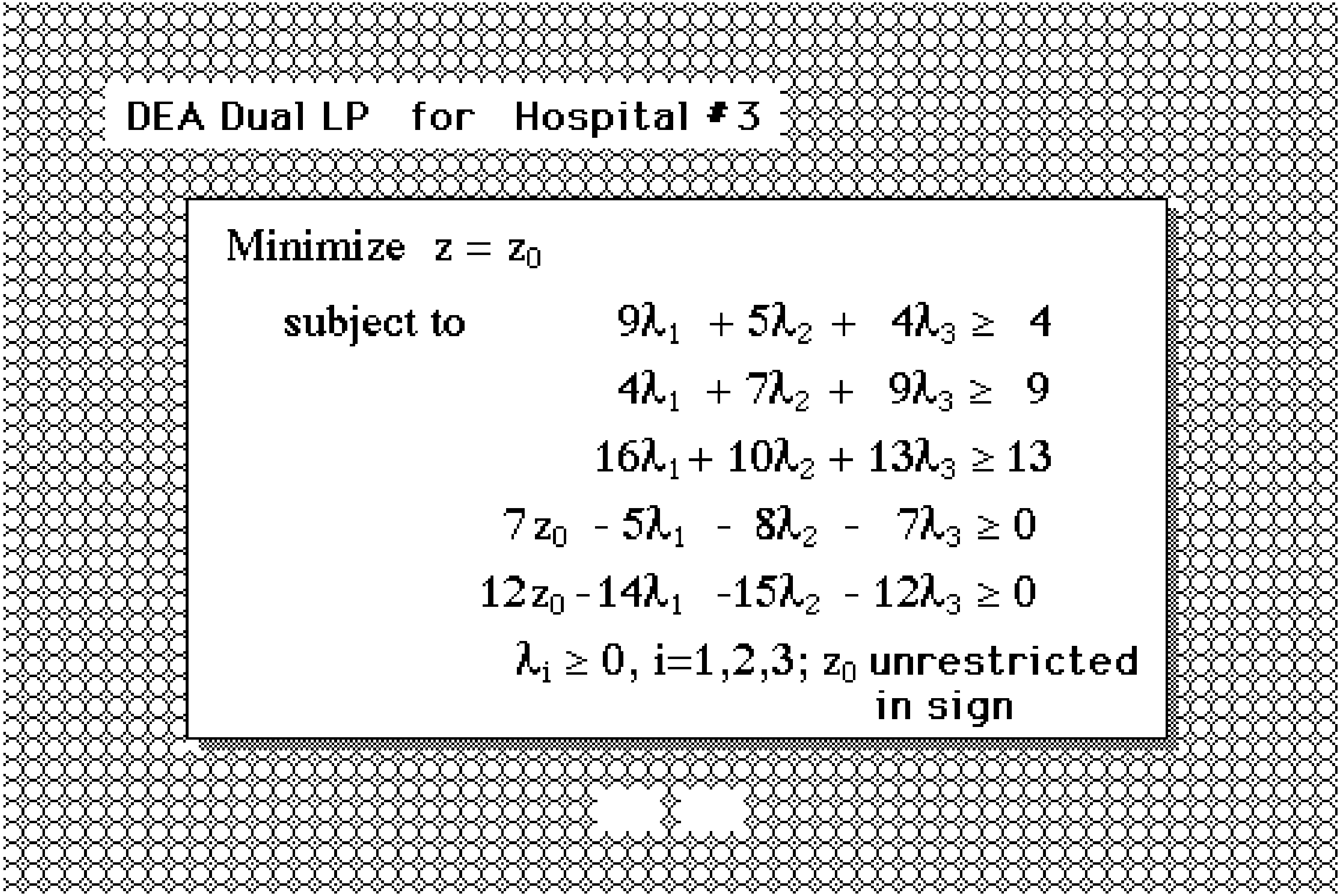
$$9v_1 + 4v_2 + 16v_3 - 5u_1 - 14u_2 \leq 0$$

$$5v_1 + 7v_2 + 10v_3 - 8u_1 - 15u_2 \leq 0$$

$$4v_1 + 9v_2 + 13v_3 - 7u_1 - 12u_2 \leq 0$$

$$7u_1 + 12u_2 = 1$$

$$u_i \geq 0,\ i = 1,2;\ v_r \geq 0,\ r = 1,2,3$$



DEA Dual LP for Hospital #3

$$\text{Minimize } z = z_0$$

$$\begin{aligned} \text{subject to} \quad & 9\lambda_1 + 5\lambda_2 + 4\lambda_3 \geq 4 \\ & 4\lambda_1 + 7\lambda_2 + 9\lambda_3 \geq 9 \\ & 16\lambda_1 + 10\lambda_2 + 13\lambda_3 \geq 13 \\ & 7z_0 - 5\lambda_1 - 8\lambda_2 - 7\lambda_3 \geq 0 \\ & 12z_0 - 14\lambda_1 - 15\lambda_2 - 12\lambda_3 \geq 0 \end{aligned}$$

