# The $64 Question

Consider the problem faced by a contestant on a TV quiz show, in which there are six stages (1,...7): the question at stage i is worth $\$2^i$.

At any stage i, the contestant may choose

- to quit and receive her accumulated winnings, or
- to continue, in which case she is presented with a question

If she correctly answers the question, she may advance to the next stage, otherwise she must quit with no payoff, i.e., she loses everything.

The questions become progressively more difficult at each stage, of course, and she estimates that the probability that she can answer the question at stage i to be $P_i$ where $P_{i+1} < P_i$.

If she correctly answers the seventh question, she receives her total winnings ($1+2+4+8+16+32+64= $127).

States: (0) "Stopped" & (1) "Active"

Decisions: (0) "Stop" & (1) "Continue"

Optimal value function:

$f_n(s_n)$ = maximum expected value of additional earnings if, at stage n, the contestant is in state $s_n$

Recursive definition of optimal value function:

$$f_n(1) = \begin{cases} \sum_{i=1}^{n-1} R_i & \text{if } x=0 \text{ "quit"} \\ p_n f_{n+1}(1) & \text{if } x=1 \text{ "continue"} \end{cases}$$

$$f_n(0) = 0$$

where

$$f_8(1) = 127$$

Data:

| *Question* | *1* | *2* | *3* | *4* | *5* | *6* | *7* |
|---|---|---|---|---|---|---|---|
| Earnings | 1 | 3 | 7 | 15 | 31 | 63 | 127 |
| P{correct} | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 |

## APL Recursive Function

```
     ∇ z←F N;t
[1]   ⍝
[2]   ⍝ Optimal Value Function for stopping problem
[3]   ⍝
[4]   :if N>NN
[5]     z←0,Earnings,-BIG
[6]   :else
[7]     ⍝ Recursive definition of optimal value function
[8]     z←((1-P[N]),P[N]) Maximize_E ((Earnings[N]×s∘.>x)∘.+0×d)
           +(F N+1)[TRANSITION s∘.×x∘.×d]
[9]    :endif
     ∇
```

Stage 7---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | **63.00** | 18.90 | 63.00 |

Stage 6---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | **31.00** | 25.20 | 31.00 |

Stage 5---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | 15.00 | **15.50** | 15.50 |

Stage 4---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | 7.00 | **9.30** | 9.30 |

Stage 3---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | 3.00 | **6.51** | 6.51 |

Stage 2---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | 1.00 | **5.21** | 5.21 |

Stage 1---

| s \ x: | 0 | 1 | Max |
|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 |
| 1 | 0.00 | **4.69** | 4.69 |

Summary:

The contestant should continue during stages 1, 2, ...6. If she reaches stage 6 without losing, she should quit, and take her current earnings, i.e., $1+2+4+8+16 = #31.