$\lambda_i \geq 0$, $i=1,2,3$; z_0 unrestricted in sign

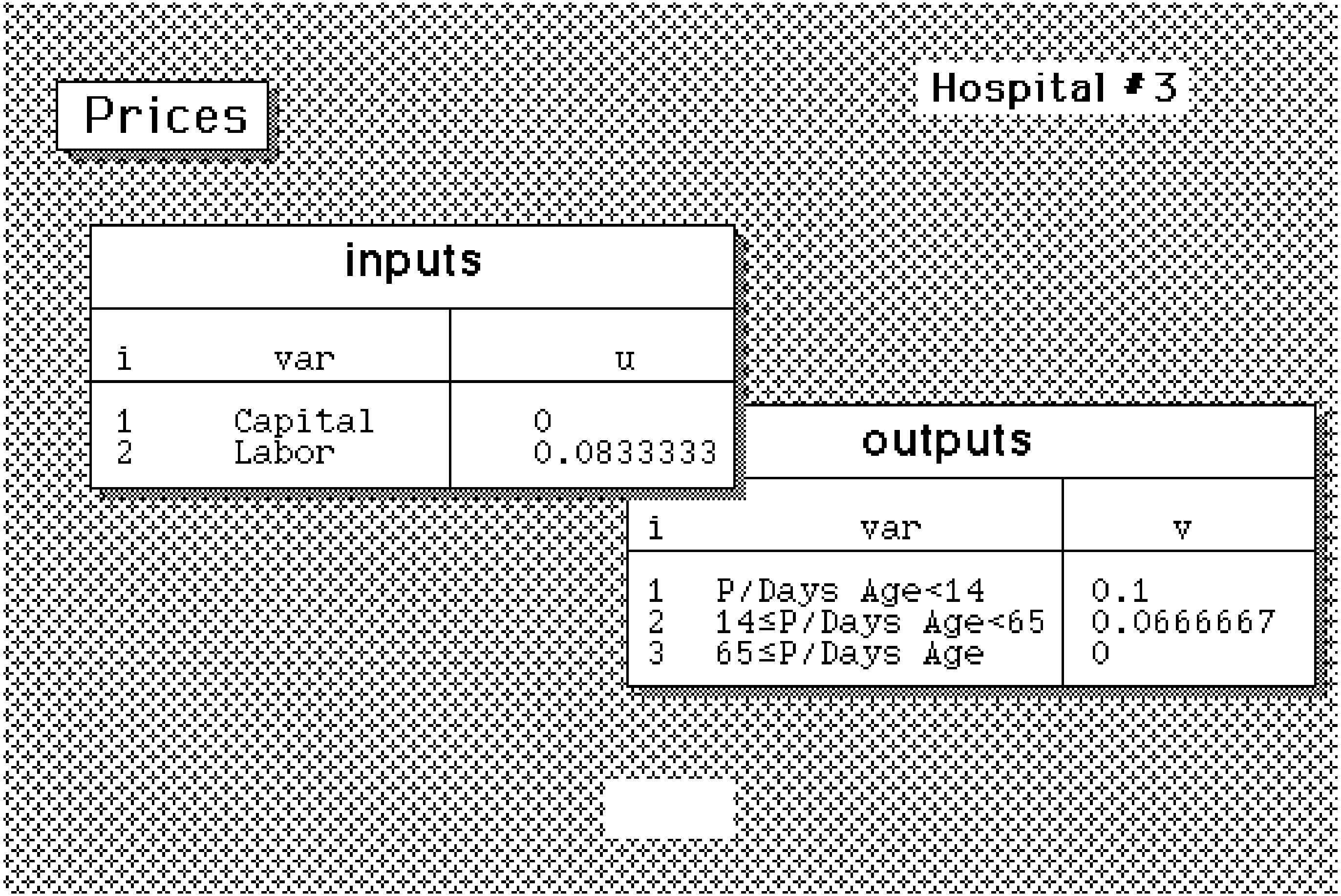
Tableau for Hospital #3 Efficiency Computation

$-z$	surplus & slack var's					z_0	λ_1	λ_2	λ_3	
1	0	0	0	0	0	1	0	0	0	0
0	¯1	0	0	0	0	0	9	5	4	4
0	0	¯1	0	0	0	0	4	7	9	9
0	0	0	¯1	0	0	0	16	10	13	13
0	0	0	0	1	0	¯7	5	8	7	0
0	0	0	0	0	1	¯12	14	15	12	0

Efficiency of Facility 3

Efficiency = 1

Slack inputs/outputs:	
Capital	¯1.33227E¯15



Prices

Hospital #3

inputs

i	var	u
1	Capital	0
2	Labor	0.0833333

outputs

i	var	v
1	P/Days Age<14	0.1
2	14≤P/Days Age<65	0.0666667
3	65≤P/Days Age	0

Efficient Facilities

Efficient facilites are: 1 3

Prices Assigned

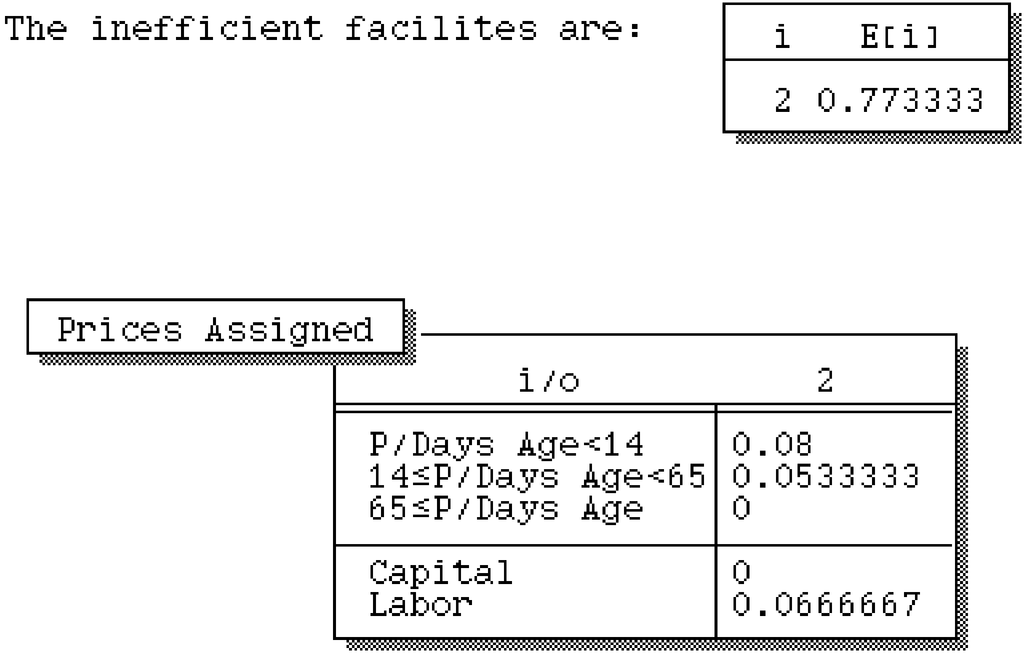
i/o	1	3
P/Days Age<14	0.0857143	0.1
14≤P/Days Age<65	0.0571429	0.0666667
65≤P/Days Age	0	0
Capital	0	0
Labor	0.0714286	0.0833333

efficient facilities

Slack inputs/outputs

i/o	1	3
P/Days Age<14	0E0	0E0
14≤P/Days Age<65	0E0	0E0
65≤P/Days Age	¯1.77636E¯15	0E0
Capital	¯2.22045E¯16	¯1.33227E¯15
Labor	0E0	0E0

≈ *zero!*



The inefficient facilites are:

i	E[i]
2	0.773333

Prices Assigned

i/o	2
P/Days Age<14	0.08
14≤P/Days Age<65	0.0533333
65≤P/Days Age	0
Capital	0
Labor	0.0666667

inefficient facility

Slack inputs/outputs

i/o	2
P/Days Age<14	0.416667
14≤P/Days Age<65	0.583333
65≤P/Days Age	3.85
Capital	1.56667
Labor	2.43333

EXAMPLE

H.D. Sherman, "Hospital Efficiency Measurement & Evaluation", Medical Care, Vol. 22 No. 10 (October '84) 922-938 (data modified)

The medical-surgical (MS) units of seven hospitals are to be evaluated using DEA. Three inputs and two outputs were identified:

INPUTS	OUTPUTS
FTEs (employees)	Patient days
Supply $s	Nurses & Interns trained
Bed-Days	

INPUTS

Hospital	FTE staff	Supply $s ($10^3$)	Bed-Days available (10^3)
A	310	134	116
B	278.5	114.3	106.8
C	165.6	131.3	65.5
D	250	316	94.4
E	206.4	151.2	102.1
F	384.6	217	153.7
G	530.4	770.8	215

OUTPUTS

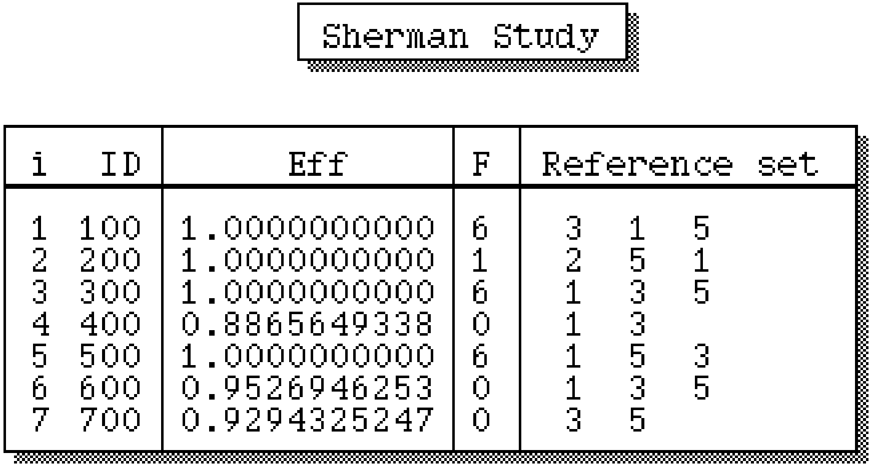
Hospital	Patient-Days (10^3)	#Student Nurses & Interns
A	104.83	338
B	93.27	159
C	58.68	167
D	75.52	181
E	87.78	239
F	130.7	377
G	178.11	233

Dual LP Tableau for Hospital #1 Efficiency

-Z	surplus & slack var's					Z_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	RHS
1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	¯1	0	0	0	0	0	104.8	93.2	58.6	75.5	87.7	130.7	178.1	104.8
0	0	¯1	0	0	0	0	338	159	167	181	239	377	233	338
0	0	0	1	0	0	¯310	310	278.5	165.6	250	206.4	384.6	530.4	0
0	0	0	0	1	0	¯134.6	134.6	114.3	131.3	316	151.2	217	770.8	0
0	0	0	0	0	1	¯116	116	106.8	65.52	94.4	102.1	153.7	215	0

Dual LP Tableau for Hospital #2 Efficiency

-Z	surplus & slack var's					z_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	RHS
1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	¯1	0	0	0	0	0	104.8	93.2	58.6	75.5	87.7	130.7	178.1	93.2
0	0	¯1	0	0	0	0	338	159	167	181	239	377	233	159
0	0	0	1	0	0	¯278.5	310	278.5	165.6	250	206.4	384.6	530.4	0
0	0	0	0	1	0	¯114.3	134.6	114.3	131.3	316	151.2	217	770.8	0
0	0	0	0	0	1	¯106.8	116	106.8	65.5	94.4	102.1	153.7	215	0



Sherman Study

i	ID	Eff	F	Reference set		
1	100	1.0000000000	6	3	1	5
2	200	1.0000000000	1	2	5	1
3	300	1.0000000000	6	1	3	5
4	400	0.8865649338	0	1	3	
5	500	1.0000000000	6	1	5	3
6	600	0.9526946253	0	1	3	5
7	700	0.9294325247	0	3	5	

Prices For Efficient Hospitals

inputs & outputs	**Hospital #**			
	1	2	3	5
Days	0.00932552	0.0105921	0.0170416	0.0111758
Staff	0.00006629	0.00007592	0	0.00007944
FTE	0.00068822	0.00231413	0.00116938	0.00082478
Supply $	0	0.00311036	0.00003723	0
Bed-days	0.00678146	0	0.0122323	0.00812698

Prices for the Inefficient Hospitals

i	E[i]
4	0.886565
6	0.952695
7	0.929433

inputs & outputs	Hospital #		
	4	6	7
Days	0.0117395	0.00728917	0.00521831
Staff	0	0	0
FTE	0.00065626	0.00050017	0.00036986
Supply $	0	0.00001592	0
Bed-days	0.00885524	0.00523211	0.00373872

Inefficient Hospitals

Slack inputs/outputs

inputs & outputs	Hospital #		
	4	6	7
Days	17.2633	0	0
Staff	112.363	0	0
FTE	0	141.969	361.529
Supply $	182.261	80.102	637.905
Bed-days	0	56.7358	146.547

"Technical Efficiency in Nursing Homes"

by John Nyman, Dennis Bricker, and David Link

Medical Care, June 1990

- 319 intermediate care facilities were included in the study
- eleven input variables were included, all of which were various categories of labor
- one output variable was included, namely number of persons residing in the facility on an arbitrary date

The "efficiency" was computed for each of the
319 facilities.

We then performed regression analysis with the efficiency as the dependent variable, and several dependent variables in order to determine which of these have an effect on efficiency, e.g.,

- for-profit vs non-profit ownership
- case-mix
- size of facility
- location of facility (rural vs urban)

etc.

RESULTS

- non-profit nursing homes used about 6% more inputs than for-profit homes.
- nursing homes with greater number of beds were more efficient up to a point (about 170 beds), after which diseconomies-of-scale became apparent.

etc.....

"Impact of Complexity of Care, Location, and Mission on the Technical Efficiency of Hospital Production",

Dennis Bricker, Thomas Gruca, and Deepika Nath
Working paper, December 1993

- 208 Illinois hospitals were used in the study.
- As in the previous paper by Nyman, Bricker, & Link, DEA was used to evaluate the efficiency of each hospital.
- Then regression analysis was performed to test the effect of other determinants on efficiency

DEA input variables

- nursing, medical, and non-medical staff
- drugs and supplies

DEA output variables

- general inpatient care (patient-days)
- intensive care
- pediatric care
- obstetric care
- surgical procedures
- outpatient & emergency care

Regression Analysis was performed with three sets of independent variables

- Complexity of Care
  - average length of stay
  - case-mix index
- Location
  - density of population in area served
- Hospital mission
  - public vs private ownership
  - teaching vs non-teaching hospital
  - system/alliance affiliation
  - religious affiliation

RESULTS of regression analysis

Complexity of care explains the greatest amount of variance in efficiency

Mission explains the least amount of variance in efficiency

Location is in the middle!

Rural hospitals are more efficient than those in large urban areas, and somewhat more than those in smaller urban areas.

"Comparing University Departments",

by J. E. Beasley, *OMEGA*, 1990.

Evaluated efficiencies of Chemistry and Physics departments in 52 universities in UK

Could be useful in identifying sources of inefficiency and suggesting strategies for improvement

Could be useful in allocating public funding to more efficient departments

- DEA input variables
  - general expenditure (majority for salaries)
  - equipment expenditure
  - research income
- DEA output variables
  - # of undergraduates
  - # of graduate students doing coursework
  - # of graduate students doing research
  - publications & citations
  - quality of research (University Grants Committee [UGC] research ranking)

Further References

WWW sites:

A Data Envelopment Analysis (DEA) Home Page
http://www.emp.pdx.edu/dea/homedea.html

Ali Emrouznejad's DEA HomePage
http://www.csv.warwick.ac.uk/~bsr1u

↩

Strengths of DEA

- can handle multiple inputs & multiple outputs
- doesn't require assumption of functional form relating inputs to outputs
- DMUs are directly compared against a peer or a combination of peers
- inputs & outputs can have very different units. E.g., X can be in units of lives saved, and X in units of dollars, without requiring an *a priori* tradeoff between the two

Weaknesses of DEA

- Since DEA is an extreme point technique, noise (even if symmetric with mean zero) such as measurement error can cause significant problems
- DEA is good at estimating "relative" efficiency of a DMU, but not its "absolute" efficiency
- Since DEA is nonparametric, statistical hypothesis tests are difficult
- Since an LP must be solved for each DMU, large problems can be computationally intensive.